

## Example 2: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes a single 1k resistor and one dc voltage source. This dc voltage source provides 5V across the leads.

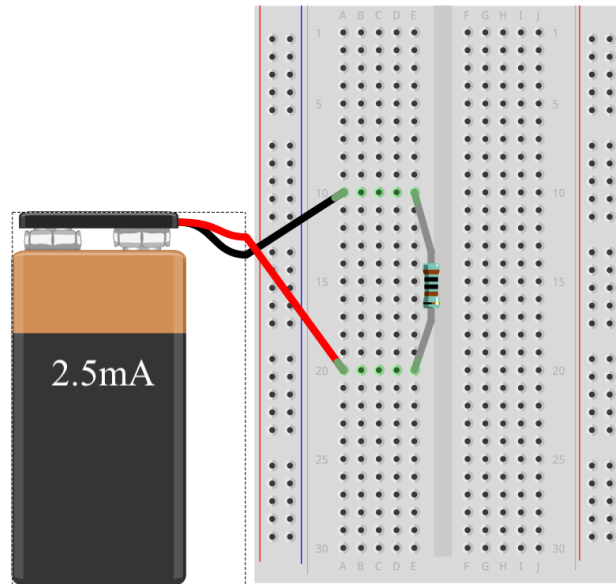


Figure 23: Digital Wiring Diagram for Physical Circuit in Example 2

## Example 2: Ideal schematic diagram

Let's take a look at the schematic diagram for our circuit in example 1, as shown in Figure 15. The ability to translate back and forth between wiring diagrams and schematic diagrams is a valuable skill and takes practice. In the work that follows, we present the background theory and material needed to understand how to develop such a schematic diagram. For now, we encourage the reader to compare and contrast the wiring diagrams in Figures 14A and 14B with the schematic diagram in Figure 15. These diagrams are drawn with the hope that the map between the real circuit elements and the ideal schematics are easy to discover.

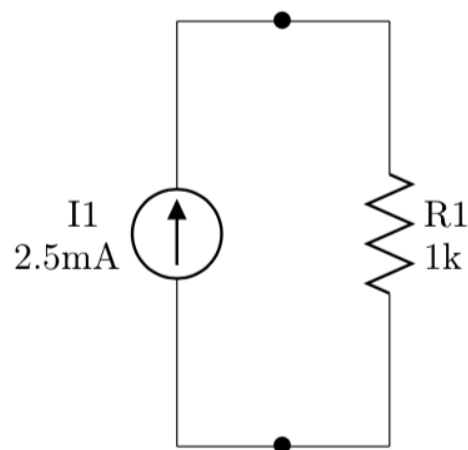


Figure 24: Ideal Circuit Schematic for Circuit in Example 2

## Example 2: Parts You Will Need

To build this circuit, you will need the following parts:

- Half-size 2.2"x 3.4" solderless breadboard
- One 2.5mA dc current sources
- One 1k resistors (brn-brn-blk-blk-brn)
- One fully-charged 9V batteries

## Example 2: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

1.  Disconnect batteries from dc sources
2.  Insert resistor R1 across e10 and e20
3.  Insert red lead of I1 in a20 and blk lead in a10

## Example 2: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the node voltage potentials at all seven nodes of the circuit and record your data in the table below.

Example 1, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
I1	$v_{i_1}$		$i_{i_1}$	

More about how to capture this data is described below. To help you build confidence and check your work, we provide a [YouTube video](#) of our voltage and current measurements for this circuit. This video can be found on the support website for this Electrify the linear-systems problem paper (CITE). We hope these resources help you build confidence in your ability to collect voltage and current measurements. As you get better at measuring node voltage potentials, we cease to provide author-generated data and instead encourage you to use mathematical analysis to validate your measurements.

In examples 1 and 2, we built our first valuable skill in circuit analysis: we now know how to use a digital multimeter to measure both voltage drops across a circuit element and the current running through a particular element. With this in mind, we should make a useful observation. When connecting a dc voltage source to our circuit, the values of the voltage drop across this source is known. In fact, engineers have designed this source to maintain the constant 5V drop across the lead regardless of the current running through the source. On the other hand, the real dc current source maintains a known 2.5mA current running from the red lead to the black lead regardless of the voltage drop across the element. In other words, voltage sources provide known voltage with unknown current. Current sources provide known current with variable, unknown voltage. We revisit this principle later in our work.

## Troubleshooting problems

The process of prototyping real circuits can be quite exciting. By the time that a practicing engineer actually sits down to prototype a proposed design, she has probably spent a number of hours thinking about and actively improving the design features of a circuit to work as intended. Thus, the prototyping stage is usually filled with a type of nervous excitement since this is where the rubber meets the road: our beloved engineer is actually going to figure out whether her initial insights bear fruit.

Just as circuit design is a valuable skill, so too is circuit prototyping. By learning how to connect components to a breadboard, we develop a useful skill that pays dividends for years to come. With this in mind, let's build the expectation now that breadboarding takes focus and attention. Moreover, we have to be willing to do the electrical equivalent of advice found in woodworking: "measure twice and cut once." Even with all of these protections in place, we may still make mistakes when putting components into our breadboard. Thus, there are a number of different checks we can run to trouble shoot a specific circuit. Below, we provide you practical advice on what to do to make sure your circuit works properly and to check for errors that might exist if the circuit you've built is not functioning as intended. These steps are listed in order of increasing complexity. Please start with the earliest steps and work your way down through the list by checking each suggestions.

### Example 3: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes two 1k resistor and one dc current source. This dc current source provides a constant 2.5mA through it's the leads in the direction from the white lead to the black lead.

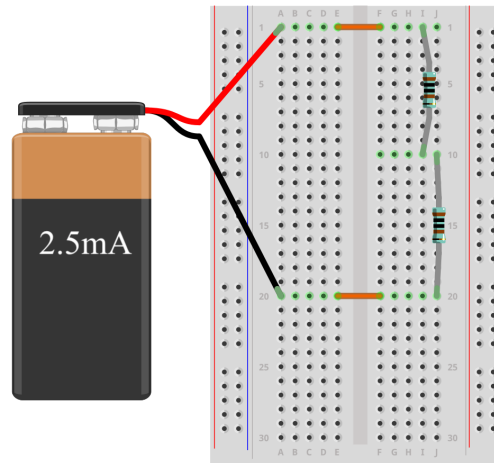


Figure 25: Digital Wiring Diagram for Physical Circuit in Example 3

### Example 3: Ideal schematic diagram

Let's take a look at the schematic diagram for our circuit in example 3, as shown in Figure 25. The ability to translate back and forth between wiring diagrams and schematic diagrams is a valuable skill and takes practice. These diagrams are drawn with the hope that the map between the real circuit elements and the ideal schematics are easy to discover.

