

## Math 2B: Applied Linear Algebra

**True/False** For the problems below, circle T if the answer is true and circle F is the answer is false.

1.    ☒ T    F    The transpose of a matrix unit is also a matrix unit.

2.    T    ☒ F    Suppose  $\mathbf{x} \in \mathbb{R}^n$  and  $\mathbf{b} \in \mathbb{R}^m$ . The matrix given by the outer product  $\mathbf{x}\mathbf{b}^T$  is an  $m \times n$  matrix.

3.    ☒ T    F    The transpose of a square  $n \times n$  shear matrix  $S_{ik}(c)$  is also a shear matrix given by  $S_{ki}(c) = (S_{ik}(c))^T$ .

4.    ☒ T    F    Let  $\mathbf{e}_j = I_n(:, j) \in \mathbb{R}^n$  be the  $j$ th column of the identity matrix  $I_n \in \mathbb{R}^{n \times n}$  for all  $j = 1, 2, \dots, n$ . If  $i, k \in [n]$  with  $i \neq k$ , then the shear matrix

$$S_{ik}(c) = I_n + c \mathbf{e}_i \mathbf{e}_k^T.$$

5.    ☒ T    F    Let  $\mathbf{e}_k = I_n(:, k) \in \mathbb{R}^n$  be the  $k$ th column of the identity matrix  $I_n \in \mathbb{R}^{n \times n}$  for all  $k = 1, 2, \dots, n$ . If  $j \in \{1, 2, 3, \dots, n\}$ , then the dilation matrix

$$D_j(c) = I_n + (c - 1) \mathbf{e}_j \mathbf{e}_j^T$$

6.    ☒ T    F    For matrices in  $\mathbb{R}^{4 \times 4}$ ,  $D_3(6) - D_3(5) = \mathbf{e}_3 \cdot \mathbf{e}_3^T$

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## Multiple Choice

 For the problems below, circle the correct response for each question.

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1. If  $A \in \mathbb{R}^{4 \times 6}$ , how many rows does the matrix  $A^T$  have?

- A. 4                      **B. 6**                      C. 0                      D. 1                      E. None of these.
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2. Consider the following expression:

$$\begin{bmatrix} 9 & 5 & 3 \\ 8 & 0 & 2 \\ 7 & -6 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} -5 & 2 & 0 \end{bmatrix}$$

Using the properties of matrix-matrix multiplication and matrix-matrix addition, which of the following represents the given expression:

- A.  $\begin{bmatrix} 9 & 5 & 3 \\ 8 & 0 & 2 \\ 7 & -6 & 1 \end{bmatrix}$     B.  $\begin{bmatrix} 3 & 5 & 3 \\ -2 & 4 & 2 \\ 7 & -6 & 1 \end{bmatrix}$     C.  $\begin{bmatrix} 9 & 5 & -3 \\ 18 & 4 & 2 \\ 7 & -6 & 1 \end{bmatrix}$     D.  $\begin{bmatrix} 9 & 5 & -3 \\ -2 & -4 & 2 \\ 7 & -6 & 1 \end{bmatrix}$     **E.  $\begin{bmatrix} 9 & 5 & -3 \\ -2 & 4 & 2 \\ 7 & -6 & 1 \end{bmatrix}$**
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3. Suppose that  $\mathbf{e}_k \in \mathbb{R}^3$  is the  $3 \times 1$  elementary basis vector with  $\mathbf{e}_k = I_3(:, k)$  for  $k = 1, 2, 3$ . Let

$$A = -2 \cdot \mathbf{e}_3 \cdot \mathbf{e}_1^T + 4 \cdot \mathbf{e}_2 \cdot \mathbf{e}_2^T + 3 \cdot \mathbf{e}_3 \cdot \mathbf{e}_3^T - \mathbf{e}_1 \cdot \mathbf{e}_2^T$$

Then, which of the following gives  $A(:, 2) \cdot A(1, :)$ ?

- A.  $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$                       B. 4                      C. 1                      **D.  $\begin{bmatrix} 0 & 1 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$**                       E.  $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
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4. Define the matrix  $B \in \mathbb{R}^{3 \times 3}$  as a sum of elementary matrices given by

$$B = D_1(2) + S_{21}(2) + S_{31}(3) - S_{13}(-4).$$

Which of the following matrices is equivalent to  $B$ ?

- A.  $\begin{bmatrix} 3 & 0 & 4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$**                       B.  $\begin{bmatrix} 2 & 0 & 4 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$                       C.  $\begin{bmatrix} 2 & 0 & -4 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$                       D.  $\begin{bmatrix} 4 & 0 & 4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$                       E.  $\begin{bmatrix} 3 & 0 & -4 \\ 2 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$