### 3.2 Anatomy of Matrices

By identifying special structure in the entries of a matrix, we can develop fast and efficient solutions to our four fundamental problems. In this section, we study a number of important features of matrix notation and categorize powerful patterns in the anatomy of matrices.

For $m, n \in \mathbb{N}$, a rectangular matrix $A \in \mathbb{R}^{m \times n}$ is an array of real numbers organized into $m$ rows and $n$ columns. Because the row dimension $m$ and the column dimension $n$ are both natural numbers, the following three options exist:
i. A matrix is tall and narrow if $m>n$. All tall and narrow matrices have more rows than columns.

$$
A=\left[\begin{array}{ccc}
a_{11} & \cdots & a_{1 n} \\
a_{21} & \cdots & a_{2 n} \\
\vdots & \ddots & \vdots \\
a_{n 1} & \cdots & a_{n n} \\
\vdots & \ddots & \vdots \\
a_{m 1} & \cdots & a_{m n}
\end{array}\right]
$$

ii. A matrix is square if $m=n$. All square matrices have the same number of rows as columns.

$$
A=\left[\begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1 n} \\
a_{21} & a_{22} & \cdots & a_{2 n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n 1} & a_{n 2} & \cdots & a_{n n}
\end{array}\right]
$$

iii. A matrix is short and wide if $m<n$. All short and wide matrices have less rows than columns.

$$
A=\left[\begin{array}{cccccc}
a_{11} & a_{12} & \cdots & a_{1 m} & \cdots & a_{1 n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
a_{m 1} & a_{m 2} & \cdots & a_{m m} & \cdots & a_{m n}
\end{array}\right]
$$

## EXAMPLE 3.2.1

Any graph, directed or not, with more nodes than edges will give rise to an incidence matrix that is tall and narrow. For example, consider th directed graph with three nodes and two edges given below.


The corresponding $4 \times 5$ incidence matrix is given by

$$
A=\left[\begin{array}{rrrrr}
1 & -1 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & -1
\end{array}\right]
$$

This matrix has more rows than columns.

## EXAMPLE 3.2.2

Vandermonde matrix with more discrete points than the degree of the polynomial (introduction to the least squares problem).

## EXAMPLE 3.2.3

Incidence matrix for a digraph with more edges than nodes

## EXAMPLE 3.2.4

Short and wide vertex matrix for a 3D polygon
We pay special attention to square matrices when considering the square linearsystems problem and the eigenvalue problem. The general linear-systems problem usually involves short and wide matrices while the full-rank least-square problem focuses on tall and narrow matrices. All three types of matrices arise in the matrixvector multiplication problem, though square matrices have special properties. One of the first steps to craft one of the four fundamental problems from a modeling context is to create a matrix and connect the physical model to the proper matrix equation. In any case, the shape of the matrix may provide guide you in deciding which techniques are applicable to solve your problem. Hence, it is very helpful to immediately identify the shape of any matrix that is written in a matrix equation.

## Subscripts on Matrices

Matrix notation is used to compress information. When referring to matrix $A \in$ $\mathbb{R}^{m \times n}$, we are actually referring to $m \cdot n$ separate scalar-valued entries. Each of these elements has a unique row and column index and is organized into our rectangular array. The power of matrix notation is to suppress all of this information and allow us to quickly encapsulate algebraic relations on the entire system with a limited number of symbols. For example, in the matrix-vector multiplication problem, we write only a few symbols $A \cdot \mathbf{x}=\mathrm{b}$ to state the entire problem. Underneath this notation, we encode the entire modeling scheme and execute many simultaneous scalar operations with very specific structure. Compressing information in this way is extremely powerful when used appropriately but can lead to confusion for the untrained observer.

With this in mind, you should get in the habit of specifically identifying the row and column dimensions of every matrix and vector you see in a matrix equation. This can be done using subscript notation. For example, a shorthand way to specify that $A$ has $m$ rows and $n$ columns is to write $A_{m \times n}$. The same subscript notation can be helpful to write dimensions on the full array.