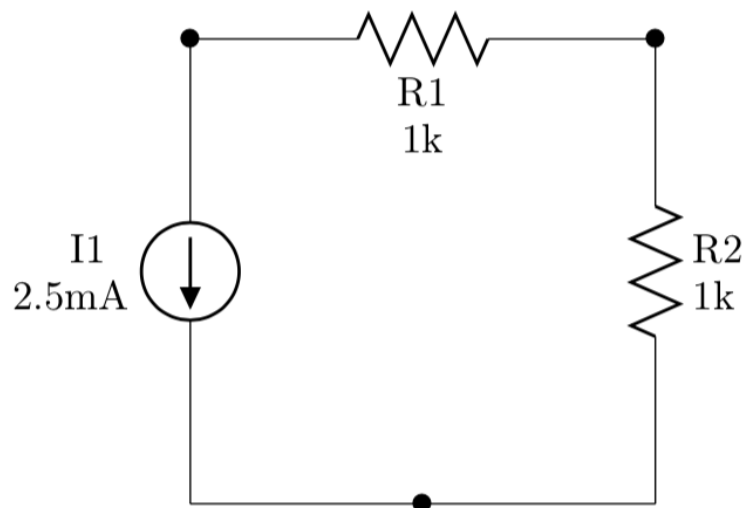


Figure 28: Ideal Circuit Schematic for Circuit in Example 3

### Example 3: Parts You Will Need

To build this circuit, you will need the following parts:

- |   |   |
|---|---|
| <input type="checkbox"/> Half-size 2.2"x 3.4" solderless breadboard | <input type="checkbox"/> One 2.5mA dc current source    |
| <input type="checkbox"/> Two orange 0.3" jumper wires               | <input type="checkbox"/> One fully-charged 9V batteries |
| <input type="checkbox"/> Two 1k resistors (brn-brn-blk-blk-brn)     |   |

### Example 3: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

- |  |   |
|--|---|
| 1. <input type="checkbox"/> Disconnect batteries from dc sources         | 4. <input type="checkbox"/> Insert resistor R1 across i1 and i10              |
| 2. <input type="checkbox"/> Insert orange 0.3" jumper across e1 and f1   | 5. <input type="checkbox"/> Insert resistor R2 across j10 and j20             |
| 3. <input type="checkbox"/> Insert orange 0.3" jumper across e20 and f20 | 6. <input type="checkbox"/> Insert white lead of I1 in a1 and blk lead in a20 |

### Example 3: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 3, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
I1	$v_{i_1}$		$i_{i_1}$	

### Example 4: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes a single 1k resistor and one dc voltage source. This dc voltage source provides 5V across the leads.

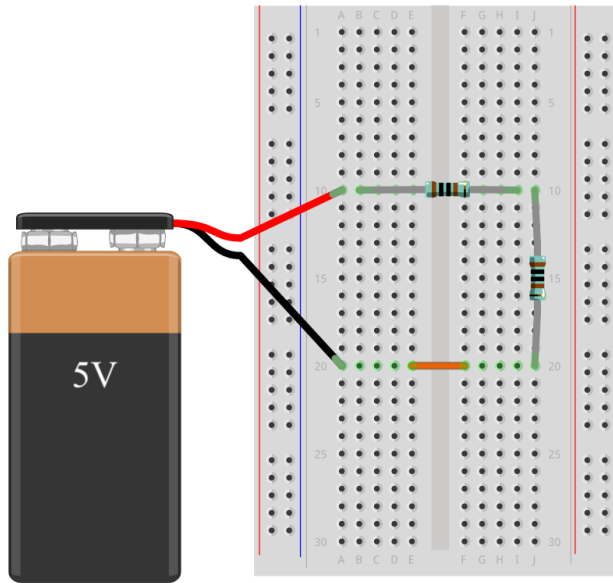


Figure 27: Digital Wiring Diagram for Physical Circuit in Example 4

### Example 4: Ideal schematic diagram

Figure 26 presents a schematic diagram for our circuit in example 3.

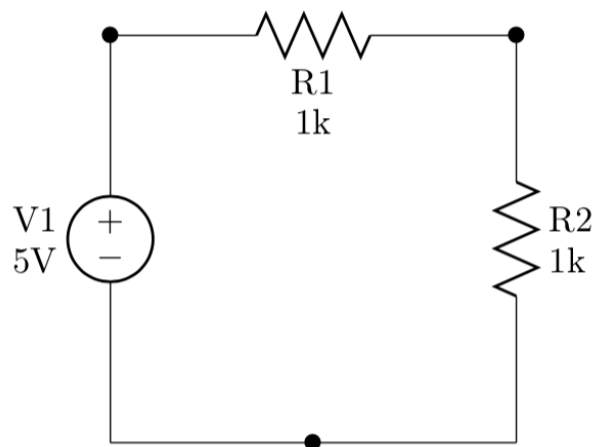


Figure 28: Ideal Circuit Schematic for Circuit in Example 4

### Example 4: Parts You Will Need

To build this circuit, you will need the following parts:

- Half-size 2.2"x 3.4" solderless breadboard
- One 5V dc voltage sources
- One 0.3"(orange) jumper wire
- One fully-charged 9V batteries
- Two 1k resistors (brn-brn-blk-blk-brn)

### Example 4: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

1.  Disconnect batteries from dc sources
2.  Insert orange 0.3" jumper across e20 and f20
3.  Insert resistor R1 across b10 and i10
4.  Insert resistor R2 across j10 and j20
5.  Insert red lead of V1 in a10 and blk lead in a20

### Example 4: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the node voltage potentials at all seven nodes of the circuit and record your data in the table below.

