

DC Voltage Sources

Figure 11A depicts a real dc voltage source. This voltage source is powered by a 9V Duracell battery and provides a constant dc voltage drop of 5V from the red lead to the black lead. The metal switch activates this source and a small, red LED light shines when the source is turned on. The support website for this paper includes much more about this type of voltage source. In Figure 11B we see a digital image of this dc voltage source labeled. Figure 5C presents the schematic for an ideal voltage source.

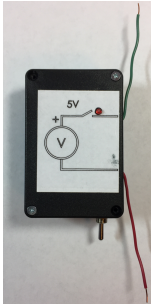


Figure 11A: Real dc voltage source

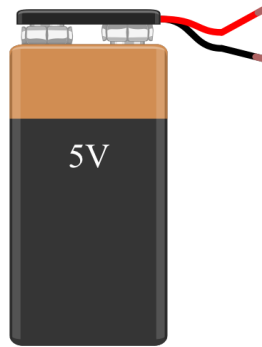


Figure 11B: Digital dc voltage source

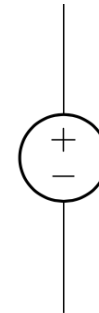


Figure 11C: Ideal dc voltage source schematic

DC Current Sources

The final category of real circuit elements we use in this modeling process are dc current sources. Figure 12A provides a photo of a dc current source that we use in this modeling activity. Almost all the same properties for the dc voltage source above apply to this dc current source. The major difference here is that this source provides a dc current running from the red lead into the green lead. In the work that follows, we use these three types of real circuit elements as building blocks to create real electric circuits. In Figure 12B we see a digital image of our dc current source. Notice that except for the label, this figure looks identical to the dc voltage source in figure 11B. However, as we discuss below, we measure current in amperes and thus we label the current source using 2.5mA to indicate that this source produces current. Finally, in Figure 12C we see the schematics for an ideal current source.

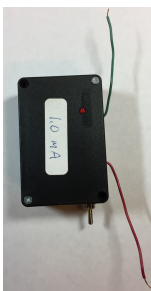


Figure 12A: Real dc current source

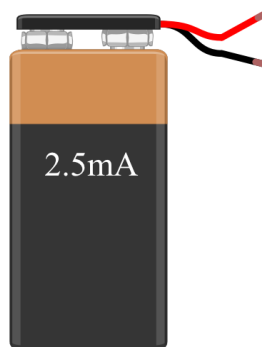


Figure 12B: Digital dc current source

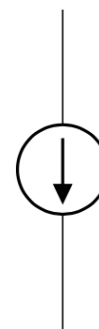


Figure 12C: Ideal dc current source schematic

We should note that both the dc voltage and current sources pictured here are custom designed for this laboratory experiment to be cheap, portable, easy-to-use, and safe so that untrained students can enjoy this modeling activity in a classroom environment. The author of this paper is currently working with a team of electrical engineers to automate the manufacturing process for these sources. The goal is to provide a usable laboratory set up that is quite affordable and does not require prior training on large, expensive equipment.

Connection “Jumper” Wires

Our learning lab includes a 350-piece jumper wire kit. This collection provides jumper wires in 14 different lengths that are color-coded for ease of reference. Moreover, the various lengths are specifically designed to correspond to the distances between contact points on a breadboard. This feature allows quick and easy connections between various electrical contact strips on the breadboard. In the standard kit, there are 25 pieces of each length of wire. We notice that the length increase by 0.1 inches for smaller pieces. Also note that the color codes for the wires follow the color codes for the resistor bands almost exactly. The only difference is that the 0.1 inch wire has no colored insulation.

