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## Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

1. T F A linear combination of vectors in the same thing as the span of these vectors
2. T F A single nonzero vector forms a linearly independent set.
3. T F Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{n}$. Then $\operatorname{Span}\{\mathbf{x}, \mathbf{y}\}$ always describes a plane passing through the origin.
4. T F If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly independent vectors, then $\mathbf{u}, \mathbf{v}, \mathbf{w} \notin \mathbb{R}^{2}$.
5. T F If $\mathbf{w}$ is a linear combination of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^{n}$, then $\mathbf{u}$ can be written as a a linear combination of $\mathbf{v}$ and $\mathbf{w}$.
6. $\mathrm{T} \quad \mathrm{F}$ Let $\mathbf{x}_{1}, \mathbf{x}_{2}, \ldots, \mathbf{x}_{n} \in \mathbb{R}^{m}$. When considering a linear combination of these vectors in the form

$$
c_{1} \mathbf{x}_{2}+c_{2} \mathbf{x}_{2}+\cdots+c_{n} \mathbf{x}_{n},
$$

the weights $c_{1}, c_{2}, \ldots, c_{n}$ cannot all be zero.
7. T F If $f(\mathbf{x})=A \mathbf{x}$ and $\mathbf{b} \in \operatorname{Rng}(f)$, then $\mathbf{b}$ is linearly independent from the columns of $A$.
8. T F If $A \in \mathbb{R}^{m \times n}$ has linearly independent rows, then $m \leq n$.
9. T F A linear combination of vectors in the same thing as the span of these vectors
10. T F Linearly independent vectors never span the space in which they come from.
11. $\mathrm{T} \quad \mathrm{F} \quad$ For vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^{n}$, the distance between $\mathbf{x}$ and $\mathbf{y}$ is given by $\|\mathbf{x}-\mathbf{y}\|_{2}$.
12. T F Any set of vectors that contains the zero vector must be linearly dependent.

Multiple Choice For the problems below, circle the correct response for each question.

1. Let $m, n \in \mathbb{N}$. Suppose $A \in \mathbb{R}^{m \times n}$ and $\mathbf{x} \in \mathbb{R}^{n}$ are given. For the solution of the matrix-vector multiplication problem given by $A \mathbf{x}=\mathbf{b}$, which of the following is false:
A. $\mathbf{b}$ is linearly dependent on the columns of $A$.
B. The vector $\mathbf{b}$ can be written as a linear combination of the columns of $A$
C. If $f(\mathbf{x})=A \mathbf{x}$, then $\mathbf{b} \in \operatorname{Rng}(f)$.
D. The columns of $A$ must be linearly independent.
E. $\mathbf{b}=\sum_{j=1}^{n} x_{j} A(:, j)$ for some $x_{1}, x_{2}, \ldots, x_{n} \in \mathbb{R}$.
2. Which of the following sets is equivalent to

$$
\operatorname{Span}\left\{\left[\begin{array}{l}
1 \\
0 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]\right\}
$$

A. $\mathbb{R}^{2}$
B. $\mathbb{R}^{3}$
C. $\left\{\left[\begin{array}{c}x_{1} \\ x_{2} \\ 0\end{array}\right]: x_{i} \in \mathbb{R}\right.$ for $\left.i=1,2\right\}$
D. $\left\{\left[\begin{array}{l}1 \\ 1 \\ 0\end{array}\right]\right\}$
E. $\left\{\left[\begin{array}{l}x \\ x \\ 0\end{array}\right]: x \in \mathbb{R}\right\}$
3. Which of the following sets of vectors is linearly dependent?
A. $\left\{\left[\begin{array}{l}1 \\ 0 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 0 \\ 0 \\ 1\end{array}\right]\right\}$
B. $\left\{\left[\begin{array}{l}1 \\ 4 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{r}0 \\ -2 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 0 \\ 1 \\ 1\end{array}\right]\right\}$
C. $\left\{\left[\begin{array}{l}1 \\ 1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{c}1 \\ -1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 0 \\ 1 \\ 1\end{array}\right]\right\}$
D. $\left\{\left[\begin{array}{l}1 \\ 1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{r}3 \\ 3 \\ -4 \\ -4\end{array}\right],\left[\begin{array}{l}0 \\ 0 \\ 1 \\ 1\end{array}\right]\right\}$
E. $\left\{\left[\begin{array}{l}1 \\ 0 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}1 \\ 1 \\ 0 \\ 0\end{array}\right],\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 0\end{array}\right]\right\}$
4. Consider the set of vectors given by

$$
\mathbf{a}_{1}=\left[\begin{array}{l}
2 \\
0 \\
2 \\
0
\end{array}\right]
$$

$\mathbf{a}_{2}=\left[\begin{array}{r}-1 \\ 0 \\ -1 \\ 0\end{array}\right]$,

$$
\mathbf{a}_{3}=\left[\begin{array}{l}
0 \\
1 \\
0 \\
1
\end{array}\right]
$$

Which of the following vectors sets is equivalent to the span of these three vectors?
A. $\mathbb{R}^{4}$
B. $\left\{\left[\begin{array}{l}x_{1} \\ x_{1} \\ x_{1} \\ x_{1}\end{array}\right]: x_{1} \in \mathbb{R}\right\}$
C. $\left\{\left[\begin{array}{l}x_{1} \\ x_{1} \\ x_{2} \\ x_{2}\end{array}\right]: x_{i} \in \mathbb{R}\right.$ for $\left.i=1,2\right\}$
D. $\left\{\left[\begin{array}{l}2 \\ 0 \\ 2 \\ 0\end{array}\right],\left[\begin{array}{r}-1 \\ 0 \\ -1 \\ 0\end{array}\right],\left[\begin{array}{l}1 \\ 0 \\ 1 \\ 0\end{array}\right],\left[\begin{array}{l}0 \\ 1 \\ 0 \\ 1\end{array}\right],\left[\begin{array}{l}2 \\ 1 \\ 2 \\ 1\end{array}\right],\left[\begin{array}{r}-1 \\ 1 \\ -1 \\ 1\end{array}\right],\left[\begin{array}{l}1 \\ 1 \\ 1 \\ 1\end{array}\right]\right\}$
E. $\left\{\left[\begin{array}{l}x_{1} \\ x_{2} \\ x_{1} \\ x_{2}\end{array}\right]: x_{i} \in \mathbb{R}\right.$ for $\left.i=1,2\right\}$