Example 1, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
V1	$v_{v_1}$		$i_{v_1}$	

More about how to capture this data is described below. To help you build confidence and check your work, we provide a [YouTube video](#) of our voltage and current measurements for this circuit. This video can be found on the support website for this Electrify the linear-systems problem paper (CITE). We hope these resources help you build confidence in your ability to collect voltage and current measurements. As you get better at measuring node voltage potentials, we cease to provide author-generated data and instead encourage you to use mathematical analysis to validate your measurements.

### Example 5: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes a single 1k resistor and one dc voltage source. This dc voltage source provides 5V across the leads.

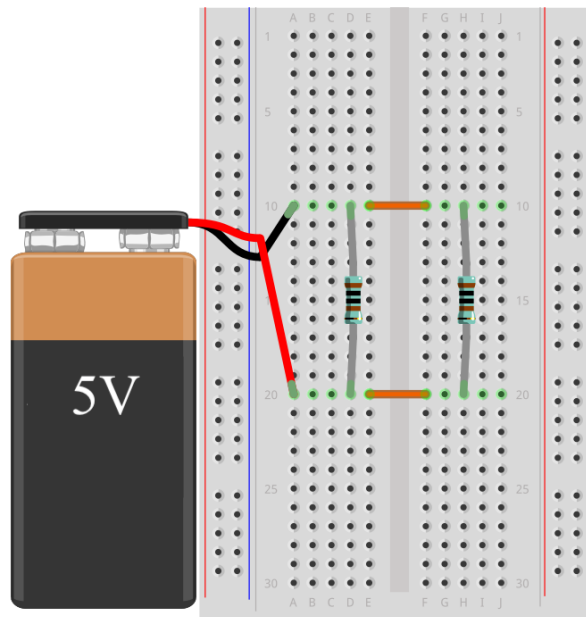


Figure 29: Digital Wiring Diagram for Physical Circuit in Example 5

### Example 5: Ideal schematic diagram

Figure 30 presents a schematic diagram for our circuit in example 5.

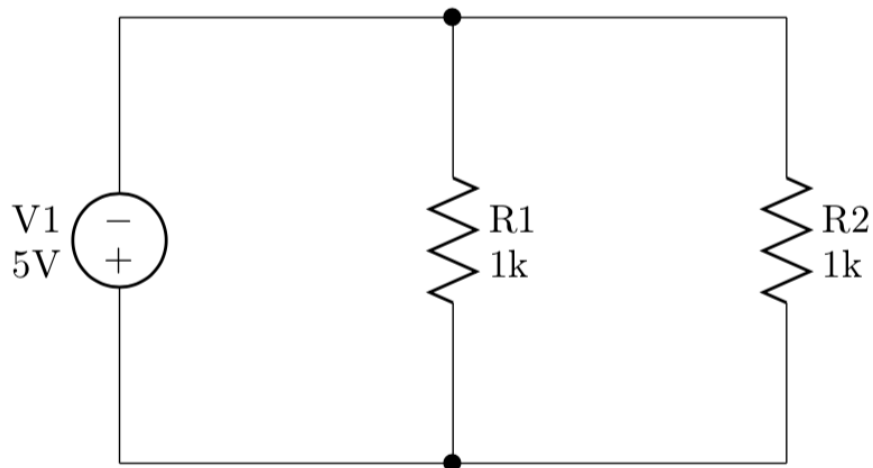


Figure 30: Ideal Circuit Schematic for Circuit in Example 5



### Example 5: Parts You Will Need

To build this circuit, you will need the following parts:

- |   |   |
|---|---|
| <input type="checkbox"/> Half-size 2.2"x 3.4" solderless breadboard | <input type="checkbox"/> One 5V dc voltage source       |
| <input type="checkbox"/> Two orange 0.3" jumper wires               | <input type="checkbox"/> One fully-charged 9V batteries |
| <input type="checkbox"/> Two 1k resistors (brn-brn-blk-blk-brn)     |   |

### Example 5: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

- |  |  |
|--|--|
| 1. <input type="checkbox"/> Disconnect batteries from dc sources         | 4. <input type="checkbox"/> Insert resistor R1 across d10 and d20            |
| 2. <input type="checkbox"/> Insert orange 0.3" jumper across e10 and f10 | 5. <input type="checkbox"/> Insert resistor R2 across h10 and h20            |
| 3. <input type="checkbox"/> Insert orange 0.3" jumper across e20 and f20 | 6. <input type="checkbox"/> Insert red lead of V1 in a20 and blk lead in a10 |

### Example 5: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 5, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
V1	$v_{v_1}$		$i_{v_1}$	

### Example 6: Get Started with a Picture of the Physical Circuit

The picture below provides a diagram of the circuit we will build in this example. This circuit includes a single 1k resistor and one dc voltage source. This dc voltage source provides 5V across the leads.

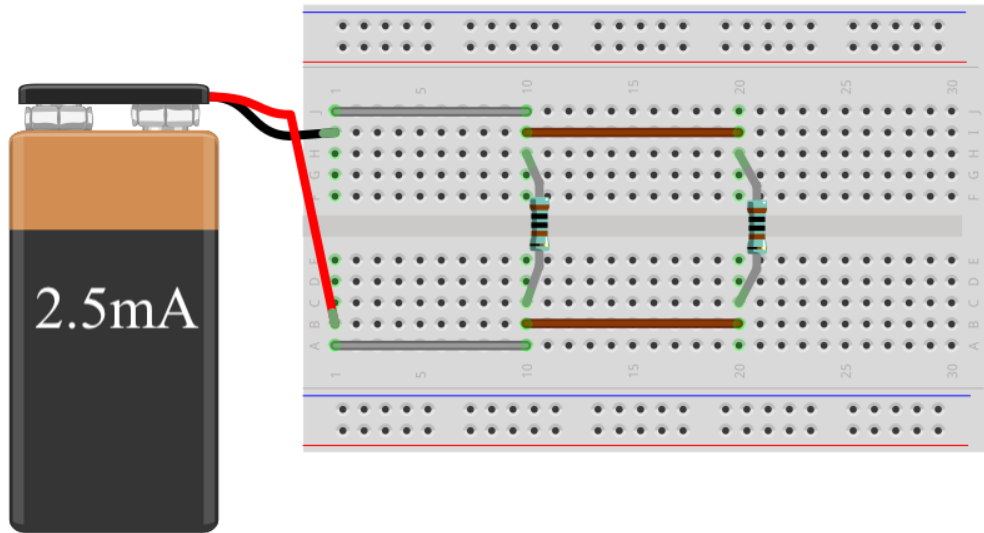


Figure 31: Digital Wiring Diagram for Physical Circuit in Example 6

### Example 6: Ideal schematic diagram

Figure 32 presents a schematic diagram for our circuit in example 6.

