

Exam 1: Practice Problems

1. (MC) Let $\mathbf{u} = \langle 1, -2 \rangle$ and $\mathbf{v} = \langle 3, 4 \rangle$. Find the $\text{Proj}_{\mathbf{v}}(\mathbf{u})$.

2. (MC) Define two lines $L_1(t)$ and $L_2(s)$ intersect at a single point in \mathbb{R}^3 , where

$$\mathbf{L}_1(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 1+t \\ 2t \\ -1+3t \end{bmatrix} \quad \text{and} \quad \mathbf{L}_2(s) = \begin{bmatrix} x(s) \\ y(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} 3+2s \\ 1+s \\ -2-s \end{bmatrix}$$

Find the point (x, y, z) of intersection. Then, find the angle θ between the lines.

3. (MC) Compute the distance from the origin $(0, 0, 0)$ to the plane $2x + y - 2z = 6$.

4. (MC) Find a parametric equation for line L in the intersection of the planes $x + 2z = 1$ and $x + y - z = 0$.

5. (MC) Let $\vec{\mathbf{A}} = \langle 3, 0, -2 \rangle$ and $\vec{\mathbf{B}} = \langle 0, -1, 1 \rangle$.

a. Find the area of the parallelogram formed by placing the vectors tail-to-tail.

b. Find an equation of the plain containing point $(1, -2, 3)$ and which is parallel to both vectors.

6. (MC) Let $z = 3x + \ln(x^2 + y)$. Compute the partial derivatives z_y, z_x and z_{xx} .

7. (MC) Find the point(s) on the following function $f(x, y) = x^3 - y^3 + 3xy$ where both $f_x = 0$ and $f_y = 0$.

8. (MC) Evaluate the following limit: $\lim_{(x,y) \rightarrow (1,-1)} \frac{1 - \sqrt{x+y+1}}{x+y}$.

9. (MC) Verify that the following limit does not exists: $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6}$.

10. (MC) Let $f(x, y) = 4 - \sqrt{y - x^2}$

(a) Determine and sketch the domain of f in 2D-space.

(b) State the range of f . BRIEFLY explain how you get your answer.