Name : ______________________ **Class Number**: _____

Final Exam Practice Problems

Part I: Multivariable Differentiation
Math 1C: Calculus III

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

What can I use on this exam?

- You may use one note card that is no larger then 11 inches by 8.5 inches. You may write on both sides of this note card. PLEASE SUBMIT YOUR NOTECARD WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of vector notation and proper use of partial derivative notation.

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

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1. T F If f has a local minimum at point $(a, b) \in \mathbb{R}^2$, then

$$D_{\mathbf{u}f(a,b)} = 0$$

for any unit vector $\mathbf{u} \in \mathbb{R}^2$.

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2. T F If $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$, then $|\mathbf{x} \cdot \mathbf{y}| \leq \|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2$

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3. T F Suppose $f : \mathbb{R}^3 \to \mathbb{R}$. If $\nabla f = 0$ at a point $\mathbf{x} \in \mathbb{R}^3$, then f has a local extreme value at point $\mathbf{x}$.

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4. T F For any vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$.

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5. T F $f_y(a, b) = \lim_{y \to b} \dfrac{f(a, y) - f(a, b)}{y - b}$.

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6. T F The set of points $\{(x, y, z) \, : \, x^2 + y^2 = 1\}$ is a circle.

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7. T F If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b),, then

$$\lim_{(x,y) \to (a,b)} f(x, y) = L.$$

Multiple Choice

For the problems below, circle the correct response for each question. After you've chosen your answer, mark your answer on your Scantron form.

8. Let $z = \sin(x \cdot y)$ and let $x = x(t)$ and $y = y(t)$ be functions of t. Suppose

$$x(1) = 0, \qquad y(1) = 1, \qquad x'(1) = 2, \qquad y'(1) = 3.$$

Find $\dfrac{dz}{dt}$ when $t = 1$.

A. 1 B. 2 C. 3 D. 4 E. 5

9. Find the direction of maximum increase of the function $f(x, y, z) = x\,e^{-y} + 3\,z$ at the point $(1, 0, 4)$.

A. $\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ C. $\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$ E. $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

10. Find the shortest distance from the origin to the surface $z^2 = 2xy + 2$

A. $\dfrac{1}{\sqrt{2}}$ B. $\sqrt{2}$ C. $\dfrac{1}{2}$ D. 2 E. 1

11. Find an equation for the line through the point $(3, -1, 2)$ and perpendicular to the plane $2x - y + z + 10 = 0$.

A. $3x - y + 2z + 10 = 0$ B. $3x - 2y + z + 10 = 0$ C. $\dfrac{x-2}{3} = \dfrac{y+1}{-1} = \dfrac{x-2}{2}$

D. $\dfrac{x-3}{2} = \dfrac{y+1}{-1} = z - 2$ E. $\dfrac{x+2}{2} = \dfrac{y-1}{-1} = z - 2$

12. Let $f(x, y) = e^{\sin(x)} + x^5 y + \ln\left(1 + y^2\right)$. Find f_{yx} :

A. $\dfrac{2y}{1+y^2}$ B. $20x^3 y$ C. $5x^4$ D. $e^{\sin(x)}\cos(x)$ E. $e^{\sin(x)}\cos(x) + x^5 + \dfrac{2y}{1+y^2}$

13. Find the area of the triangle with vertices at the points $(0,0,0)$, $(1,0,-1)$ and $(1,-1,2)$.

A. $\dfrac{\sqrt{11}}{2}$ B. $\dfrac{\sqrt{6}}{2}$ C. 1 D. $\sqrt{11}$ E. $\sqrt{6}$

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14. Given $f(x,y) = \sqrt{x^2+y^2}$, find f_{xx} :

A. $\dfrac{y}{\left(x^2+y^2\right)^{1/2}}$ B. $\dfrac{xy}{\left(x^2+y^2\right)^{1/2}}$ C. $\dfrac{x}{\left(x^2+y^2\right)^{1/2}}$ D. $\dfrac{x^2}{\left(x^2+y^2\right)^{3/2}}$ E. $\dfrac{y^2}{\left(x^2+y^2\right)^{3/2}}$

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15. Find the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^4y^2}{x^4+3y^4}$

A. 0 B. 2 C. $\dfrac{1}{2}$ D. $\dfrac{2}{3}$ E. Does NOT exist

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16. Consider the vectors

$$\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = 2\mathbf{i} + 3\mathbf{k}, \qquad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} = \mathbf{j} - \mathbf{k}.$$

Which of the following vectors gives $\mathbf{x} \times \mathbf{y}$?