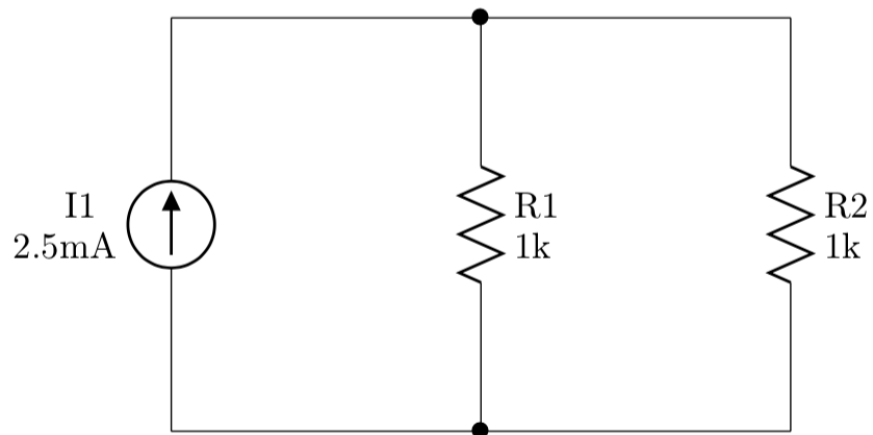


Figure 32: Ideal Circuit Schematic for Circuit in Example 6

### Example 6: Parts You Will Need

To build this circuit, you will need the following parts:

- |   |   |
|---|---|
| <input type="checkbox"/> Half-size 2.2"x 3.4" solderless breadboard | <input type="checkbox"/> Two 1k resistors (brn-brn-blk-blk-brn) |
| <input type="checkbox"/> Two grey 0.8" jumper wires                 | <input type="checkbox"/> One 2.5mA dc current source            |
| <input type="checkbox"/> Two brown 1.0" jumper wires                | <input type="checkbox"/> One fully-charged 9V batteries         |

### Example 6: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

- |   |  |
|---|--|
| 1. <input type="checkbox"/> Disconnect batteries from dc sources        | 5. <input type="checkbox"/> Insert brown 1.0" jumper across i10 and i20      |
| 2. <input type="checkbox"/> Insert grey 0.8" jumper across a1 and a10   | 6. <input type="checkbox"/> Insert resistor R1 across c10 and h10            |
| 3. <input type="checkbox"/> Insert grey 0.8" jumper across j1 and j10   | 7. <input type="checkbox"/> Insert resistor R2 across c20 and h20            |
| 4. <input type="checkbox"/> Insert brown 1.0" jumper across b10 and b20 | 8. <input type="checkbox"/> Insert white lead of I1 in b1 and blk lead in i1 |

### Example 6: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 6, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
I1	$v_{i_1}$		$i_{i_1}$	

## Canonical ideal circuit elements

For our purposes, we present three ideal circuit elements corresponding to the three types of real devices we use to build physical circuits. These include linear resistors, dc voltage sources, and dc current sources. All three of these ideal devices are created from a generic template for a canonical ideal circuit element, seen in Figure 27.

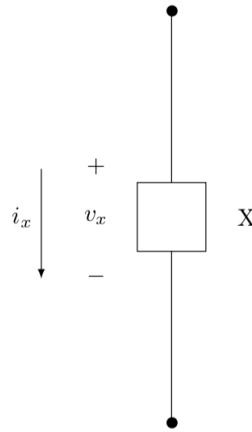


Figure 27: Template for ideal circuit element

This template has four major features. First we begin by drawing a stylized sketch, sometimes called a *schematic*, which is a pictorial representation of the ideal device. Second, we assign a unique label and an associated value to each circuit element. Such a label is also known as a *component designator* or *reference designator*. In our template, we've labeled our element  $X$ . Third, for each circuit element we define two signed variables that represent the voltage across and the current running through each element. Finally, we adjoin a set of mathematical equations that describe the relationship between our two signed variables. These equations are known as *branch constitutive relations* (BCR) and are ideal mathematical descriptions of the electrical properties observed in the associated real circuit device to be modeled using each ideal element.

Let's begin our discussion by focusing on the first of these features. When sketching an ideal element, we always use the same format which can be roughly stated as line, body, line. The empty box in between the two lines is called the *element body*. For each of the three circuit elements considered here, the element body has a specific shape determined by well-established conventions in electrical engineering (CITE), see Figures 9C, 11C, and 12C. The two lines on either side of the empty rectangle are known as the leads of this element (pronounced "leads"). These lines are meant to symbolize wires that connect real circuit elements together in a physical circuit.

We assume these leads are ideal conductors that allow current to flow with no resistance. In other words, the only mathematical role of these leads is to connect ideal elements together. With this singular purpose in mind, we assume that the shape of the leads does not affect circuit behavior. We also assume that we can bend these leads however we want or draw the leads to be any length without affecting the electrical properties of a circuit. The small black dots at the end of the leads are called terminals. These terminals are the locations where we attach the leads to other ideal elements to form a circuit. In our work, we restrict ourselves to the use of *two-terminal elements*, which are ideal elements that have exactly two leads.

Let's move onto to discussing the reference designators and associated values. When we define and label a circuit element, each part is referenced using a letter and number combination (R1, V3, I2). The letter indicates the type of component we use and the number uniquely identifies each individual component of this type that is used in the design. In our work, we use the standard letter codes for resistors, voltage sources, and current sources, as seen in Table 1 below. To ensure readability and ease of reference, each component designator appears next to the associated component.