Of course, subscript notation should not be a substitute for the matrix definition using set theory $A \in \mathbb{R}^{3 \times 4}$. However, this can be a convenient tool when analyze a matrix equation. Seasoned linear algebraists usually identify the dimensions of matrices as a pre-requisite to analyzing any matrix equation.

## Entries of a Matrix

A zero entry of a matrix is an entry $a_{i k}=0$ while a nonzero entry is given by $a_{i k} \neq 0$. By studying patterns in the locations of zero and nonzero entries, we can customize our solution techniques to exploit the structure of a given matrix.

## Definition 3.5: Leading Entry

The leading entry of a row of a matrix is the first nonzero element in that row when reading from left to right.

We use leading entries in our solution techniques for linear-systems problems, least-squares problems and eigenvalue problems.

## EXAMPLE 3.2.5

Identify the leading entry of row 4 for a incidence matrix of a given di-graph.
The total number of elements of our matrix is given by

$$
\operatorname{numel}(A)=m n
$$

since there are $m$ rows each of which contains $n$ entries. The total number of nonzero entries in a matrix, denoted as $\operatorname{nnz}(A)$, is the number of nonzero entries of our matrix. To count the total number of zero entries of a matrix, we calculate

$$
\operatorname{numel}(A)-\operatorname{nnz}(A)
$$

## $1-\star-\times-0$ Notation

Many beautiful features of matrix algebra rely on specifically exploiting special structure in the location of nonzero entries of a matrix. We identify the sparsity structure of a matrix by specifying the exact location of all zero entries, nonzero entries and entries that can take any value. Below are some examples of general patterns we use to identify the sparsity structure of matrices.

$$
\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
\times & 1 & 0 & 0 \\
\times & \times & 1 & 0 \\
\times & \times & \times & 1
\end{array}\right],\left[\begin{array}{ccccc}
\star & \times & \times & \times & \times \\
0 & \star & \times & \times & \times \\
0 & 0 & \star & \times & \times \\
0 & 0 & 0 & \star & \times \\
0 & 0 & 0 & 0 & \star
\end{array}\right],\left[\begin{array}{ccccc}
\times & \times & \times & \times & \times \\
\star & \times & \times & \times & \times \\
0 & \star & \times & \times & \times \\
0 & 0 & \star & \times & \times \\
0 & 0 & 0 & \star & \times \\
0 & 0 & 0 & 0 & \star
\end{array}\right],\left[\begin{array}{cccc}
\times & \times & 0 & 0 \\
\times & \times & \times & 0 \\
0 & \times & \times & \times \\
0 & 0 & \times & \times
\end{array}\right]
$$

The 1 represents entries of the matrix that must be equal to one while the $\star$ 's designate nonzero entries. The $\times$ symbols represent entries that may be any real number including zero. Finally, in each of these cases, the symbol 0 specifies the location of the zero entries. Some authors also leave all entries that are zero blank, as in the example below

$$
\left[\begin{array}{ccccc}
\star & 0 & 0 & 0 & 0 \\
0 & \star & 0 & 0 & 0 \\
0 & 0 & \star & 0 & 0 \\
0 & 0 & 0 & \star & 0 \\
0 & 0 & 0 & 0 & \star
\end{array}\right]=\left[\begin{array}{lllll}
\star & & & & \\
& \star & & & \\
& & \star & & \\
& & & \star & \\
& & & & \star
\end{array}\right]
$$

In this text, we will explicitly identify all zero entries using the symbol 0 unless otherwise stated.

## EXAMPLE 3.2.6

Given a few matrices, draw the sparsity structure using the $1-\star-\times-0$ notation. Focus on identifying patterns in the sparsity structure rather than focusing on the exact location of zeros.

## Main Diagonal

The main diagonal entries of $A$ are elements with equal row and column indices. In other words, we say that $a_{i k}$ is on the main diagonal of $A$ if $i=k$.

