Name : ____________________ Math 1C: Calculus III

Exam 2: Extra Practice Problems

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1. (FR) Find an equation to the tangent plane to the surface $4x^2 - y^2 + 3z^2 = 10$ at the point $(2, -3, 1)$.

2. (MC) Consider the function $f(x, y) = 1 + x^2 + y^2$.

   A. Find the equation for the tangent plane to $f(x, y)$ at point $(1, 2)$.

   B. Use the tangent plane to approximate f as (x, y) moves a distance of $\dfrac{1}{10\sqrt{5}}$ units toward the origin.

3. (FR) Find the points on the surface defined by

$$(x - y)^2 + y^2 + (y + z)^2 = 1$$

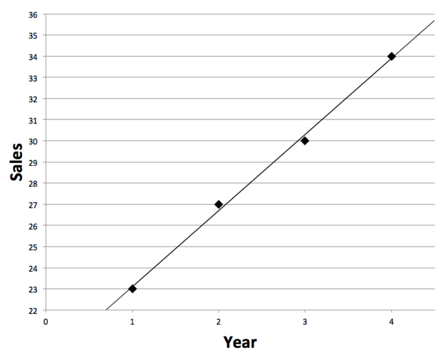
   at which the tangent plane is parallel to the xz-plane

4. (MC) Find all local extreme values and any saddle point(s) of the function $f(x, y) = 4xy - x^4 - y^4 + \dfrac{1}{16}$.

5. (MC) Find the extreme value(s) of the function $f(x, y) = 2x + 3y + 4$ on the circle $x^2 + y^2 = 1$

6. (FR) A small bike company selling utility bicycles for daily commuting has been in business for four years. This company has recorded annual sales (in tens of thousands of dollars) as follows:

Year	Sales
1	23
2	27
3	30
4	34



   This data is plotted in the figure next to the table above. Although the data do not exactly lie on a straight line, we can create a linear model to fit this data.

   a. Set up the least squares problem to fit this data to a linear model.

   b. Explicitly identify the unknown variables.

   c. Explain how you would use multivariable calculus to find the line of best fit.

   d. What exactly is being optimized in the least squares problem?

7. (FR) Start at the origin and move 4 units along the positive y-axis. Turn 90 degrees to the right and move 75% of your last distance. Turn 90 degrees to the right and move 75% of your last distance. Turn 90 degrees to the right and move 75% of your last distance. Continue this processes forming a “spiral with square corners.” Determine the **y-coordinate** for the point (x, y) where the spiral “ends.”

8. (EC) The following series converges. Determine the **exact** value of this infinite seres.

$$\frac{1}{1} + \frac{2}{2} - \frac{3}{2^2} + \frac{4}{2^3} + \frac{5}{2^4} - \frac{6}{2^5} + \frac{7}{2^6} + \frac{8}{2^7} - \frac{9}{2^8} + \cdots$$

9. (MC) Determine whether each of the following series converges or diverges. Write clear and complete solutions including the name of the series test that you use and what your final answer is.

   (a) $\sum_{n=1}^{\infty} \frac{n+2}{3n+5}$

   (b) $\sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^n$

   (c) $\sum_{n=2}^{\infty} \sqrt{\frac{n}{n^4+3}}$

   (d) $\sum_{n=3}^{\infty} \frac{(\ln(n))^2}{n}$

   (e) $\sum_{n=1}^{\infty} \frac{5^{n+1}}{(2n)!}$

   (f) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+\sqrt{n}}$

   (g) $\frac{1}{1^4} + \frac{1}{2^4} - \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} - \frac{1}{6^4} + \frac{1}{7^4} + \frac{1}{8^4} - \frac{1}{9^4} + \cdots$

   (h) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$

10. (MC) The alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+7}}$ converges. What should N be so that the partial sum

$$s_N = \sum_{n=1}^{N} (-1)^{n+1} \frac{1}{\sqrt{n+7}}$$

estimates the exact value of the series with absolute error at most 0.001?

11. (MC) The series $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln(n))^2}$ converges. What should N be so that the partial sum

$$s_N = \sum_{n=1}^{N} \frac{1}{n(1+\ln(n))^2}$$

estimates the exact value of the series with absolute error at most 0.1?