Letters for component labels

<b>Letter Code</b>	<b>Element Type</b>
R	Resistor
V	Voltage Source
I	Current Source

Let's look back at Figure 26 that presents our circuit schematic for Example 3 and notice that each component designator also includes an associated value. More specifically, our first and only voltage source V1 in Figure 26 has an associated value of 5V while our second resistor R2 is a 1k resistor. These values signal the exact parts we use when building the circuit and are crucial for modeling said parts using the appropriate BCR, as discussed below.

### Example 7: Get Started with a Picture of the Physical Circuit

In Example 7, we build a circuit that includes a three 1k resistor and one dc voltage source in series, as indicated in Figure 33 below.

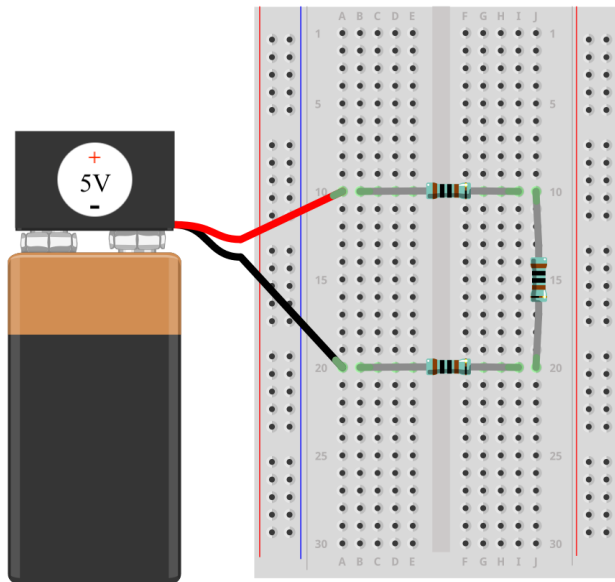


Figure 33: Digital Wiring Diagram for Physical Circuit in Example 7

### Example 7: Ideal schematic diagram

Figure 34 presents a schematic diagram for our circuit in example 7.

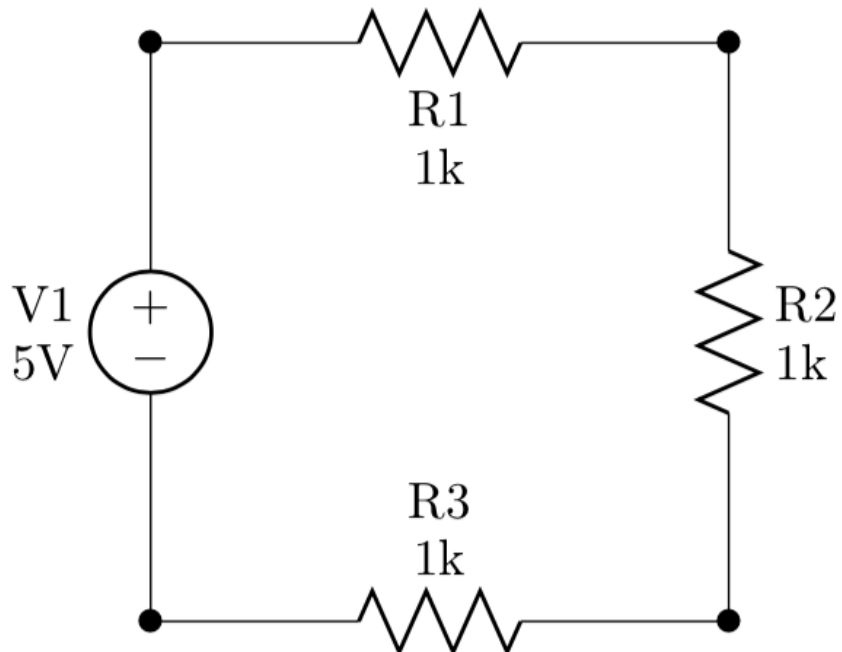


Figure 34: Ideal Circuit Schematic for Circuit in Example 7

### Example 7: Parts You Will Need

To build this circuit, you will need the following parts:

- |   |   |
|---|---|
| <input type="checkbox"/> Half-size 2.2"x 3.4" solderless breadboard | <input type="checkbox"/> One 5V dc voltage source       |
| <input type="checkbox"/> Three 1k resistors (brn-brn-blk-blk-brn)   | <input type="checkbox"/> One fully-charged 9V batteries |

### Example 7: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

- |   |  |
|---|--|
| 1. <input type="checkbox"/> Disconnect battery from dc source     | 4. <input type="checkbox"/> Insert resistor R3 across b20 and i20            |
| 2. <input type="checkbox"/> Insert resistor R1 across b10 and i10 | 5. <input type="checkbox"/> Insert red lead of V1 in a10 and blk lead in a20 |
| 3. <input type="checkbox"/> Insert resistor R2 across j10 and j20 |  |

### Example 7: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 7, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
R3	$v_{r_3}$		$i_{r_3}$	
V1	$v_{v_1}$		$i_{v_1}$	

Example 7, Table 2:  
Node potential measurements

Node	Node Variable	Measured value (V)
1	$u_1$	
2	$u_2$	
3	$u_3$	
4	$u_4$	

**Example 7: Create a Digraph Model of the Circuit**

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**Example 7: Write the Voltage Drop Variables in Terms of Node Potentials**

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**Example 7: Identify the Currents Entering/Leaving Each Node**



### Example 8: Get Started with a Picture of the Physical Circuit

In Example 8, we build a circuit that includes a three 1k resistor and one dc voltage source in series, as indicated in Figure 33 below.

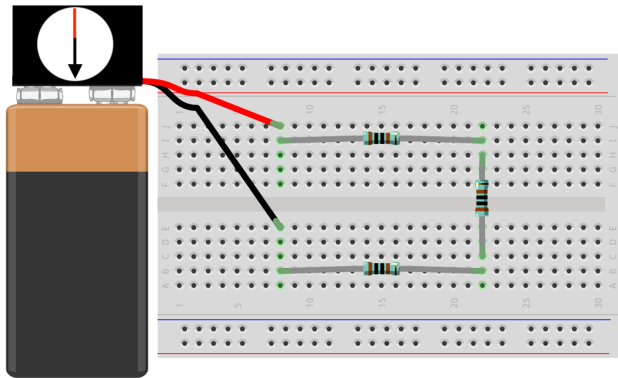


Figure 35: Digital Wiring Diagram for Physical Circuit in Example 8

### Example 8: Ideal schematic diagram

Figure 36 presents a schematic diagram for our circuit in Example 8.

