Math 1C.01: Summer Session 2018

Lesson 7: In-class Problems List

Lesson 7: Graphs and level curves (53 min)

Problem 5 Statement

Consider the functon

$$z = f(x,y) = x^2 + y^2 - 16.$$

☐ Part A: Find a parametric equation $\vec{r} : D \subseteq \mathbb{R} \longrightarrow \mathbb{R}^2$ with $\vec{r}(t) = \langle x(t), y(t) \rangle = \vec{r}_0 + t \cdot \vec{v}$ to the tangent line of level curve

$$L_{z_0}(f) = L_0(f) = \{ (x,y) : z = z_0 = 0 = f(x,y) \}$$
$$= \{ (x,y) : 0 = x^2 + y^2 - 16 \}$$

at point $(x, y, z) = (-2\sqrt{2}, -2\sqrt{2}, 0)$.

☐ Part B: Graph the level curve from part A.

(12 min) Wed. 7/18/2018 @ 11:23am – 11:35am

Problem 5: solution to part A

We Know $L_{z_0}(f) = L_0(f)$ is defined as

$$L_0(f) = \{(x,y) : 0 = f(x,y)\}$$

$$= \{(x,y) : 0 = x^2 + y^2 - 16\}$$

$\Rightarrow$ we want to collect ordered pairs (x,y) s.t.

$$x^2 + y^2 = 16$$

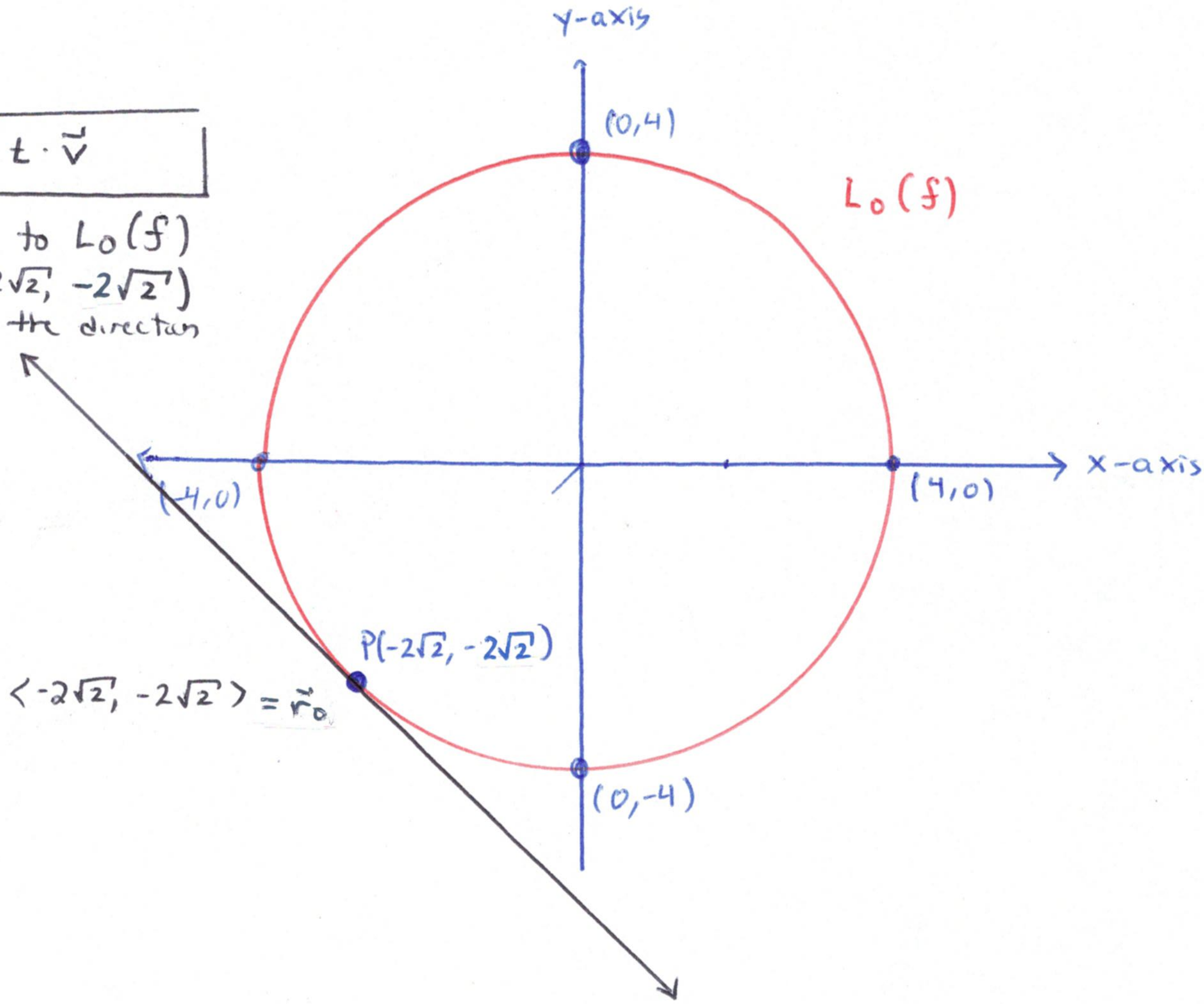
$\Rightarrow$ we can visualize $L_0(f)$ as a circle centered at $(0,0)$ with radius 4:

(21 min)

Wed 7/18/2018 @ 11:36am - 11:57am

$$\boxed{\vec{r}(t) = \vec{r}_0 + t \cdot \vec{v}}$$

tangent line to $L_0(f)$ at point $(-2\sqrt{2}, -2\sqrt{2})$ where $\vec{v}$ is the direction of this line.