The main diagonal of $A$ is the set of all diagonal entries of $A$.


The non-diagonal entries of $A$ are entries that are not on the main diagonal. Thus, $a_{i k}$ is a non-diagonal entry if $i \neq k$.

## Definition 3.6: Diagonal Matrix

Let $D \in \mathbb{R}^{n \times n}$ be a given, square matrix. We say that $D$ is diagonal if $d_{i k}=0$ for all $i \neq k$. Diagonal matrices take the form

$$
D=\left[\begin{array}{cccc}
d_{11} & 0 & \cdots & 0 \\
0 & d_{22} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & d_{n n}
\end{array}\right]
$$

In this case, the entries on the main diagonal $d_{i i}$ are any real numbers and are not necessarily zero.

## Definition 3.7: Identity Matrix

Let $I_{n} \in \mathbb{R}^{n \times n}$ be $n \times n$ identity matrix given by

$$
I_{n}=\left[\begin{array}{cccc}
1 & 0 & \cdots & 0 \\
0 & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & 1
\end{array}\right]
$$

In this case, we can define the identity matrix using the individual coefficients as follows

$$
I_{n}(i, k)= \begin{cases}1 & \text { if } i=k \\ 0 & \text { if } i \neq k\end{cases}
$$

## Lower Triangular Entries

The lower-triangular entries of a matrix $A$ are all entries on or below the main diagonal. Thus, we say that element $a_{i k}$ is a lower-triangular entry if and only if $i \geq k$.


The strictly lower-triangular entries of a matrix are all entries $a_{i k}$ with $i>k$.
$\left[\begin{array}{llllll}a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}\end{array}\right]$

## Definition 3.8: Lower-Triangular Matrix

Let $L \in \mathbb{R}^{n \times n}$ be a given, square matrix. We say that $L$ is lower-triangular if $\ell_{i k}=0$ for all $i<k$. Lower-triangular matrices take the form


## Definition 3.9: Unit Lower-Triangular Matrix

Let $L \in \mathbb{R}^{n \times n}$ be a given, square matrix. We say that $L$ is a unit lowertriangular matrix if $\ell_{i k}=0$ for all $i<k$ and $\ell_{i k}=1$ for all $i=k$. Unit lower-triangular matrices take the form

$$
L=\left[\begin{array}{cccc}
1 & 0 & \cdots & 0 \\
\ell_{21} & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
\ell_{n 1} & \ell_{n 2} & \cdots & 1
\end{array}\right]
$$

## Upper Triangular Entries

The upper-triangular entries of a matrix $A$ are all entries on or above the main diagonal. Thus, we say that element $a_{i k}$ is a upper-triangular entry if and only if $i \leq k$.


The strictly upper-triangular entries of a matrix are all entries $a_{i k}$ with $i<k$.


## Definition 3.10: Upper-Triangular Matrix

Let $U \in \mathbb{R}^{n \times n}$ be a given, square matrix. We say that $U$ is uppertriangular if $u_{i k}=0$ for all $i>k$. In other words, upper-triangular matrices take the form

$$
U=\left[\begin{array}{cccc}
u_{11} & u_{12} & \cdots & u_{1 n} \\
0 & u_{22} & \ddots & u_{2 n} \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & u_{n n}
\end{array}\right]
$$

## Bands of a Matrix

A diagonal band of a matrix $A \in \mathbb{R}^{m \times n}$ is a set of entries with a constant difference between the row and column indices. In other words, for a given $d \in$ $\{-(n-1),-(n-2), \ldots,-1,0,1, \ldots,(m-2),(m-1)\}$, we call the $d$ th band of $A$ the set of all $a_{i k}$ such that $i-k=d$.