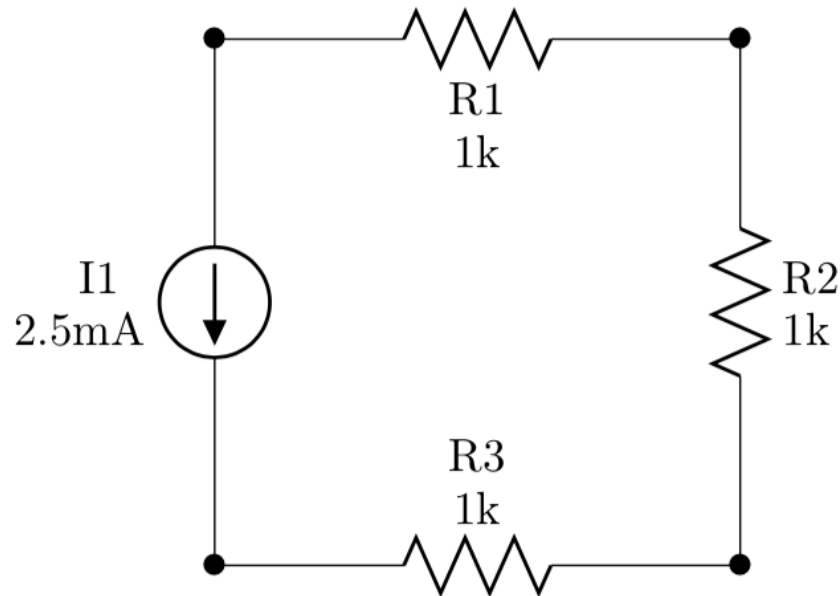


Figure 36: Ideal Circuit Schematic for Circuit in Example 8

### Example 8: Parts You Will Need

To build this circuit, you will need the following parts:

- |   |   |
|---|---|
| <input type="checkbox"/> Half-size 2.2"x 3.4" solderless breadboard | <input type="checkbox"/> One 5V dc current source       |
| <input type="checkbox"/> Three 1k resistors (brn-brn-blk-blk-brn)   | <input type="checkbox"/> One fully-charged 9V batteries |

### Example 8: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

- |   |  |
|---|--|
| 1. <input type="checkbox"/> Disconnect battery from dc source     | 4. <input type="checkbox"/> Insert resistor R3 across b8 and b22             |
| 2. <input type="checkbox"/> Insert resistor R1 across i8 and i22  | 5. <input type="checkbox"/> Insert white lead of I1 in j8 and blk lead in e8 |
| 3. <input type="checkbox"/> Insert resistor R2 across h22 and c22 |  |

### Example 8: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 8, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
R3	$v_{r_3}$		$i_{r_3}$	
I1	$v_{i_1}$		$i_{i_1}$	

Example 8, Table 2:  
Node potential measurements

Node	Node Variable	Measured value (V)
1	$u_1$	
2	$u_2$	
3	$u_3$	
4	$u_4$	

**Example 8: Create a Digraph Model of the Circuit**

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**Example 8: Write the Voltage Drop Variables in Terms of Node Potentials**

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**Example 9: Identify the Currents Entering/Leaving Each Node**

### Example 9: Get Started with a Picture of the Physical Circuit

In Example 9, we build a circuit that includes a three 1k resistor and one dc voltage source in series, as indicated in Figure 33 below.

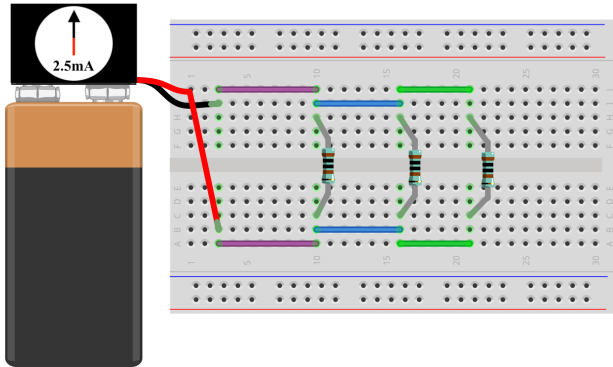


Figure 37: Digital Wiring Diagram for Physical Circuit in Example 9

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### Example 9: Ideal schematic diagram

Figure 38 presents a schematic diagram for our circuit in Example 9.

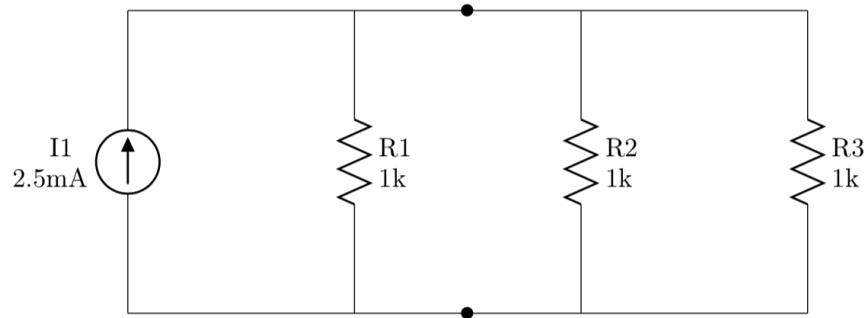


Figure 38: Ideal Circuit Schematic for Circuit in Example 9

### Example 9: Parts You Will Need

To build this circuit, you will need the following parts:

- Half-size 2.2"x 3.4" solderless breadboard
- Two violet 0.7" jumper wires
- Two blue 0.6" jumper wires
- Two green 0.5" jumper wires
- Three 1k resistors (brn-brn-blk-blk-brn)
- One 5V dc current source
- One fully-charged 9V batteries

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### Example 9: Build the Circuit

To build the circuit, grab the half-size solderless breadboard. Then, follow the instructions below:

1.  Disconnect battery from dc source
2.  Insert violet 0.7" jumper across a3 and a10
3.  Insert violet 0.7" jumper across j3 and j10
4.  Insert blue 0.6" jumper across b10 and b16
5.  Insert blue 0.6" jumper across i10 and i16
6.  Insert green 0.5" jumper across a16 and a20
7.  Insert green 0.5" jumper across j16 and j20
8.  Insert resistor R1 across c10 and h10
9.  Insert resistor R2 across c16 and h16
10.  Insert resistor R3 across c20 and h20
11.  Insert white lead of I1 in b3 and blk lead in i3

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### Example 9: Create a Digraph Model of the Circuit

### Example 9: Test the Circuit

Before connecting the 9V batteries to the dc source, please check the circuit for errors. Once you're sure that all the components are properly installed, connect the batteries to the dc sources and be careful that the leads of the sources stay firmly in place on the breadboard. Now, use a multimeter to measure the values for all circuit variables.

