Name : ________________ **Lesson 5 Warm Up Quiz** **Class Number:** _____

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	T	F	If $\mathbf{x} \in \mathbb{R}^n$, then the inner product of $\mathbf{x}$ with itself is given by $\mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|_2^2$
2.	T	F	$\sqrt{\mathbf{x} \cdot \mathbf{x}} = \|\mathbf{x}\|_2$
3.	T	F	If $f(\mathbf{x}, \mathbf{y}) = \mathbf{x} \cdot \mathbf{y} = \mathbf{x}^T \mathbf{y}$, then the codomain of f is $\mathbb{R}$.
4.	T	F	Suppose $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. $\mathbf{x} \cdot \mathbf{y} = \sum_{i=1}^{n} x_i \, y_i$
5.	T	F	$\mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|_2^2$
6.	T	F	$\mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|_2$
7.	T	F	$\|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2 = \mathbf{x} \cdot \mathbf{y} \cos(\theta)$ where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$.
8.	T	F	If $f(\mathbf{x}) = \|\mathbf{x}\|_2$, then $\mathrm{Rng}(f) = (0, \infty) \subseteq \mathbb{R}$.
9.	T	F	$\mathbf{x} \cdot \mathbf{y} = \|\mathbf{x}\|_2 \, \|\mathbf{y}\|_2 \cos(\theta)$ where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$.

Multiple Choice

For the problems below, circle the correct response for each question.

1. Let $E = \{\mathbf{e}_j\}_{j=1}^4$ be the standard basis for $\mathbb{R}^4$:

$$\mathbf{e}_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \qquad \mathbf{e}_2 = \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \qquad \mathbf{e}_3 = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \qquad \mathbf{e}_4 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

Define the vector $\mathbf{x}$ by taking a linear combination of elements of E below

$$\mathbf{x} = 2\,\mathbf{e}_1 - 5\,\mathbf{e}_2 - 3\,\mathbf{e}_3 + 4\,\mathbf{e}_4.$$

Which of the following gives the value of the dot product $(\mathbf{e}_4 - \mathbf{e}_2) \cdot \mathbf{x}$:

A. 9 B. -9. C. -2 D. 1 E. -1

2. Consider the following two column vectors

$$\mathbf{x} = \begin{bmatrix}-1\\1\\1\\-1\end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix}1\\-1\\1\\1\end{bmatrix}$$

Find the angle θ between these vectors:

A. $-\frac{1}{2}$ B. π C. $\frac{2\pi}{3}$ D. $\frac{\pi}{3}$ E. $\frac{5\pi}{6}$

3. Define vectors

$$\mathbf{x} = \begin{bmatrix}t\\-4\\2\\t\end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix}-t\\t\\5\\1\end{bmatrix}$$

Find <u>all</u> values of scalar t so that the inner product $\mathbf{x} \cdot \mathbf{y} = 0$

A. $t = -2$ B. $t = 5$ and $t = -2$ C. $t = 5$ and $t = 2$ D. $t = -5$ and $t = 2$ E. $t = 5$

Free Response

1. State and prove each of the following the pythagorean theorem

2. State and prove the Law of Cosines (BOTH acute and obtuse case)

3. State and prove the cosine formula for the inner product. Using the cosine formula for the inner product, discuss when two vectors are orthogonal "perpendicular" to each other.

4. (Optional, Extra Credit, Challenge Problem) Let $\mathbf{x} \in \mathbb{R}^n$ be a column vector. Recall that we defined the 2-norm of $\mathbf{x}$ to be

$$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$$

We can create a different norm, called the $\infty-$norm (read "infinity norm"), using the following definition

$$\|\mathbf{x}\|_\infty = \max_{1 \le i \le n} |x_i|$$

which takes the maximum value of the absolute values of all entries of $\mathbf{x}$. Using these definitions, prove

$$\|\mathbf{x}\|_\infty \le \|\mathbf{x}\|_2 \le \sqrt{n} \cdot \|\mathbf{x}\|_\infty$$

5. (Optional, Extra Credit, Challenge Problem)

Let $\mathbf{x} \in \mathbb{R}^n$ be a column vector. Recall that we defined the 2-norm of $\mathbf{x}$ to be

$$\|\mathbf{x}\|_2 = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2}$$

This is one example of a much larger class of vector norms, known as $p-$norms. To create a $p-$norm, we choose a real number $p \ge 1$ and set

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p}$$

Using this definition, we can set $p = \infty$ and define the $\infty-$norm (read "infinity norm"), using the following definition

$$\|\mathbf{x}\|_\infty = \max_{1 \le i \le n} |x_i|$$

Prove $\lim_{p \to \infty} \|\mathbf{x}\|_p = \|\mathbf{x}\|_\infty$