

# Applied Linear Algebra Fundamentals

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This is a draft table of contents for Jeff Anderson's *Applied Linear Algebra Fundamentals* textbook project. For more about this project, check out any of the following links:

- [Applied Linear Algebra Fundamentals textbook project homepage](#)
- [Applied Linear Algebra Fundamentals laboratory activities](#)
- [Applied Linear Algebra Fundamentals textbook project description](#)

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