Example 9, Table I: Observed Circuit Variable Values

Circuit Element	Voltage Variable	Measured voltage (V)	Current Variable	Measured current (mA)
R1	$v_{r_1}$		$i_{r_1}$	
R2	$v_{r_2}$		$i_{r_2}$	
R3	$v_{r_3}$		$i_{r_3}$	
I1	$v_{i_1}$		$i_{i_1}$	

Example 9, Table 2:  
Node potential measurements

Node	Node Variable	Measured value (V)
1	$u_1$	
2	$u_2$	

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**Example 9: Write the Voltage Drop Variables in Terms of Node Potentials**

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**Example 9: Identify the Currents Entering/Leaving Each Node**

## Branch Constituent Relations Experiment

Remember that resistors resist the flow of electrical current. Let's study the

### Circuit elements and their equations

In addition to a stylized sketch, our generic template for a canonical ideal circuit element also includes two other features which are a collection of mathematical variables and corresponding equations, so-called BCR's. Thus, the mathematical variables we assign to each ideal element should mirror the specific measurements we collect from each device in our physical circuit. Next, let's move onto marking each element with an individual identifier.

One useful habit is to assign a unique label for each branch in our circuit. In the template for our canonical element, the lower case letter  $x$  to the right of the element body plays the role of a general label. When working with an ideal circuit that has a numerous elements of the same type as well as multiple types of elements, element labels encode two different categories of information: element type and enumeration within type. Such labels include both lower case letters and a numeric subscript. The lower case letters specify the type of ideal element and the numeric subscript counts the number of unique instances that occur within each element type.

In addition to the element label, we also define the signed variables  $v_x$  and  $i_x$  to each ideal element. In our analysis of electric circuits, we assume that the electrical behavior of each ideal circuit element is completely described by two mathematical variables. The variable  $v_x$  represents the voltage drop across that element. At any point in time, to calculate the value of  $v_x$  we take the difference between the voltage potentials of each lead on either side of the circuit element. The calculated value of our voltage drop  $v_x$  is a signed quantity and may be negative, zero or positive. The reference direction for the voltage drop across the end terminals occurs in the direction starting from the  $+$  node and going to the  $-$  node. Using this general template, let's build a library of ideal elements corresponding to specific real circuit devices.

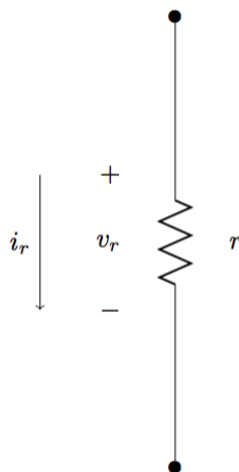


Figure 20A: Ideal linear resistor

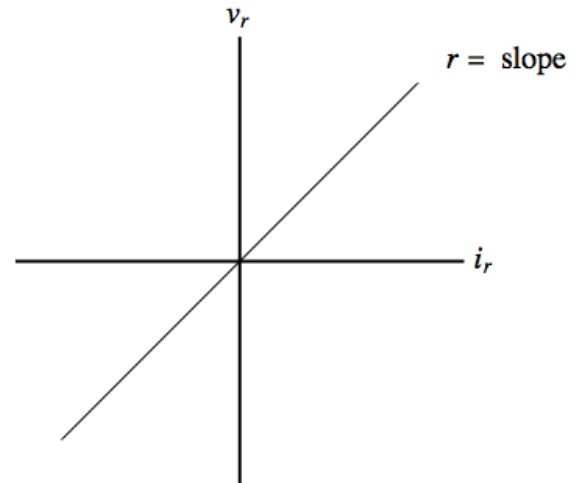


Figure 20B: Graph of branch constitutive relation for resistor

Figure 2B shows the ideal circuit schematic used to model the electric behavior of the corresponding real resistor from Figure 2A. The circuit symbol  $r$  attached to this ideal element plays two roles. First, this symbol acts as a label for our element. In a circuit with many resistors, we often include an enumerated subscript like  $r_1$  or  $r_6$ , representing the first and sixth resistor in our circuit, respectively. The second use of this label is to encode the resistance value of the resistor. In the case of the precision resistor, we have the  $r = 1\text{k}\Omega$ . In general, these values are encoded using the color bands on each resistor in the circuit. Figure 2C presents a graphical representation of the relationship between current running through a resistor and the



voltage drop across the resistor. *Resistance* is a measurable electrical property of materials that quantifies how much the material impedes the flow of electric current. We use resistors to set voltage levels or to limit current flow at different points in a circuit.