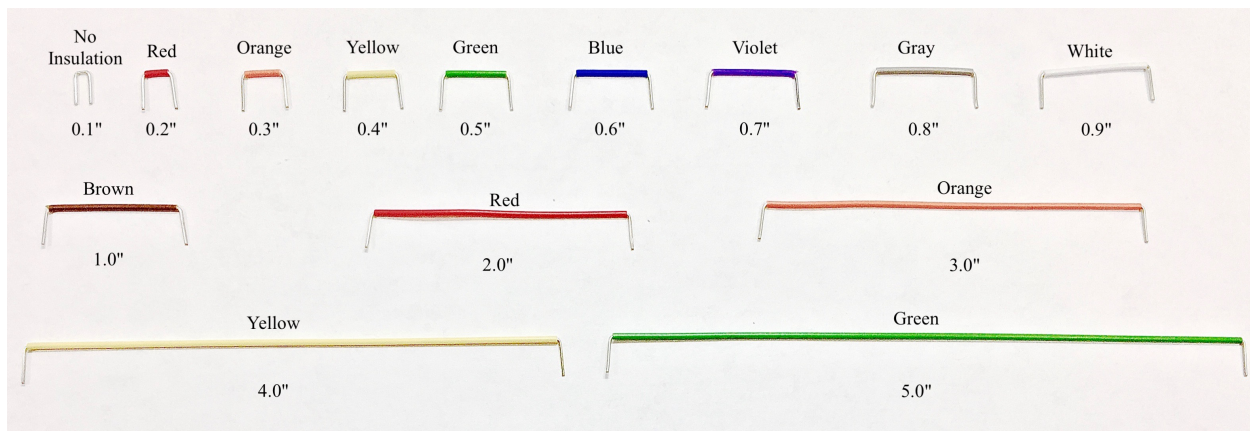


Figure 13: The jumper wire kit provides wires

With this, we have finished reviewing all of the different components found in the electronics learning lab. Let’s move on to learn how to connect a collection of physical components and study the electrical properties that arise from the chosen interconnections.

Real circuits from individual circuit elements

A *real circuit* is a collection of real elements, sometimes called *branches*, that are linked together so that each lead of any real circuit element is connected to the lead of at least one other element. Throughout this activity, we use solderless breadboards to connect the leads of our elements and form our example circuits. In the work that follows, we demonstrate the experimental framework to build and test real circuits containing only resistors, dc voltage sources, and dc current sources. We provide TWO other examples of such circuits including measured node voltages. Figure 14A and 14B presents a photo of our first real circuit that we analyze in this paper. Our example circuit includes eight $1\text{k}\Omega$ precision resistors, two dc voltage sources, and two dc current sources connected together on a breadboard. By the end of this process, we transform this circuits into a valuable linear-systems problems and verify the accuracy of our model by comparing measured and computed data. This is the exact activity that our students can complete.

Example 1: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes a single 1k resistor and one dc voltage source. This dc voltage source provides 5V across the leads.

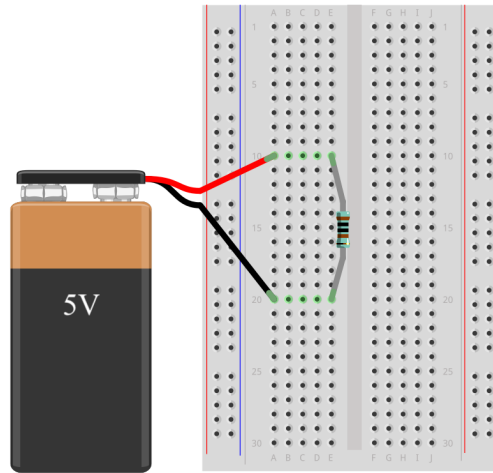


Figure 14A: Digital Wiring Diagram for Physical Circuit in Example 1

Example 1: Ideal schematic diagram

Let's take a look at the schematic diagram for our circuit in example 1, as shown in Figure 15. The ability to translate back and forth between wiring diagrams and schematic diagrams is a valuable skill and takes practice. In the work that follows, we present the background theory and material needed to understand how to develop such a schematic diagram. For now, we encourage the reader to compare and contrast the wiring diagrams in Figures 14A and 14B with the schematic diagram in Figure 15. These diagrams are drawn with the hope that the map between the real circuit elements and the ideal schematics are easy to discover.

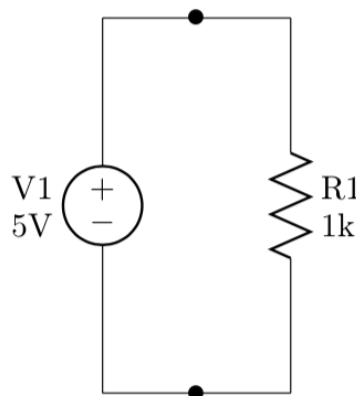


Figure 15: Ideal Circuit Schematic Diagram for Example 1

Example 1: Parts You Will Need

To build this circuit, you will need the following parts:

- Half-size 2.2"x 3.4" solderless breadboard
- One 1k resistors (brn-brn-blk-blk-brn)
- One 5V dc voltage sources
- One fully-charged 9V batteries

Example 1: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

1. Disconnect batteries from dc sources
2. Insert resistor R1 across e10 and e20
3. Insert red lead of V1 in a10 and blk lead in a20

Example 1: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the node voltage potentials at all seven nodes of the circuit and record your data in the table below.

