Name : ______________________ **Class Number:** _____

Math 1C: INA Lesson 1 Suggested Problems

Theoretic Problems: Discussed in-class

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1. **Define the two major problems we work on in sequences and series**

   A. Please describe the two types of problems we will study in the "Introduction to Numerical Analysis" (INA) part of this class.

   B. Please recall the definition of a sequence. Then, explain how are sequences are related to the two problems you discussed above.

   C. Please recall the three different representations of a sequence that we discussed in class.

Problems Solved in Jeff's Handwritten Notes

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2. Example 8.1.2 p. 598: Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the recurrence relation

$$a_1 = 1, \qquad a_{n+1} = 2 \cdot a_n + 1$$

   A. Represent this sequence as an ordered set by finding the first six terms of this sequence.

   B. Find an explicit formula for a_n in terms of the index variable n.

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3. Example 8.1.3a p. 598: Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the ordered set

$$\{-2, 5, 12, 19, ...\}$$

   A. Find the next two terms of this sequence.

   B. Represent this sequence as a one-term recurrence relation.

   C. Find an explicit formula for a_n in terms of the index variable n.

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4. Example 8.1.3b p. 598: Suppose we have a sequences $\{b_n\}_{n=1}^{\infty}$ defined by the explicit formula

$$b_n = 3 \cdot 2^{n-1}$$

   A. Find the first six terms of this sequence and represent this sequence as an ordered set.

   B. Represent this sequence as a one-term recurrence relation.

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5. Example 8.2.1a p. 608: Define the sequence $\{a_n\}_{n=1}^{\infty}$ using the explicit formula

$$a_n = \frac{3\,n^3}{n^3 + 1}$$

   Find $\lim_{n\to\infty} a_n$ using theorem 8.1 on page 607

6. Example 8.1.5 p. 599: Define the sequence $\{a_n\}_{n=1}^{\infty}$ using the explicit formula

$$a_n = \frac{4\,n^3}{n^3 - 1}$$

Find $\lim\limits_{n\to\infty} a_n$ using theorem 8.2 on page 607.

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7. Example p. 609: Define the sequence $\{a_n\}_{n=1}^{\infty}$ using the explicit formula

$$a_n = 1 - \frac{1}{n}$$

   A. Show that this sequence is increasing.
   B. Show that this sequence is bounded.
   C. Show that this sequence is nondecreasing.
   D. Show that this sequence is monotonic.

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8. Example 8.2.3 p. 610: Define the sequence $\{a_n\}_{n=1}^{\infty}$ using the explicit formula

$$a_n = \alpha \cdot r^{n-1}$$

where $r \in \mathbb{R}$. For each case below, et $\alpha = r$. Then, plot the sequence and find the $\lim\limits_{n\to\infty} a_n$:

   A. Set $r = 0.75$
   B. Set $r = -0.75$
   C. Set $r = 1.15$
   D. Set $r = -1.15$

Suggested Problems

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9. Define the sequence $\{a_n\}_{n=1}^{\infty}$ using the explicit formula

$$a_n = 10 + \frac{1}{n}$$

   A. Show that this sequence is decreasing.
   B. Show that this sequence is bounded.
   C. Show that this sequence is nonincreasing.
   D. Show that this sequence is monotonic.

10. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ defined by the recurrence relation $a_1 = 3, a_{n+1} = 0.5\, a_n$.

   i. Find the first 5 terms of this sequence.
   ii. Find an explicit formula for a_n in terms of n.
   iii. Find the $\lim_{n \to \infty} a_n$

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11. Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the ordered set

$$\left\{5, \frac{5}{2}, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \ldots\right\}$$

   A. Find the next two terms of this sequence.
   B. Represent this sequence as a one-term recurrence relation.
   C. Find an explicit formula for a_n in terms of the index variable n.
   D. Find the $\lim_{n \to \infty} a_n$

Suppose we have a sequences $\{a_n\}_{n=1}^{\infty}$ defined by the recurrence relation

$$a_1 = 7, \qquad a_{n+1} = -1 \cdot a_n$$

   A. Represent this sequence as an ordered set by finding the first six terms of this sequence.
   B. Find an explicit formula for a_n in terms of the index variable n.
   C. Find the $\lim_{n \to \infty} a_n$

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12. Evaluate the limit or state that the limit does not exists for the sequence

$$a_n = \int_1^n x^{-2} dx$$

Optional Challenge Problems

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13. Prove theorem 8.3 p. 611: The limit of a geometric series (this is done for you in Jeff's notes)

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14. Exercise 8.1.82 p. 606

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15. Exercise 8.2.60 p. 616

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16. Exercise 8.2.95 p. 618