A. $3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ B. $-3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ C. $-3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ D. $-3\mathbf{k}$ E. -3

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17. Find an equation of the line through the point $(1,2,3)$ and parallel to the plane $x - y + z = 100$:

A. $x - y + z - 2 = 0$ B. $x - 1 = \dfrac{y+1}{2} = \dfrac{z-1}{3}$ C. $x + 2y + 3z = 100$

D. $x - 1 = 2 - y = z - 3$ E. $x - y + z + 2 = 0$

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18. The equation of the sphere with center $(4,-1,3)$ and radius $\sqrt{5}$ is

A. $(x+4)^2 + (y-1)^2 + (z+3)^2 = 5$

B. $(x-4)^2 + (y-1)^2 + (z-3)^2 = 5$

C. $(x-4)^2 + (y+1)^2 + (z-3)^2 = 25$

D. $(x-4)^2 + (y+1)^2 + (z-3)^2 = \sqrt{5}$

E. $(x-4)^2 + (y+1)^2 + (z-3)^2 = 5$

19. Given $\mathbf{x} = (2, 0, 1)$ and $\mathbf{v} = (4, 1, 2)$, what is the area of the parallelogram formed by the vectors $\mathbf{x}$ and $\mathbf{y}$?

A. $2\sqrt{5}$ B. $\sqrt{5}$ C. $2\sqrt{3}$ D. $3\sqrt{2}$ E. $4\sqrt{2}$

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20. Find values of $b \in \mathbb{R}$ such that the vectors $\begin{bmatrix} -11 \\ b \\ 2 \end{bmatrix}$ and $\begin{bmatrix} b \\ b^2 \\ b \end{bmatrix}$ are orthogonal.

A. $0, 3, -3$ B. $0, 11, -3$ C. $0, 2, -2$ D. $0, -11, 2$ E. $0, 11, 2$

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21. Find the parametric equations of the intersection of the planes $x - z = 0$ and $x - y + 2z + 3 = 0$

A. The line given by $x(t) = 1 + t, y(t) = 6$ and $z(t) = 1 - t$.
B. The line given by $x(t) = 1 + t, y(t) = 6 - t$ and $z(t) = 1 + 2t$.
C. The line given by $x(t) = -t, y(t) = 3 - 3t$ and $z(t) = -t$.
D. The plane $3x + 3y - 3z + 3 = 0$
E. The line given by $x(t) = -2 - t, y(t) = 1 - 3t$ and $z(t) = -t$.

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22. Given $\mathbf{x} = (4, 0)$ and $\mathbf{y} = (5, 2)$, which of the following is the projection of vector $\mathbf{x}$ onto the vector $\mathbf{y}$?

A. $\left(\frac{40}{27}, \frac{16}{27}\right)$ B. $(5, 0)$ C. $(4, 2)$ D. $\left(\frac{100}{29}, \frac{40}{29}\right)$ E. $\left(\frac{100}{\sqrt{29}}, \frac{40}{\sqrt{29}}\right)$

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23. Which of the following is a unit vector point in the direction of vector $\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$?

A. $\frac{1}{\sqrt{13}} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ B. $\begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ D. $\frac{1}{\sqrt{13}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ E. $\frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

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24. Find the distance between the point $(-1, -1, -1)$ and the plane $x + 2y + 2z - 1 = 0$

A. 6 B. 2 C. 0 D. -2 E. -6

25. Consider the vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}.$$

Which of the following vectors gives $\mathbf{x} \cdot \mathbf{y}$?

A. 14 B. 10 C. -14 D. $-12\mathbf{k}$ E. -10

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26. Find the minimum value of the function $f(x, y) = x\,y$ subject to the constraint that $x^2 + y^2 = 2$:

A. 1 B. 2 C. -1 D. $\frac{3}{2}$ E. $-\frac{3}{2}$

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27. Find the directional derivative of the function

$$f(x, y) = y^2 \cdot \ln(x)$$

at the point $(1, 2)$ in the direction of the vector $(3, 4) = 3\mathbf{i} + 4\mathbf{j}$:

A. $\frac{5}{16}$ B. 12 C. $\frac{5}{12}$ D. $\frac{16}{5}$ E. $\frac{12}{5}$

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28. Find an equation of the tangent plane to the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 4$ at the point $(4, 1, 1)$.