Example 1, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	v_{r_1}		i_{r_1}	
V1	v_{v_1}		i_{v_1}	

More about how to capture this data is described below. To help you build confidence and check your work, we provide a [YouTube video](#) of our voltage and current measurements for this circuit. This video can be found on the support website for this Electrify the linear-systems problem paper (CITE). We hope these resources help you build confidence in your ability to collect voltage and current measurements. As you get better at measuring node voltage potentials, we cease to provide author-generated data and instead encourage you to use mathematical analysis to validate your measurements.

Notice that the reported value of the voltage source is not identical to the measured value of the voltage source. In other words, we claimed that the voltage source was supposed to produce a 5V drop across its lead. But, during measurement, we found a measured drop not exactly equal to 5V. This discrepancy leads us to develop some valuable intuition that is best summed up in Eistein's famous quote: "in theory, theory and practice are the same, but in practice they are different." When working with real circuit elements, the reported behavior of a particular element is not identical to the actual measured behavior. To account for this difference, manufacturers provide a specification tolerance which indicates a range of values that the element is known to take under testing.

Operational definition of Measurement

During our work with circuit analysis, we define many variables to describe the behavior of each real circuit element. We do so using an *operational definition* that consists of two specifications: first, we indicate the instrument we use to measure each quantity and second we provide a detailed procedure for using this instrument to collect individual data readings.

Before we describe the operational definitions of the voltages drop across an element and the current running through an element, let's consider a more intuitive illustration of this process. Suppose we study the behavior of a mass-spring system. To create this system, we connect an one end of extension spring to a fixed position. Describe entire experiment and provide a YouTube link to corresponding video.

To study this system, we'll provide an operational definition of a fundamental quantity that we'll call *displacement*. To define such a variable we must specify the instrument we use and the exact procedure we run through to measure a single displacement value.

We begin our operational definition by specifying our measurement instrument. In this case, we use a metric measuring tape, seen in Figure 16 and on the left-hand side of Figures 17A, 17B, and 17C.

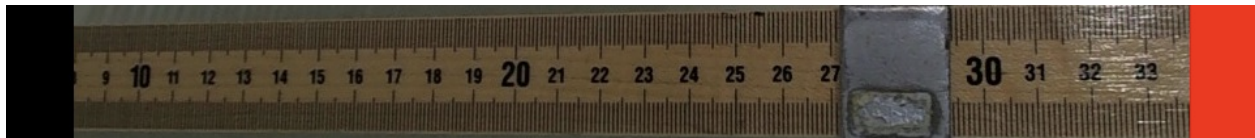


Figure 16: Metric measuring tape with polarity labeled

Our procedure to collect position data begins by setting a reference direction for distance measurements. First we place the end of our measuring tape at a chosen location along the path of the mass-spring system. Next, we pull the tape straight along this path and connect it to the locations where the spring attaches to the apparatus. To make a position measurements, we need to determine the location of a reference position along the tape and mark the location of the center of mass at this position.

The position of each center of mass is given as a signed measurement. These positions are calculated as the number of units between the ground position and the marked location of each center of mass. Notice that in this paradigm, position can be negative if the location of our reference ground location is not the same as the location of the zero end of our measuring tape



Figure 17A: Reference Position

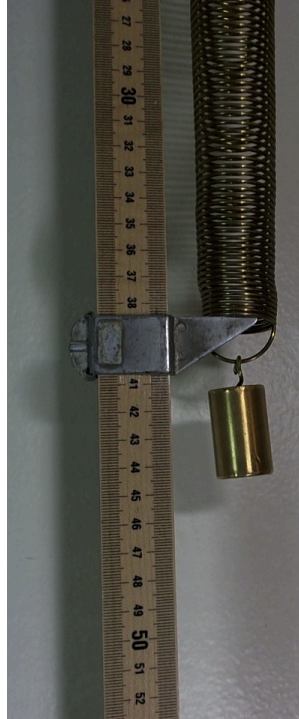


Figure 17B: Positive Points Down



Figure 17C: Positive Points Up

This example illustrates two important concepts. First, we see that position measurements are relative to a chosen reference point. Often in mathematics, we call this reference point the “origin” and assume its value to be zero. However, in an experimental framework, no such assumption is necessary. As long as we clearly indicate our chosen reference, the location of our zero point can be any arbitrary point in our system.

