Name : ________________ **Lesson 7 Warm Up Quiz** **Class Number:** _____

Math 2B: Applied Linear Algebra

Multiple Choice

For the problems below, circle the correct response for each question.

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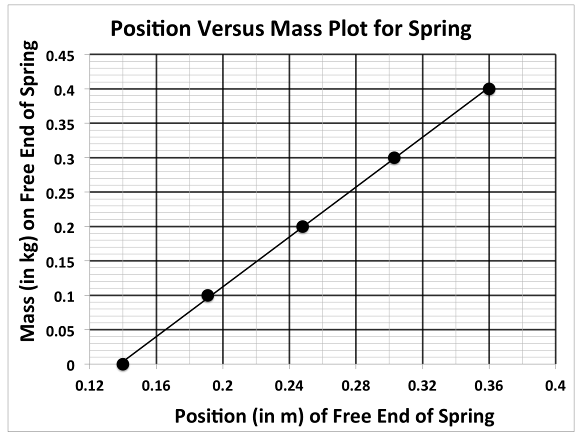
1. Hooke's Law is a principle of physics stating that the force needed to extend or compress a spring by some distance is proportional to that distance. Recall from class that we can set up an experiment to verify Hooke's law using a spring, various masses, a scale, and a measuring stick. Below are five collected data points relating to Hooks Law.

Measurement Number	Position x in Meters (m)	Applied mass m in kilograms
1	0.140	0.000
2	0.191	0.100
3	0.248	0.200
4	0.303	0.300
5	0.360	0.400



From this data, we can calculate u_i, the displacement of movable end of spring in measurement i. We can also create a mathematical model in the form

$$y_i = b + k \cdot u_i$$

where y_i is the modeled force associated with displacement u_i. Choose the correct matrix-vector model for generating vector $\mathbf{y} \in \mathbb{R}^5$ given any choice of $b, k \in \mathbb{R}$.

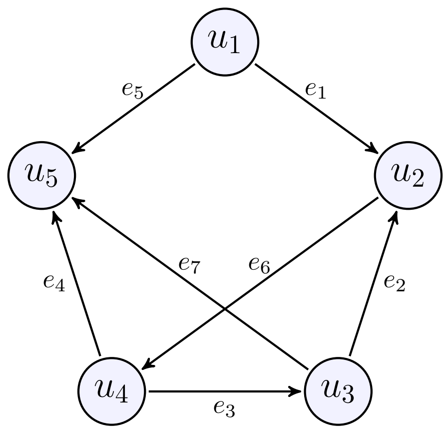
A. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & 0.140 \\ 1 & 0.191 \\ 1 & 0.248 \\ 1 & 0.303 \\ 1 & 0.360 \end{bmatrix} \begin{bmatrix} b \\ k \end{bmatrix}$

B. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & 0.0 \\ 1 & 0.1 \\ 1 & 0.2 \\ 1 & 0.3 \\ 1 & 0.4 \end{bmatrix} \begin{bmatrix} b \\ k \end{bmatrix}$

C. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 & 0.000 \\ 1 & 0.051 \\ 1 & 0.108 \\ 1 & 0.163 \\ 1 & 0.220 \end{bmatrix} \begin{bmatrix} b \\ k \end{bmatrix}$

D. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0.140 & 0.0 \\ 0.191 & 0.1 \\ 0.248 & 0.2 \\ 0.303 & 0.3 \\ 0.360 & 0.4 \end{bmatrix} \begin{bmatrix} b \\ k \end{bmatrix}$

Consider the directed graph given below. Use this graph to fill in the corresponding incidence matrix. Use your entries for the incidence matrix to identify the correct answer for problems **2 - 3**.



