## Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	Т	F	A linear combination of vectors in the same thing as the span of these vectors
2.	Т	F	A single nonzero vector forms a linearly independent set.
3.	Т	F	Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ . Then Span $\{\mathbf{x}, \mathbf{y}\}$ always describes a plane passing through the origin.
4.	Т	F	If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly independent vectors, then $\mathbf{u}, \mathbf{v}, \mathbf{w} \notin \mathbb{R}^2$ .
5.	Т	F	If <b>w</b> is a linear combination of vectors <b>u</b> and <b>v</b> in $\mathbb{R}^n$ , then <b>u</b> can be written as a a linear combination of <b>v</b> and <b>w</b> .
6.	Т	F	Let $\mathbf{x}_1, \mathbf{x}_2,, \mathbf{x}_n \in \mathbb{R}^m$ . When considering a linear combination of these vectors in the form
			$c_1\mathbf{x}_2 + c_2\mathbf{x}_2 + \dots + c_n\mathbf{x}_n,$
			the weights $c_1, c_2,, c_n$ cannot all be zero.
7.	Т	F	If $f(\mathbf{x}) = A\mathbf{x}$ and $\mathbf{b} \in \operatorname{Rng}(f)$ , then $\mathbf{b}$ is linearly independent from the columns of $A$ .
8.	Т	F	If $A \in \mathbb{R}^{m \times n}$ has linearly independent rows, then $m \leq n$ .
9.	Т	F	A linear combination of vectors in the same thing as the span of these vectors
10.	Т	F	Linearly independent vectors never span the space in which they come from.
11.	Т	F	For vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ , the distance between $\mathbf{x}$ and $\mathbf{y}$ is given by $\ \mathbf{x} - \mathbf{y}\ _2$ .
12.	Т	F	Any set of vectors that contains the zero vector must be linearly dependent.

Multiple Choice For the problems below, circle the correct response for each question.

- 1. Let  $m, n \in \mathbb{N}$ . Suppose  $A \in \mathbb{R}^{m \times n}$  and  $\mathbf{x} \in \mathbb{R}^n$  are given. For the solution of the matrix-vector multiplication problem given by  $A\mathbf{x} = \mathbf{b}$ , which of the following is false:
  - A. **b** is linearly dependent on the columns of A.
  - B. The vector  ${\bf b}$  can be written as a linear combination of the columns of A
  - C. If  $f(\mathbf{x}) = A\mathbf{x}$ , then  $\mathbf{b} \in \operatorname{Rng}(f)$ .
  - D. The columns of A must be linearly independent.

E. 
$$\mathbf{b} = \sum_{j=1}^{n} x_j A(:, j)$$
 for some  $x_1, x_2, ..., x_n \in \mathbb{R}$ .

2. Which of the following sets is equivalent to

$$\operatorname{Span}\left\{ \begin{bmatrix} 1\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\0 \end{bmatrix} \right\}$$
A.  $\mathbb{R}^2$  B.  $\mathbb{R}^3$  C.  $\left\{ \begin{bmatrix} x_1\\x_2\\0 \end{bmatrix} : x_i \in \mathbb{R} \text{ for } i = 1, 2 \right\}$  D.  $\left\{ \begin{bmatrix} 1\\1\\0 \end{bmatrix} \right\}$  E.  $\left\{ \begin{bmatrix} x\\x\\0 \end{bmatrix} : x \in \mathbb{R} \right\}$ 

3. Which of the following sets of vectors is linearly dependent?

$$A. \left\{ \begin{bmatrix} 1\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\1\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\0\\1 \end{bmatrix} \right\} \qquad B. \left\{ \begin{bmatrix} 1\\4\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\-2\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1\\1 \end{bmatrix} \right\} \qquad C. \left\{ \begin{bmatrix} 1\\1\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 1\\-1\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\0\\1\\1\\1 \end{bmatrix} \right\} \right\}$$

4. Consider the set of vectors given by

$$\mathbf{a}_1 = \begin{bmatrix} 2\\0\\2\\0 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} -1\\0\\-1\\0 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0\\1\\0\\1 \end{bmatrix},$$

Which of the following vectors sets is equivalent to the span of these three vectors?

A. 
$$\mathbb{R}^{4}$$
 B.  $\left\{ \begin{bmatrix} x_{1} \\ x_{1} \\ x_{1} \\ x_{1} \end{bmatrix} : x_{1} \in \mathbb{R} \right\}$  C.  $\left\{ \begin{bmatrix} x_{1} \\ x_{1} \\ x_{2} \\ x_{2} \end{bmatrix} : x_{i} \in \mathbb{R} \text{ for } i = 1, 2 \right\}$   
D.  $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$  E.  $\left\{ \begin{bmatrix} x_{1} \\ x_{2} \\ x_{1} \\ x_{2} \end{bmatrix} : x_{i} \in \mathbb{R} \text{ for } i = 1, 2 \right\}$