Second, we notice that each position measurement has its own reference direction, also known as the polarity of the measurement system. In other words, for each measured position value corresponding to individual components of our system, we introduce a reference direction for positive and negative values of position. Notice that the exact same displacement measurement has either a positive or negative sign depending on how we orient our measurement instrument. In Figures 17B and 17C, the physical dynamics of the system have not changed one bit. We capture the exact same displacement data in the same experiment. The difference between the two measurements lies in our choice of polarity.

In Figure 17B we decide to measure positive positions in the downward position, producing a reference polarity that is positive in the downward direction. Thus, when gravity acts on the mass attached to the spring, we measure a positive displacement since the movement of the mass is in the same direction of as our chosen polarity. On the other hand, in Figure 17C we decide to measure positive positions in the upwards direction. This reference polarity is the exact negative of our work in 17B since positive measurements are captured in the upward direction. Even though gravity has an identical effect on the mass and the mass does not move in our system, we end up recording a negative displacement in this second measurement paradigm.

This is a very important in specifying the procedure to capture measurements on our device. Not only does the method of connecting our device matter but we also must identify and track the corresponding polarity implicit in our connection. Each variable that we measure produces a signed value with reference to that polarity and we must have access to this information in order to accurately interpret any reported data from the system. The same concepts apply to our measurements of real circuit elements.

Digital Multimeter

To study the behavior of the real devices in this paper, we define and measure only two fundamental quantities: voltage and current. We ignore time in this activity because our voltage and current sources operate in a dc state yielding algebraic rather than differential equations.

With this in mind, we assume that the electrical behavior of each real circuit element is fully determined by the *voltage drop across each element* and the *current running through each element*. We describe each of these fundamental quantities using an *operational definition* that consists of two specifications. First, we indicate the instrument we use to measure each quantity. Second, we provide a detailed procedure for using this instrument to collect individual data readings. To capture all the data measurements we need in this work, we use a powerful modern measurement instrument called a digital multimeter (DMM), shown in Figure 18.



Figure 18: Digital multimeter

This multimeter includes a circular function dial to select the type of measurement we want to make. We find the two probes that are attached to this device; one of these probes is black and the other is red. To make a single measurement, we connect each of these probes to a different point in the circuit. The red probe is associated with a positive reference value of the measurement device while the black probe represents the negative reference value. The motivated reader can reference great tutorials online to learn how to take voltage readings using a multimeter (CITE).

Measuring voltage

To measure the value of the voltage drop across an element, we use an instrument called a voltmeter, which we access by setting the function dial of our multimeter to the voltage reading, which is labeled with a V. Now that we've completed the first step of our operational definition of voltage by specifying a measurement instrument, let's detail the procedure we use to measure the voltage drop across a real circuit element. To do so, we attach the red probe of our multimeter to one lead of the circuit element and the black probe of the lead to the other lead. The element body should sit between these two probes. Assuming that the elements are connected in such a way that we form a complete circuit and that we have dc sources that provide power to the circuit, our voltmeter will display a voltage reading. The digital display of this instrument returns a four-digit number associated with each voltage measurement. This number is the measured value of the voltage drop across our chosen circuit element.

We have now completed our operational definition of the voltage drop across each circuit element. Let's take a look at how to measure voltages on real circuits. Figures 19A and 20A show two different measurements of the voltage drop across our resistor in the circuit from Example 1. In Figure 19A, we attach the red probe of our voltmeter to the top lead of our resistor and the black probe to the bottom lead.

