Name : ______________________ **Class Number**: ______

Math 1C: INA Lesson 2 Suggested Problems

Theoretic Problems: Discussed in-class

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1. **Geometric series test** Consider the infinite series $\sum_{k=1}^{\infty} a \cdot r^{k-1}$.

   A. Derive the geometric sum formula given by $S_n = \sum_{k=1}^{n} a \cdot r^{k-1} = a \cdot \dfrac{1-r^n}{1-r}$

   B. Use part A and the limit of a geometric sequence to derive the geometric series test.

Problems Solved in Jeff's Handwritten Notes

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2. Example 1: Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the explicit formula $a_n = n$. Consider the sequence of partial sums associated with $\{a_n\}_{n=1}^{\infty}$ given by

$$S_n = \sum_{k=1}^{n} a_k = 1 + 2 + 3 + \cdots + n$$

   A. Show that $S_n = \dfrac{n \cdot (n+1)}{2}$

   B. Find $\lim_{n\to\infty} S_n$

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3. Example 8.3 Quick Check 3 p. 621: Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the explicit formula $a_n = \dfrac{1}{2^n}$. Consider the sequence of partial sums associated with $\{a_n\}_{n=1}^{\infty}$ given by

$$S_n = \sum_{k=1}^{n} a_k = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

   A. Find the first six terms of the sequence of partial sums.

   B. Show that $S_n = 1 - \dfrac{1}{2^n}$.

   C. Find $\lim_{n\to\infty} S_n$.

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4. Example 8.3.1a p. 621: Determine if the series

$$\sum_{n=1}^{\infty} 2^{2n} \cdot 3^{1-n}$$

   converges or diverges.

5. Example 8.3.1c p. 621: Evaluate the following series or state that the series diverges:

$$\sum_{k=2}^{\infty} 3 \cdot (-0.75)^k$$

6. Example 8.3.2 p. 622: Write the repeating type II decimal number

$$y = 2.3\overline{17} = 2.317171717...$$

as a ratio of integers. Please make sure to use the geometric series test in your solution.

7. Example 8.3.3b p. 622: Show that the telescoping series

$$\sum_{k=1}^{\infty} \frac{1}{k \cdot (k+1)}$$

converges and find the sum.

8. Exercise 8.3.67 p. 624: Find the limit of the infinite (telescoping) series

$$\sum_{k=1}^{\infty} \frac{1}{16k^2 + 8k - 3}$$

converges and find the sum.

Suggested Problems

9. Example 8.3.52 p. 623: Write the repeating decimal $1.00\overline{39} = 1.0039393939...$ as a geometric series. Then, use the geometric series test to write this repeating decimal as a fraction (a ratio of two integers).

10. Example 8.3.58 p. 623: Find the limit of the series: $\sum_{k=3}^{\infty} \frac{1}{9k^2 + 15k + 4}$

11. Example 8.3.33 p. 623: Find the limit of the series: $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k \cdot 5^{3-k}$

Optional Challenge Problems

12. Exercise 8.3.83 p. 625
13. Exercise 8.3.87 p. 625
14. Exercise 8.3.88 p. 625