Recall, I want to draw a tangent line to the red curve $L_0(f)$

Side note:

$$\|\langle -2\sqrt{2}, -2\sqrt{2}\rangle\|_2^2 = (-2\sqrt{2})^2 + (-2\sqrt{2})^2$$

$$= 8 + 8$$

$$= 16$$

$$\Rightarrow \|\langle -2\sqrt{2}, -2\sqrt{2}\rangle\|_2 = \sqrt{16} = 4 \checkmark$$

(9 min)

Wed 7/18/2018 @ 11:57 am - 12:06pm

To find the direction vector $\vec{v}$, let's use implicit differentiation from Math 1A (see video 7.6). Let's start at the defining implicit equation for curve $L_0(f) \subseteq \mathbb{R}^2$:

$$x^2 + y^2 = 16$$

Now, we will differentiate both sides w/r to x:

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[16]$$

$$\Rightarrow \frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = 0$$

$$\Rightarrow 2x + 2 \cdot y \cdot y' = 0$$

Side note:

Here, we pretend that

$$y = y(x)$$

and use chain rule:

$$\Rightarrow \frac{d}{dx}[y^2] = \frac{d}{dx}[y(x)]^2$$

$$= 2[y(x)] \cdot \frac{d}{dx}[y(x)]$$

$$= 2 \cdot y \cdot y'$$

(5 min)

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$$\Rightarrow \quad 2x + 2y \cdot y' = 0$$

$$\Rightarrow \quad y' = \frac{-2x}{2y} = \frac{-x}{y}$$

$$\Rightarrow \quad y' \Big|_{P(-2\sqrt{2},\ -2\sqrt{2})} = \frac{-(-2\cdot\sqrt{2})}{-2\sqrt{2}}$$

$$= \frac{+2\sqrt{2}}{-2\sqrt{2}}$$

$$= \frac{-1}{1} \begin{array}{l} \leftarrow \text{rise } (\Delta y) \\ \hline \leftarrow \text{run } \quad \Delta x \end{array}$$

$$\Rightarrow \quad y' = -1 \quad \Rightarrow \quad \vec{v} = \langle 1, -1 \rangle$$

$$\Rightarrow \quad \vec{r}(t) = \vec{r}_0 + t \cdot \vec{v}$$

$$= \langle -2\sqrt{2}, -2\sqrt{2} \rangle + t \langle 1, -1 \rangle$$

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$\Rightarrow \quad \vec{r}(t) = \langle -2\sqrt{2} + t, \ -2\sqrt{2} - t \rangle$

$= \langle x(t), \ y(t) \rangle$

$$\Rightarrow \begin{cases} x(t) = -2\sqrt{2} + t \\ y(t) = -2\sqrt{2} - t \end{cases}$$

$\Rightarrow \quad y = mx + b \quad$ or $\quad y - y_1 = m(x - x_1)$

$\Rightarrow \quad y = -1(x + 2\sqrt{2}) - 2\sqrt{2}$

$\Rightarrow \quad \boxed{y = -x - 4\sqrt{2}}$ ✓

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Total: 12 min

Math 1C.01: Summer 2018

Lesson 8: In-class Problem List

Lesson 8: Limits & Continuity

Problem 5 Statement

Evaluate the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2}{x^4+y^2}$

Problem 5 solution

Let's try two path's:

path 1: $y=0$

path 4: $y=x^2$

Let's travel along path 1: Set $y=0$

$\Rightarrow \displaystyle\lim_{(x,y)\to(0,0)} \frac{y^2}{x^4+y^2} = \lim_{(x,0)\to(0,0)} \frac{0^2}{x^4+0^2}$ since $y=0$ by my assumpto

$\displaystyle = \lim_{(x,0)\to(0,0)} \frac{0}{x^4+0} = 0$ by williams ~~genious~~ sweat

$\displaystyle = \lim_{x\to 0} \frac{0}{x^4} = \lim_{x\to 0} \underbrace{\boxed{0\cdot\frac{1}{x^4}}}_{0 \text{ if } x\neq 0} = \lim_{x\to 0}$

(8 min)

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Let's travel along path 4: set $y = x^2$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{y^2}{x^4+y^2} = \lim_{(x,x^2)\to(0,0)} \frac{(x^2)^2}{x^4+(x^2)^2}$$

$$= \lim_{(x,x^2)\to(0,0)} \frac{x^4}{x^4+x^4}$$

$$= \lim_{x\to 0} \frac{1\cdot x^4}{2\cdot x^4}$$

$$= \lim_{x\to 0} \frac{1}{2}\cdot\frac{x^4}{x^4}$$

$$= \frac{1}{2}\cdot\underbrace{\lim_{x\to 0}\frac{x^4}{x^4}}$$

$$= \frac{1}{2} \quad 1$$

by mathemagics (math 1A)

we see along two paths we get two different limit values.

By the two path test on page, the limit in this problem DNE.

(4 min) Wed 7/18/2018 @ 12:30pm - 12:34pm