Incidence Matrix

	u_1	u_3	u_3	u_4	u_5
e_1					
e_2					
e_3					
e_4					
e_5					
e_6					
e_7					

2. Let$A$ represent the 7×5 incidence matrix. Then the entry a_{53} is given by which of the following:

A. $a_{53} = 2$ B. $a_{53} = -1$ C. $a_{53} = 0$ D. $a_{53} = 1$ E. $a_{53} = e_7$

3. Let A represent the 7×5 incidence matrix. Then $[(A(6,:)]^T$ is given by which of the following:

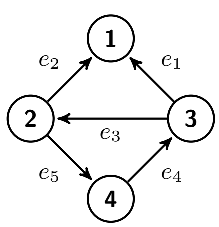
A. $\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ C. $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ D. $\begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ E. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

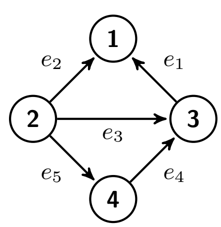
4. Let the following matrix $A \in \mathbb{R}^{5\times 4}$ be the incidence matrix for a directed graph:

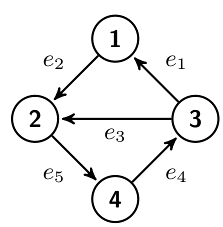
Incidence Matrix

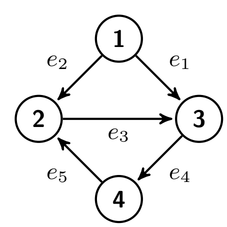
	u_1	u_3	u_3	u_4
e_1	-1	0	1	0
e_2	-1	1	0	0
e_3	0	-1	1	0
e_4	0	0	-1	1
e_5	0	1	0	-1

This matrix corresponds to which of the following directed graphs:

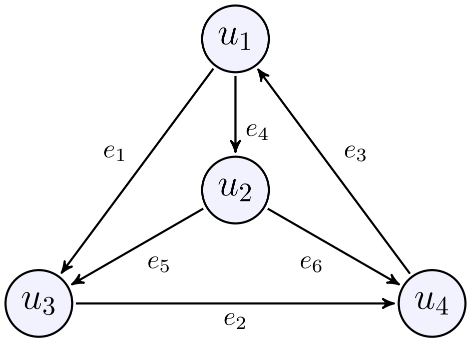
A. 

B. 

C. 

D. 

Consider the following directed graph. Use this graph to find the correct answer for problem 6.



Incidence Matrix

	u_1	u_3	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				
e_6				

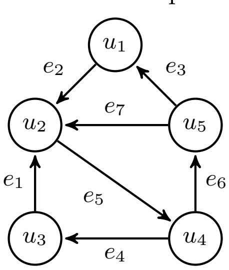
5. Let A represent the 6×4 incidence matrix. Find $[A(2,:)]^T \cdot [A(5,:)]^T$:

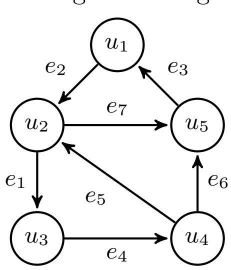
A. 2 B. 1 C. 0 D. -1 E. -2

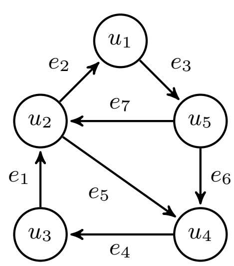
6. Let the following 5×7 matrix be the incidence matrix for a directed graph:

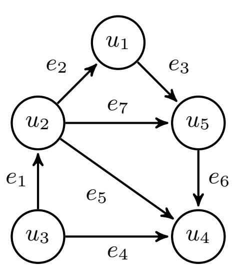
$$A = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

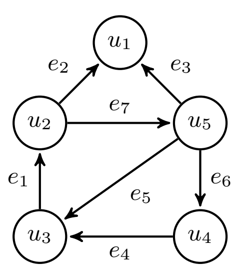
This matrix corresponds to which of the following directed graphs:

A.


B.