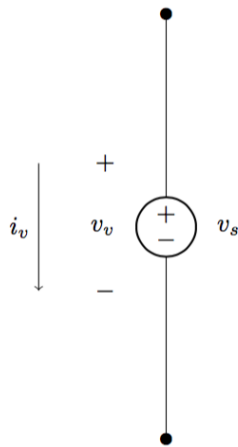


Figure 21A: Ideal dc voltage source

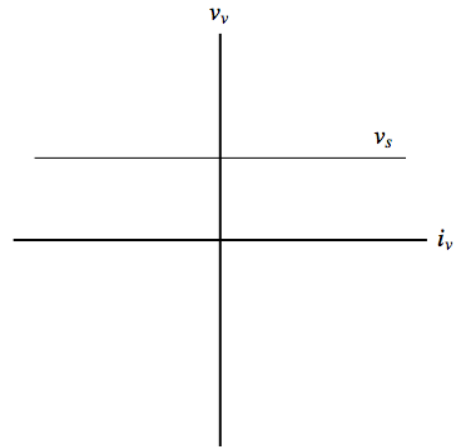


Figure 21B: Branch constitutive relation for voltage source

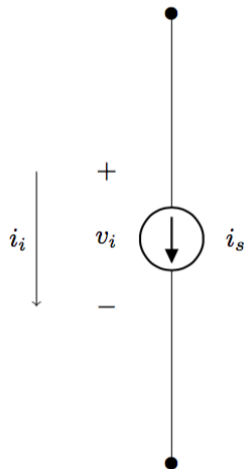


Figure 22A: Ideal dc current source

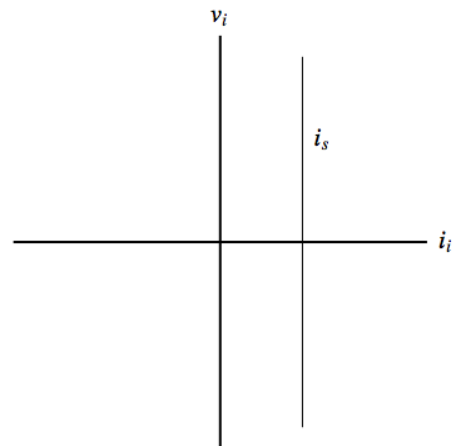


Figure 22B: Branch constitutive relation for current source

## Kirchoff's Current Law Experiment

Remember that resistors resist the flow of electrical current. Let's study the

## Loop Version of Kirchoff's Voltage Law Experiment

Remember that resistors resist the flow of electrical current. Let's study the

### The nodes of an ideal circuit

An *ideal circuit* is a collection of ideal elements that are linked together so that each lead of an element is connected to the lead of at least one other element. Figure 18A presents an example of a circuit including 12 two-terminal elements. On the other hand, Figure 18B illustrates a set of elements that does not form a circuit since the element on the very bottom of this diagram has a detached lead that is not connected to the lead of any other element.

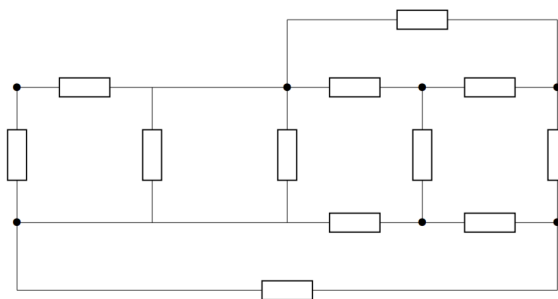


Figure 18A: An ideal circuit with 12 two-terminal elements

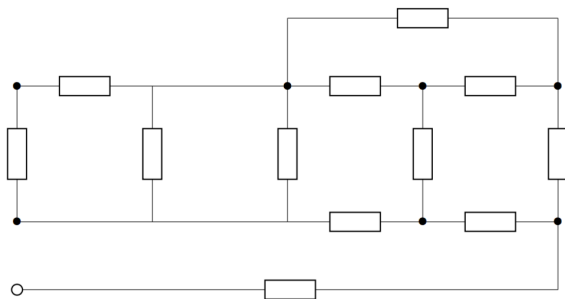


Figure 18B: A group of elements that does not form a circuit

When considering the leads of a given element, the only detail that matters is whether or not the specific lead in question makes contact with another lead of a different element. Ideal circuit schematics, such as the one seen in Figure 18, provide a detailed map of the interconnections between the leads of each element, known as the *topology* of the circuit. To completely specify these interconnections, we identify all locations where the leads of the individual elements in the circuit are joined together.

A circuit's *nodes* are the individual connected segments of wire where the terminals of two or more circuit elements make contact. Because we assume that the leads of each element are ideal conductors with zero resistance that model physical wires, we can think of a node as the collections of all fragments of wire that sit between element bodies. A powerful heuristic to identify the nodes of a circuit is to erase all the element bodies in a ideal circuit diagram without erasing any part of the leads. The connected chunks of wire that remain after erasing the element bodies are the nodes of the circuit. In Figure 19, we demonstrate how to identify the nodes of the circuit shown in Figure 2A by erasing all element bodies in this circuit.

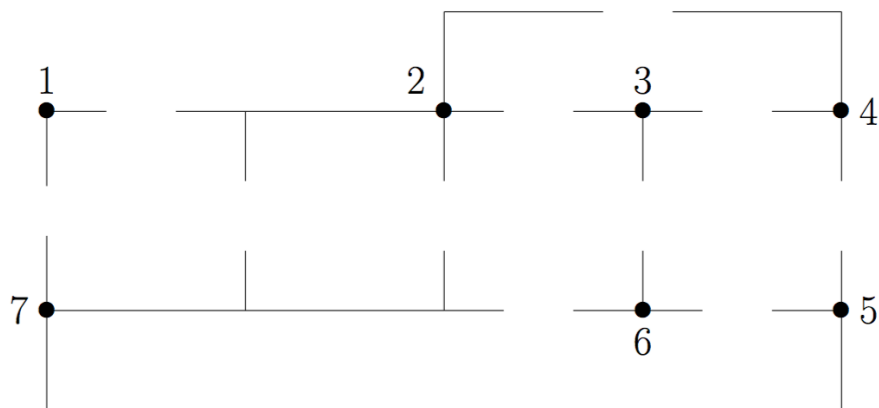


Figure 19: Heuristic to identify the nodes of circuit in Figure 2A

To define the fundamental variables that govern circuit behavior, we refer back to the nodes of the circuit. Thus, we find it advantageous to enumerate these nodes for a shared sense of circuit topology. In Figure 3, we choose an arbitrary enumeration scheme. The only enumeration rule that matters is to assign each and every node a unique identifier. Enumerating the nodes with positive integers sets us up nicely to model this circuit using matrices because the row and column indices for matrix entries follow the same pattern. Finally, the natural numbers  $m$  represent the total number of elements in our ideal circuit. The positive integers  $n_g$  counts the total number of nodes in our circuit. This notation is special: the lower case  $n$  is suggestive of the idea that we are counting nodes and the inclusion of the subscript  $g$  is intended to indicate that the ground node included in our count. Based on our work in Figure 3, we see that the circuit in Figure 2A has  $m = 12$  branches and  $n_g = 7$  nodes.

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**Draw the circuit schematic and label variables**