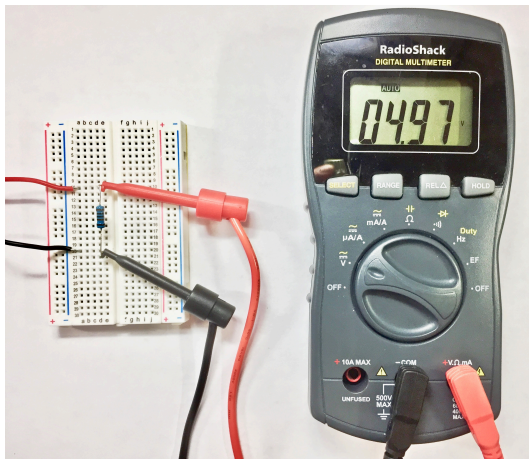


Figure 19A: Photo of voltage drop measurement with positive polarity

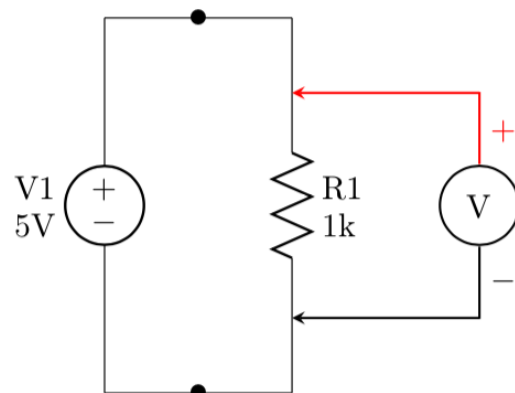


Figure 19B: Schematic of voltage drop measurement with positive polarity

On the other hand, in Figure 20A, we switch the location of our probes. This change produces a different voltage measurement for the same underlying voltage.

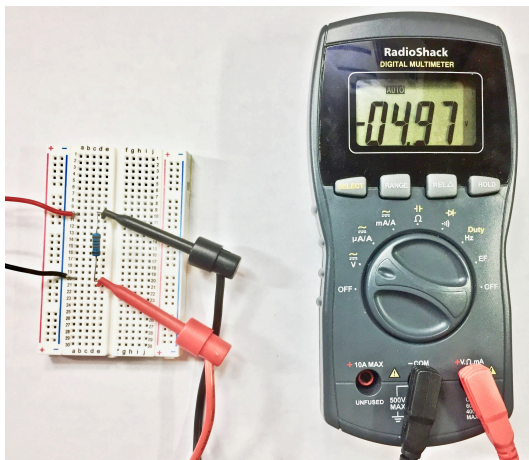


Figure 20A: Photo of voltage drop measurement with negative polarity

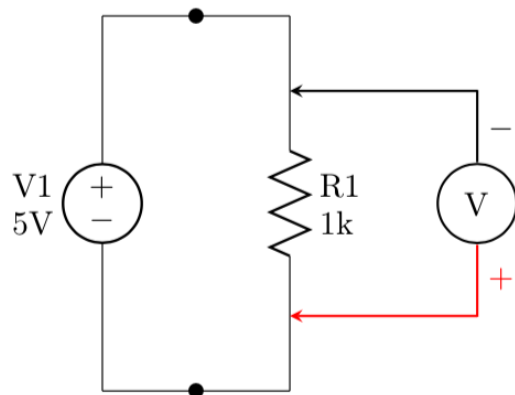


Figure 20B: Schematic of voltage drop measurement with negative polarity

Each voltage measurement contains three unique pieces of information including a nonnegative real number magnitude (such as 4.97), the sign of the measurement (either positive + or negative -) and a unit (such as V). In order to properly interpret the sign of each measurement, we track the reference direction of our measurement.

It is worth noting that the location of the red and black probes on the two leads of our device automatically determine a reference direction for the voltage drop across the element. If the measured value that results from this reading is positive, the voltage drops from the location of the red lead to the black lead by the indicated voltage reading.

In both cases, we measure the exact same underlying quantity and yet we produce two different voltage measurements. We see that regardless of polarity, the magnitude of our measurement is the same. The difference in the two cases lies in the sign. This observation leads to a key idea in circuit theory: for each voltage variable we define in a circuit, we need to specify a reference direction for that variable to indicate the meaning of signed voltages. We revisit this principle of assigning reference directions for voltage drop in our discussion of ideal circuit schematics.

Measuring current

To measure the current running through an element, we use an instrument known as an *ammeter*. The good news is that this functionality is built into the same multimeter we use to measure voltage drops. However, to measure current, we set the dial of the multimeter to the current reading. Note that we measured voltage drops *across* an element by connecting one probe to each lead on either side of the element body. When using an ammeter, we measure the current running *through* a conductor.

The procedure to measure current proceeds by inserting the ammeter in parallel along a specific segment of conductor. More specifically, we disconnect the conductor through which we want to measure the current and imbed the ammeter between this lead and the point to which the lead was previously connected. To do so, we connect the red probe of the ammeter to the free lead of the element. We also attach the black probe to the other point in the circuit to which the lead had previously been linked. The probes of the ammeter now reconnect the circuit and the current flows through the meter. As this happens, a four-digit number associated with each current measurement is broadcast on the digital display of this instrument.