## EXAMPLE 3.2.7

The main diagonal of a matrix is the 0 band of a matrix

## EXAMPLE 3.2.8

The +1 band of a matrix. The -1 band of a matrix.
The upper-triangular bands are all bands of a matrix such that $i-k \leq 0$.
There are a total of $(m-1)$ strictly lower triangular bands of a matrix, 1 diagonal band and $(n-1)$ strictly upper-triangular bands.

The lower-triangular bands are all bands of a matrix such that $k>0$.
The lower bandwidth of a matrix is the number $d_{\ell}$ such that if $a_{i k}=0$ for all $i-k>d_{\ell}$

The upper bandwidth of a matrix is the number $d_{u}$ such that if $a_{i k}=0$ for all $i-k<d_{u}$

| Type <br> of Matrix | Lower <br> Bandwidth | Upper <br> Bandwidth |
| :--- | :---: | :---: |
| diagonal | 0 | 0 |
| upper-triangular | 0 | $n-1$ |
| lower-triangular | $m-1$ | 0 |
| tridiagonal | 1 | 1 |

## Colon Notation

## Definition 3.11: Colon Notation

A handy way to specify individual columns or rows of a matrix is to use colon notation. If $A \in \mathbb{R}^{m \times n}$, then $A(:, k) \in R^{m \times 1}$ designates the $k$ th column of $A$ :

$$
A(:, k)=\left[\begin{array}{c}
a_{1 k} \\
a_{2 k} \\
\vdots \\
a_{m k}
\end{array}\right]
$$

for $k \in\{1,2, \ldots, n\}$. Similarly, $A(i,:) \in \mathbb{R}^{1 \times n}$ designates the $i$ th row of $A$ :

$$
A(i,:)=\left[\begin{array}{llll}
a_{i 1} & a_{i 2} & \cdots & a_{i n}
\end{array}\right]
$$

for $i \in\{1,2, \ldots, m\}$
This notation originated from the MATLAB computing language. As we will see, we can use this notation is extremely useful in focusing on vector-level computational issues.

Throughout this text, we will use $i$ to representing row indices, $k$ to represent column indices, and $j$ to represent auxiliary indexing variables.

## EXAMPLE 3.2.9

Let's take a look at a different digraph with 4 nodes and 6 edges:


The incidence matrix $A$ for this directed graph can be read from the table

| Incidence Matrix |  |  |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | $e_{1}$ | $e_{2}$ | $e_{3}$ | $e_{4}$ | $e_{5}$ | $e_{6}$ |
| $N_{1}$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $N_{2}$ | 0 | -1 | 1 | 1 | 0 | -1 |
| $N_{3}$ | 0 | 0 | -1 | 0 | 1 | 0 |
| $N_{4}$ | -1 | 0 | 0 | -1 | -1 | 1 |

Here, $A \in \mathbb{R}^{4 \times 6}$ since there are 4 vertices and 6 edges of the graph. Using colon notation, we focus on the second node of this graph $A(2,:)=\left[\begin{array}{llllll}0 & -1 & 1 & 1 & 0 & -1\end{array}\right]$ to see that edges 2 and 6 enter node 2 and edges 3 and 4 leave node 2 . Similarly,
we conclude that edge 4 leave node 2 and enters node 4 since

$$
A(:, 4)=\left[\begin{array}{r}
0 \\
1 \\
0 \\
-1
\end{array}\right]
$$

## Rows of a Matrix

## Definition 3.12: Row Partition of a Matrix

We can also write $A \in \mathbb{R}^{m \times n}$ using a row paritition: Let

$$
\{A(1,:), A(2,:), \ldots, A(m,:)\} \subseteq \mathbb{R}^{1 \times n}
$$

be a collection of $m$ separate $1 \times n$ row vectors. Organize each row vector $A(i,:)$ one on top of the other to form the rectangular array:

$$
A=\left[\begin{array}{c}
A(1,:) \\
A(2,:) \\
\vdots \\
A(m,:)
\end{array}\right]
$$

In this row partition, $A(i,:)$ is the $i$ th row of matrix $A$ as is given by

$$
A(i,:)=\left[\begin{array}{llll}
a_{i 1} & a_{i 2} & \cdots & a_{i n}
\end{array}\right]
$$

Given an $m \times n$ matrix $A$, each row of $A$ is a $1 \times n$ short and wide matrix. We denote our rows as

$$
A(i,:)=A_{i *}=\left[\begin{array}{llll}
a_{i 1} & a_{i 2} & \cdots & a_{i n}
\end{array}\right]
$$

The $n$ individual real-valued entries in any row have identical row index values.