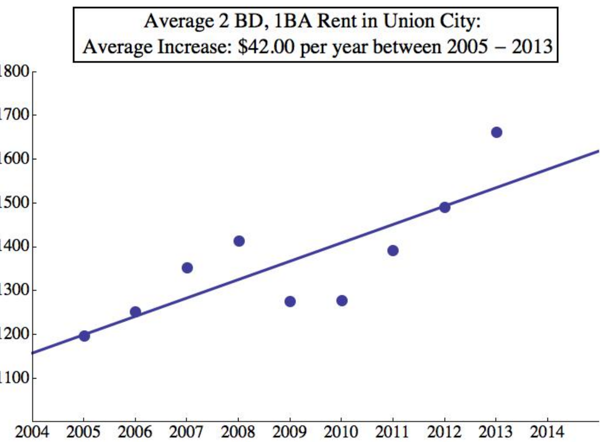
C.


D.


E.


7. Consider the following data set that describes the average rent levels for a rental unit with 2 bedrooms and 1 bathroom (2Bd/1Ba) in Union City, CA.

Index i	Year t_i	Average Rent Level R (in \$)
1	2005	\$1,197
2	2006	\$1,252
3	2007	\$1,353
4	2008	\$1,413
5	2009	\$1,275
6	2010	\$1,277
7	2011	\$1,392
8	2012	\$1,490
9	2013	\$1,663



From this data, we can model the rent for a 2Bd/1Ba unit using a linear function in the form

$$R_i = R(t_i) = b + m \cdot t_i$$

where R_i is the modeled monthly rent during year t_i. Choose the correct matrix-vector model for generating vector $\mathbf{R} \in \mathbb{R}^9$ given any choice of $b, m \in \mathbb{R}$.

A. $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{bmatrix} = \begin{bmatrix} 1 & 2005 \\ 1 & 2006 \\ 1 & 2007 \\ 1 & 2008 \\ 1 & 2009 \\ 1 & 2010 \\ 1 & 2011 \\ 1 & 2012 \\ 1 & 2013 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix}$

B. $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{bmatrix} = \begin{bmatrix} 2005 & 1197 \\ 2006 & 1252 \\ 2007 & 1353 \\ 2008 & 1413 \\ 2009 & 1275 \\ 2010 & 1277 \\ 2011 & 1392 \\ 2012 & 1490 \\ 2013 & 1663 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix}$

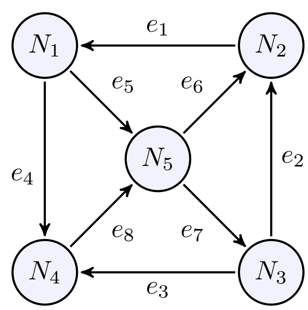
C. $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{bmatrix} = \begin{bmatrix} 1 & 1197 \\ 1 & 1252 \\ 1 & 1353 \\ 1 & 1413 \\ 1 & 1275 \\ 1 & 1277 \\ 1 & 1392 \\ 1 & 1490 \\ 1 & 1663 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix}$

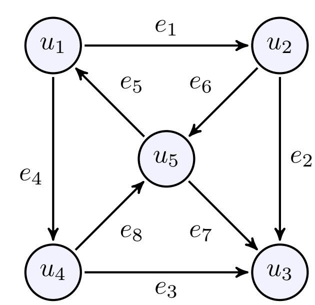
D. $\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \\ R_6 \\ R_7 \\ R_8 \\ R_9 \end{bmatrix} = \begin{bmatrix} 1 & 2005 \\ 2 & 2006 \\ 3 & 2007 \\ 4 & 2008 \\ 5 & 2000 \\ 6 & 2010 \\ 7 & 2011 \\ 8 & 2012 \\ 9 & 2013 \end{bmatrix} \begin{bmatrix} b \\ m \end{bmatrix}$

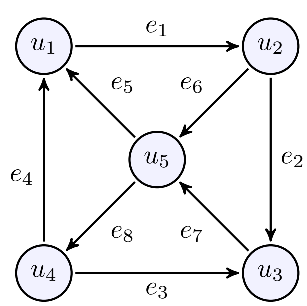
8. Let the following matrix $A^T \in \mathbb{R}^{8\times 5}$ be the incidence matrix for a directed graph:

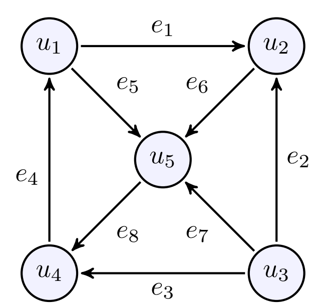
$$A^T = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Then, this is the incidence matrix for which of the following directed graphs:

A. 