The current measurement is a signed number, meaning it can be positive or negative. As discussed previously, we need to introduce a reference direction to properly interpret signed measurements. For each current measurement we introduce a reference direction using an arrow. The orientation of the probes of the ammeter determines the reference direction completely. The red probe corresponds to tail of the arrow while the black probe is associated with the head of the arrow. The reference direction for current flow moves from the red probe to the black probe.

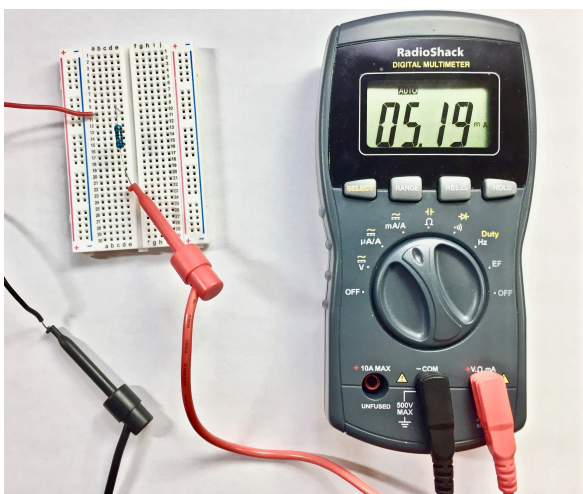


Figure 21A: Photo of current measurement with positive polarity

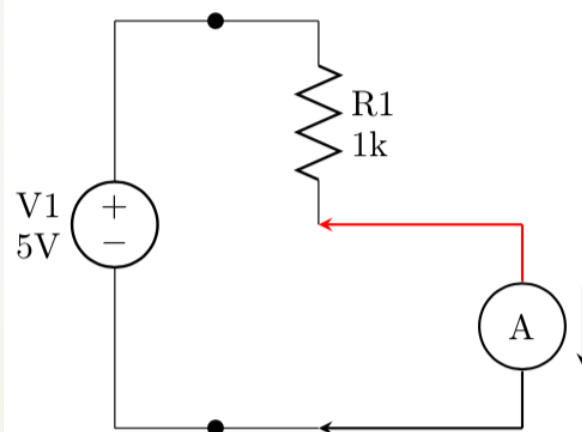


Figure 21B: Schematic of current measurement with positive polarity

On the other hand, if the measurement is negative, the voltage rises from the red lead to the black lead.

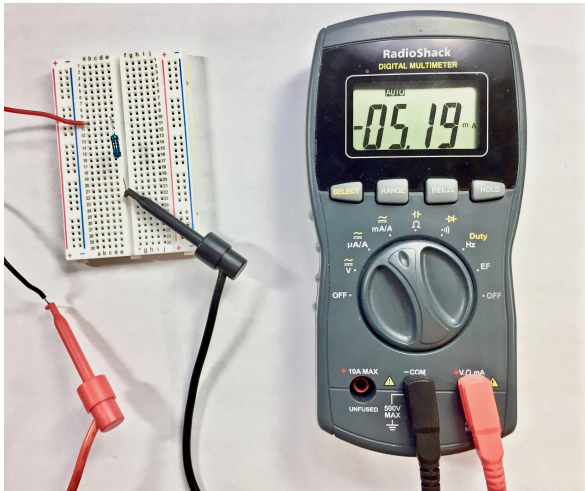


Figure 22A: Positive polarity Measurement

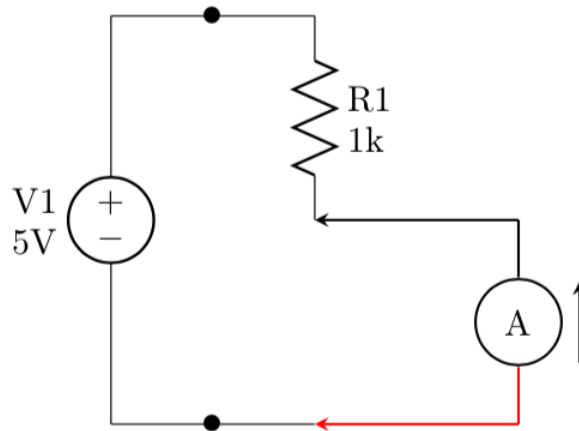


Figure 21B: Schematic of current measurement with negative polarity

Notice that for each and every real circuit element, we measure two quantities: the voltage drop across the element and the current running through each element. Recall that our circuit in example 1 has a total of two circuit elements including one resistor and one voltage source. In this case, we take four separate measurements corresponding to two measurements for each real circuit element.