The Row operator takes in a row index $i$ and a matrix to produce the $i$ th row of the given matrix.

## Columns of a Matrix

## Definition 3.14: Column Partition of a Matrix

Consider a matrix $A \in R^{m \times n}$. The column partition of $A$ is a description of the matrix $A$ in terms of the column vectors that make up the matrix. Let $\{A(:, 1), A(:, 2), \ldots, A(:, n)\} \subseteq \mathbb{R}^{m \times 1}$ be a collection of the $n$ separate $m \times 1$ column vectors. Organize these vectors side by side to form the rectangular array:

$$
A=\left[\begin{array}{llll}
A(:, 1) & A(:, 2) & \cdots & A(:, n)
\end{array}\right]
$$

In this column partition, $A(:, k)$ is the $k$ th column of matrix $A$ given by

$$
A(:, k)=\left[\begin{array}{c}
a_{1 k} \\
a_{2 k} \\
\vdots \\
a_{m k}
\end{array}\right]
$$

The column partition of a matrix is another way to organize and define a matrix. The column partition of a matrix requires that we define $n$ separate vectors, each one having dimension $m \times 1$. This partition will be very helpful when we interpret the matrix-vector product $A \mathbf{x}$ as linear combinations of the columns of the matrix $A$ with scaling weights defined by the entries of $\mathbf{x}$.

In contrast, to use the entry-by-entry definition of the matrix $A$, we must define each of the $m \cdot n$ in the entire matrix individually. This specification can be very helpful in creating matrix models from disparate data and when storing matrices in computers. However, we will tend away from using the entry-by-entry definition to interpret the matrix-vector multiplication operations.

Every column vector is an $m \times 1$ tall and narrow matrix. We use column vectors extensively to describe our four fundamental problems and to model many diverse phenomenon. Whenever we refer to a vector without explicitly stating it's dimensions, we always mean a column vector.

We denote each column vector as

$$
A(:, k)=A_{* k}=\left[\begin{array}{r}
a_{1 k} \\
a_{2 k} \\
\vdots \\
a_{m k}
\end{array}\right]
$$

The $m$ individual real-valued entries in any column have identical column indices.

## Definition 3.15

Let $A \in \mathbb{R}^{m \times n}$. Define the map

$$
\operatorname{Column}_{k}(A)=A(:, k)
$$

for $1 \leq k \leq n$.

## EXAMPLE 3.2.10

Let's look back on Definition 3.7, which gave the entry-by-entry definition of the identity matrix. We can define also define the $n \times n$ identity matrix using a column partition. We write

$$
I_{n}=\left[\begin{array}{llll}
\mathbf{e}_{1} & \mathbf{e}_{2} & \cdots & \mathbf{e}_{n}
\end{array}\right]
$$

where the $k$ th column of the identity matrix $I_{n}(:, k)=\mathbf{e}_{k}$ is given by the $k$ th elementary basis vector, as defined in Example 3.3.1.


