Name : ____________________ **Class Number**: ______

Exam 1, Version 5A

Math 1C: Calculus III

What are the rules of this exam?

- PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO!
- It is a violation of the Foothill Academic Integrity Code to, in any way, assist another person in the completion of this exam. Please keep your own work covered up as much as possible during the exam so that others will not be tempted or distracted. Thank you for your cooperation.
- No notes (other than your note card), books, or classmates can be used as resources for this exam.
- Please turn off your cell phones during this exam. No cell phones will be allowed on your desk.

How long is this exam?

- This exam is scheduled for a 110 minute class period.
- Make sure you have 4 sheets of paper (8 pages front and back) including this cover page.
- There are a total of 5 separate questions (50 points) on this exam including:
  - 4 Free-Response Questions (50 points)
  - 1 Optional, Extra Credit Challenge Problem (5 points)

What can I use on this exam?

- You may use up to three note sheets that are no larger then 11 inches by 8.5 inches. You may write on both sides of these note sheets. PLEASE SUBMIT ALL OF YOUR NOTE SHEETS WITH YOUR EXAM.
- You are allowed to use calculators for this exam. Examples of acceptable calculators include TI 83, TI 84, and TI 86 calculators. You are not allowed to use any calculator with a Computer Algebra System including TI 89 and TI NSpire. If you have a question, please ask your instructor about this.

How will I be graded on the Free-Response Questions?

- Read the directions carefully. Show all your work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting a good score on this exam. Neatness and organization are IMPORTANT!
- You will be graded on proper use of vector notation.

Free Response

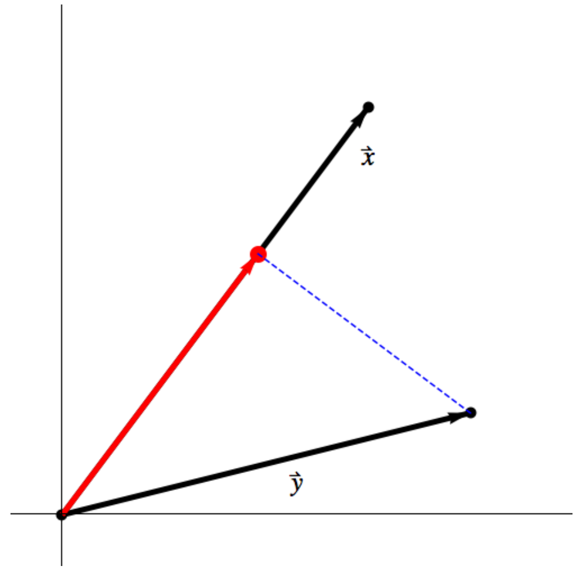
1. (8 points) Please explain your understanding of the dot product between two vectors in $\mathbb{R}^n$ below:

   A. Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. Derive the cosine formula for the dot product:

   $$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2 \, \cos(\theta)$$

   You don't have to prove the pythagorean theorem nor the law of cosines in this derivation. However, please explain your work and specifically identify the steps you took to arrive at your final answer.

B. Suppose $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$. Using the diagram below, derive an equation for the projection of vector $\mathbf{y}$ onto $\mathbf{x}$. Be sure to identify the projection vector $\mathbf{p}$ and the residual vector $\mathbf{r}$ on the diagram below. Please explain your answer and specifically identify the steps you took to arrive at your final answer.



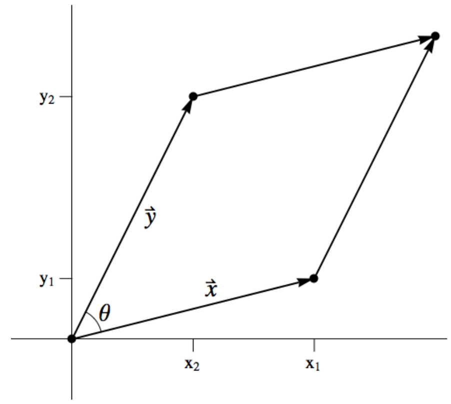
2. (8 points) Find the distance from the point $P(1, -2, 4)$ to the plane $3x + 2y + 6z = 5$. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

3. (8 points) For vectors $\mathbf{x} = \langle 1, 1, 2 \rangle$ and $\mathbf{y} = \langle -2, 3, 1 \rangle$, express the vector $\mathbf{x}$ as the sum of two vectors $\mathbf{x} = \mathbf{p} + \mathbf{r}$ where $\mathbf{p}$ is parallel to $\mathbf{y}$ and $\mathbf{r}$ is orthogonal to $\mathbf{y}$. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

4. (6 points) Find two unit vectors orthogonal to both $\langle 3, 2, 1\rangle$ and $\langle -1, 1, 0\rangle$. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

5. (12 points) Below, please explain your understanding of the cross product between two vectors in $\mathbb{R}^3$.

   A. Let $\mathbf{x} = \langle x_1, y_1 \rangle$ and $\mathbf{y} = \langle x_2, y_2 \rangle$ be two vectors in $\mathbb{R}^2$. Using the diagram below, derive an equation for the area of the parallelogram formed by vectors $\mathbf{x}$ and $\mathbf{y}$ based only on the components of these vectors (note: this equation should NOT be based on the angle θ between these vectors). Please explain your answer and specifically identify the steps you took to arrive at your final answer.



   B. Under the same assumptions in problem A. ,suppose that θ is the angle between above with $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$. Derive a formula for the area of the parallelogram parallelogram formed by vectors $\mathbf{x}$ and $\mathbf{y}$ as a function of θ and the two norms of these vectors. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

C. How do your answers to parts A. and B. on the last page relate to the component form of the cross product of the vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^3$ where

$$\mathbf{a} = \langle a_1, b_1, c_1 \rangle, \qquad \mathbf{b} = \langle a_2, b_2, c_2 \rangle,$$

Make sure to explicitly state the component form of the cross product in your explanation.

D. Using the assumptions from part C above, derive the sine formula for the cross product:

$$\|\mathbf{a} \times \mathbf{b}\|_2 = \|\mathbf{a}\|_2 \, \|\mathbf{b}\|_2 \, |\sin(\theta)|$$

6. (8 points) Find the parametric equation for a line in the intersection of the planes $x + y + z = 1$ and $x - 2y + 3z = 1$. Please explain your answer and specifically identify the steps you took to arrive at your final answer.

Challenge Problem

7. (Optional, Extra Credit, Challenge Problem) Derive the general equation for an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$