B. 

C. 

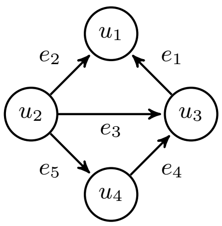
D. 

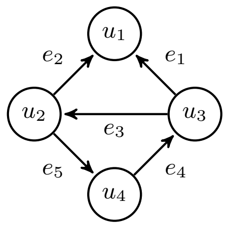
9. Let the following matrix $A \in \mathbb{R}^{4\times 5}$ be the incidence matrix for a directed graph:

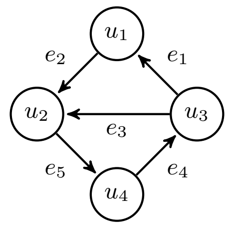
Incidence Matrix

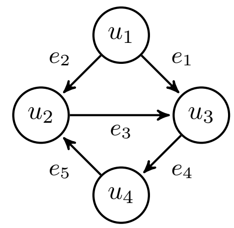
	u_1	u_3	u_3	u_4
e_1	1	0	-1	0
e_2	1	-1	0	0
e_3	0	1	-1	0
e_4	0	0	1	-1
e_5	0	-1	0	1

This matrix corresponds to which of the following directed graphs:

A. 

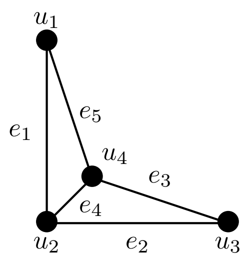
B. 

C. 

D. 

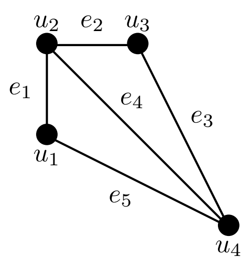
Free Response

1. For each of the undirected graphs below, find the associated incidence matrix



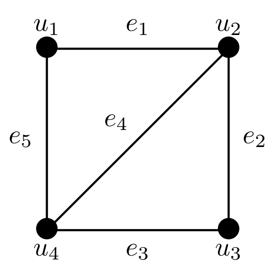
Undirected Incidence Matrix Table

	u_1	u_2	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				



Undirected Incidence Matrix Table

	u_1	u_2	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				



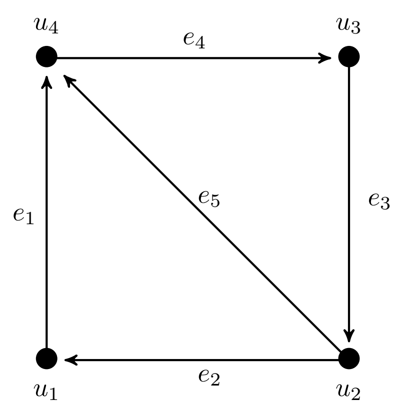
Undirected Incidence Matrix Table

	u_1	u_2	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				

(See Reverse)

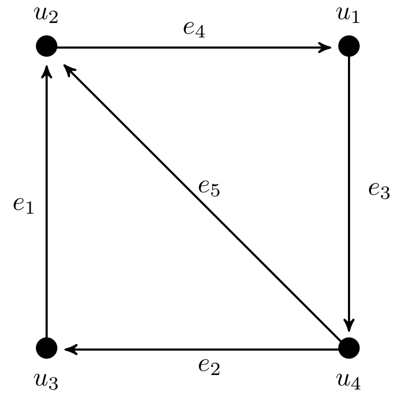
2. What do you notice about each of the incidence matrices from question 1 above? How does your observation relate to the graphs in this example?

3. For each of the directed graphs below, find the associated incidence matrix



Directed Incidence Matrix Table

	u_1	u_2	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				



Directed Incidence Matrix Table

	u_1	u_2	u_3	u_4
e_1				
e_2				
e_3				
e_4				
e_5				