### 3.3 Matrices from Outer Products

## Definition 3.16: Outer Product of Vectors

Let $\mathbf{x} \in \mathbb{R}^{m \times 1}$ and $\mathbf{y} \in \mathbb{R}^{n \times 1}$. Then, the outer product between $\mathbf{x}$ and $\mathbf{y}$ is the $m \times n$ matrix given by

$$
\mathbf{x y}^{T}=\left[\begin{array}{c}
x_{1} \\
x_{2} \\
\vdots \\
x_{m}
\end{array}\right]\left[\begin{array}{llll}
y_{1} & y_{2} & \cdots & x_{n}
\end{array}\right]=\left[\begin{array}{cccc}
x_{1} y_{1} & x_{1} y_{2} & \cdots & x_{1} y_{n} \\
x_{2} y_{1} & x_{2} y_{2} & \cdots & x_{2} y_{n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m} y_{1} & x_{m} y_{2} & \cdots & x_{m} y_{n}
\end{array}\right]
$$

Notice that row $i$ of the outer product $\mathbf{x y}^{T}$ is given by $x_{i} \mathbf{y}^{T}$ for $i=$ $1,2, \ldots, m$ while column $k$ of the outer product is given by $y_{k} \mathbf{x}$ for $k=$ $1,2, \ldots, n$.

The outer product between two vectors can be very useful in computing many multiplications between real numbers simultaneously. These matrices are also extremely helpful in generating new matrices.

## EXAMPLE 3.3.1

Let $\left\{\mathbf{e}_{i}\right\}_{i=1}^{n} \subseteq \mathbb{R}^{n \times 1}$ be the set of elementary basis vectors for $\mathbb{R}^{n}$. We will define each vector $\mathbf{e}_{i}$ component-wise as follows

$$
\mathbf{e}_{i}(j, 1)= \begin{cases}1 & \text { if } i=j \\ 0 & \text { otherwise }\end{cases}
$$

For $n=3$, we see

$$
\mathbf{e}_{1}=\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right], \quad \mathbf{e}_{2}=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right], \quad \mathbf{e}_{3}=\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right]
$$

Using these vectors and the outer product, we can create $n \times n$ matrix units

$$
E_{i k}=\mathbf{e}_{i} \mathbf{e}_{k}^{T}
$$

with all zero entries except the entry in row $i$ and column $k$, which is equal to 1 . In the $3 \times 3$ case, there are a total of nine matrix units given as follows:

$$
\begin{aligned}
& E_{11}=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right], \quad E_{12}=\left[\begin{array}{lll}
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right], \quad E_{13}=\left[\begin{array}{lll}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right], \\
& E_{21}=\left[\begin{array}{lll}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right], \quad E_{22}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right], \quad E_{23}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{array}\right], \\
& E_{31}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{array}\right], \quad E_{32}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{array}\right], \quad E_{33}=\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{array}\right] .
\end{aligned}
$$

Notice, we can actually define matrix units in the $m \times n$ case, although to do so we would need to take an outer product between vectors of different sizes. For now we focus on the square matrix units as we will use these most frequently.

## EXAMPLE 3.3.2

Lets suppose we want to create a $5 \times 5$ matrix $T$ having the following structure:

$$
T_{2}=\left[\begin{array}{rrrrr}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & 0 \\
0 & -2 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 0
\end{array}\right]
$$

Then, we can define a $5 \times 1$ vector

$$
\boldsymbol{\tau}=\left[\begin{array}{r}
0 \\
0 \\
3 \\
-2 \\
4
\end{array}\right]
$$

and we can write $T=\boldsymbol{\tau} \mathbf{e}_{2}^{T}$, where $\mathbf{e}_{2}$ is the second elementary basis vector in $\mathbb{R}^{5}$.

Lesson 9, Part 1: Matrices from Outer Products- Suggested Problems

For all the problems below, be sure to explicitly state the dimensions of the matrices you use for each model.

1. Create each of the following matrices using an outer product between two vectors. Specifically state the two vectors you are using to write the outer product:

$$
A=\left[\begin{array}{rrrrr}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
1 & -3 & 2 & 5 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right], \quad B=\left[\begin{array}{rrrr}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -7 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right], \quad C=\left[\begin{array}{rrr}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 4 & 0 \\
0 & -3 & 0 \\
0 & 2 & 0
\end{array}\right]
$$

