Answers **Lesson 6 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	T	**F**	A linear combination of vectors in the same thing as the span of these vectors
2.	**T**	F	A single nonzero vector forms a linearly independent set.
3.	T	**F**	Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. Then Span$\{\mathbf{x}, \mathbf{y}\}$ always describes a plane passing through the origin.
4.	**T**	F	If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ are linearly independent vectors, then $\mathbf{u}, \mathbf{v}, \mathbf{w} \notin \mathbb{R}^2$.
5.	**T**	F	If $\mathbf{w}$ is a linear combination of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ can be written as a a linear combination of $\mathbf{v}$ and $\mathbf{w}$.
6.	T	**F**	Let $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n \in \mathbb{R}^m$. When considering a linear combination of these vectors in the form $$c_1\mathbf{x}_2 + c_2\mathbf{x}_2 + \cdots + c_n\mathbf{x}_n,$$ the weights $c_1, c_2, ..., c_n$ cannot all be zero.
7.	T	**F**	If $f(\mathbf{x}) = A\mathbf{x}$ and $\mathbf{b} \in \text{Rng}(f)$, then $\mathbf{b}$ is linearly independent from the columns of A.
8.	**T**	F	If $A \in \mathbb{R}^{m\times n}$ has linearly independent rows, then $m \leq n$.
9.	T	**F**	A linear combination of vectors in the same thing as the span of these vectors
10.	T	**F**	Linearly independent vectors never span the space in which they come from.
11.	**T**	F	For vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the distance between $\mathbf{x}$ and $\mathbf{y}$ is given by $\|\mathbf{x} - \mathbf{y}\|_2$.

12. **T** F Any set of vectors that contains the zero vector must be linearly dependent.

Multiple Choice

For the problems below, circle the correct response for each question.

1. Let $m, n \in \mathbb{N}$. Suppose $A \in \mathbb{R}^{m\times n}$ and $\mathbf{x} \in \mathbb{R}^n$ are given. For the solution of the matrix-vector multiplication problem given by $A\mathbf{x} = \mathbf{b}$, which of the following is false:

   A. $\mathbf{b}$ is linearly dependent on the columns of A.

   B. The vector $\mathbf{b}$ can be written as a linear combination of the columns of A

   C. If $f(\mathbf{x}) = A\mathbf{x}$, then $\mathbf{b} \in \text{Rng}(f)$.

   D. The columns of A must be linearly independent.

   E. $\mathbf{b} = \sum_{j=1}^{n} x_j A(:, j)$ for some $x_1, x_2, ..., x_n \in \mathbb{R}$.

2. Which of the following sets is equivalent to

$$\text{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$$

   A. $\mathbb{R}^2$ B. $\mathbb{R}^3$ **C.** $\left\{\begin{bmatrix}x_1\\x_2\\0\end{bmatrix} : x_i \in \mathbb{R} \textbf{ for } i = 1, 2\right\}$ D. $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}\right\}$ E. $\left\{\begin{bmatrix}x\\x\\0\end{bmatrix} : x \in \mathbb{R}\right\}$

3. Which of the following sets of vectors is linearly dependent?

   A. $\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$ B. $\left\{\begin{bmatrix}1\\4\\0\\0\end{bmatrix}, \begin{bmatrix}0\\-2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\right\}$ C. $\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\right\}$

   D. $\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}3\\3\\-4\\-4\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}\right\}$ E. $\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}\right\}$

4. Consider the set of vectors given by

$$\mathbf{a}_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

Which of the following vectors sets is equivalent to the span of these three vectors?

A. $\mathbb{R}^4$

B. $\left\{ \begin{bmatrix} x_1 \\ x_1 \\ x_1 \\ x_1 \end{bmatrix} : x_1 \in \mathbb{R} \right\}$

C. $\left\{ \begin{bmatrix} x_1 \\ x_1 \\ x_2 \\ x_2 \end{bmatrix} : x_i \in \mathbb{R} \text{ for } i = 1, 2 \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_1 \\ x_2 \end{bmatrix} : x_i \in \mathbb{R} \textbf{ for } i = 1, 2 \right\}$