A. $2x + y - z = 1$
B. $x + 2y + 2z = 8$
C. $x - 2y + 4z = 0$
D. $x + y + z = 6$
E. $2x + y + z = 10$

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29. Determine how many critical points the function $f(x, y) = x\,y - x^2\,y - x\,y^2$ has:

A. 1 B. 2 C. 3 D. 4 E. 5

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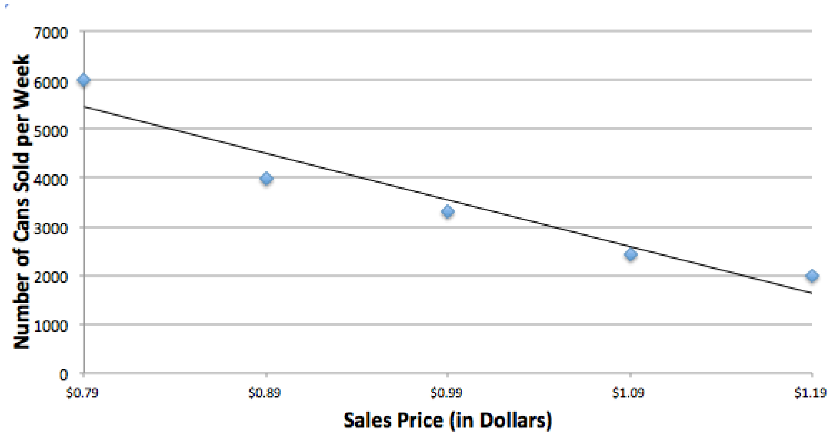
30. Let $f(x, y) = \frac{x}{y} + \frac{y}{x}$. Find the gradient vector ∇f:

A. $\begin{bmatrix} 2y \\ 2x \end{bmatrix}$ B. $\begin{bmatrix} x \\ y \end{bmatrix}$ C. $\begin{bmatrix} \frac{1}{y} - \frac{y}{x^2} \\ \frac{1}{x} - \frac{x}{y^2} \end{bmatrix}$ D. $\begin{bmatrix} -y/x^2 \\ -x/y^2 \end{bmatrix}$ E. $\begin{bmatrix} y \\ x \end{bmatrix}$

Free Response

31. (10 points) A company test-markets a new canned energy drink made of all natural ingredients in 5 cities of equal size on the West Coast of the US. The selling price (in dollars) and the number of drinks sold per week is each of the cities is listed as follows

City	Price	Sales/Week
1	0.79	6000
2	0.89	3980
3	0.99	3300
4	1.09	2440
5	1.19	1990



This data is plotted in the figure next to the table above. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.

A. Set up the least squares problem to fit this data to a linear model

$$S(p) = c_1 + c_2 p$$

where S is the sales per week and p is the price. Explain all choices that you made in setting up this model and describe why you made these choices.

B. Explain how you would use multivariable calculus to find the line of best fit.

32. (10 points) Let $f : \mathbb{R}^2 \to R$ be a differentiable, multivariable function. Let $\mathbf{u}$ be a unit vector.

A. Derive the dot product formula for the limit definition of the directional derivative $D_{\mathbf{u}}f$.

B. Prove that the gradient vector is in the direction of maximum increase.

33. Find the points on the surface defined by

$$(x-y)^2 + y^2 + (y+z)^2 = 1$$

at which the tangent plane is parallel to the xz-plane

34. Let $z = z(x, y)$ be defined implicitly by equation

$$x^2 + y^2 - z^2 = 3xyz$$

Compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at point $(3, 1, 1)$.

35. Consider the function $f(x, y) = 1 + x^2 + y^2$.

A. Find the equation for the tangent plane to $f(x, y)$ at point $(1, 2)$.

B. Use the tangent plane to approximate f as (x, y) moves a distance of $\dfrac{1}{10\sqrt{5}}$ units toward the origin.

36. A. Use a scalar projection to show that the distance from point $P(x_1, y_1)$ to line $ax + by + c = 0$ is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Draw a diagram and explain your reasoning in detail using full sentences.

B. Use this formula to find the distance from the point $(-2, 3)$ to the line $3x - 4y + 5 = 0$.

Challenge Problem

37. (Optional, Extra Credit, Challenge Problem)

Let A be the area of the region in the first quadrant of the cartesian plane bounded by the line $y = \frac{1}{2}x$, the x-axis, and the ellipse $\dfrac{1}{9}x^2 + y^2 = 1$. Find the positive number m such that A is equal to the area of the region in the first quadrant bounded by the line $y = mx$, the y-axis and the ellipse $\dfrac{1}{9}x^2 + y^2 = 1$.

