Answers **Lesson 3 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T **F** Any $n \times 1$ column vector $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ is equal to its transpose $\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$.

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2. **T** F Vectors in $\mathbb{R}^4$ correspond to a list of four real numbers that are represented as a stacked column with four rows.

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3. T **F** The vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ are equal.

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4. **T** F Any list of n real numbers can be represented as a vector in $\mathbb{R}^n$ or a vector in $\mathbb{R}^{1\times n}$.

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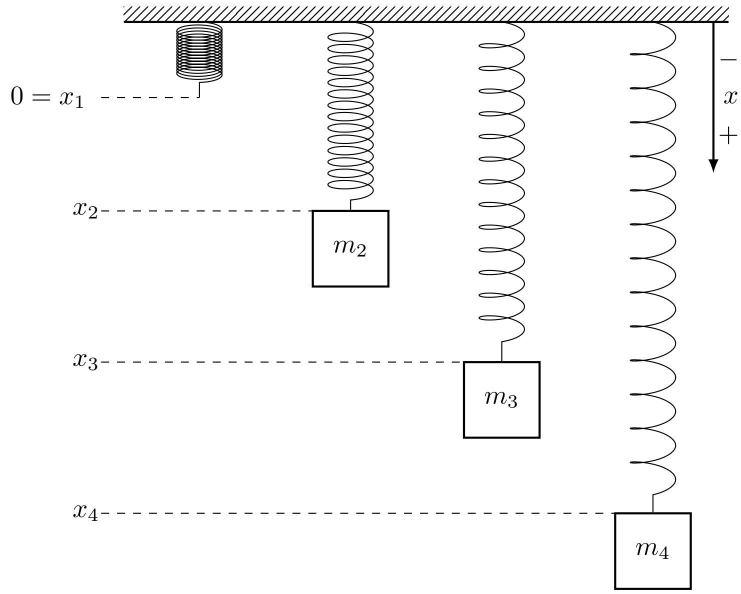
5. T **F** Since all entries of the vectors $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ are zero, these vectors are equal.

Multiple Choice For the problems below, circle the correct response for each question.

1. Consider the experiment below. Suppose we hang three masses on the same spring and record the position data for that spring. Assume the spring constant is known to be $k = 5$ N/m. Assume also that the acceleration due to earth's gravity is $g = 9.8N/kg$. Finally, suppose that the mass of the spring is zero and that this spring satisfy Hooke's law exactly.



In order to model the relationship between the displacement of the movable end of the spring and the internal force stored in the spring, we introduce two 4×1 vectors given by

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Each entry m_i is measured in kg. The entries of the position vector x_i, measured in meters. We know $x_1 = 0$m and the other entries $x_2, x_3, x_4 \in \mathbb{R}$ can be calculated from our knowledge of vector $\mathbf{m}$ and Hooke's Law. Which of the following gives the vector $\mathbf{x}$ in this situation?

A. $\begin{bmatrix} 0.0 \\ 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}$ **B.** $\begin{bmatrix} 0.000 \\ 0.196 \\ 0.392 \\ 0.588 \end{bmatrix}$ C. $\begin{bmatrix} 0.00 \\ 0.02 \\ 0.04 \\ 0.06 \end{bmatrix}$ D. $\begin{bmatrix} 0.0 \\ 0.5 \\ 1.0 \\ 1.5 \end{bmatrix}$ E. $\begin{bmatrix} 0.0 \\ 4.9 \\ 9.8 \\ 14.7 \end{bmatrix}$

Free Response

1. Suppose you are enrolled in a math course in which your final percent score is calculated as a weighted average. Below is a table that describes the important details of this class's grading scheme:

Grade Category on Syllabus	Total Points Available	Percentage Weight
Homework	200	10%
Projects	500	15%
Exam 1	100	20%
Exam 2	100	20%
Final Exam	100	35%

Suppose the teacher of this class does NOT have a grade replacement policy for your exam scores. With this in mind, respond to the following three questions.

A. Set up a vector model $\mathbf{g} \in \mathbb{R}^5$ that encodes all aspects of your course grade. Define each entry of $\mathbf{g}$ and describe your choices.

Solution: To create this vector model, we will define a 5×1 vector given by

$$\mathbf{g} = \begin{bmatrix} \dfrac{h}{200} \\ \dfrac{p}{500} \\ \dfrac{e_1}{100} \\ \dfrac{e_2}{100} \\ \dfrac{e_3}{100} \end{bmatrix}$$

In this case, we will set

$$h = \text{total points earned in homework grade category}$$

$$p = \text{total points earned in project grade category}$$

$$e_1 = \text{total points earned on exam 1}$$

$$e_2 = \text{total points earned on exam 2}$$

$$e_3 = \text{total points earned on the final exam}$$

This grade vector stores the percent score earned in each grade category for this course.

B. Demonstrate how to use the inner-product operation to calculate your final grade in this class.

Solution: In order to calculate the final percent score in this class, we consider the following inner product:

$$\mathbf{g} \cdot \mathbf{c} = \begin{bmatrix} \frac{h}{200} \\ \frac{p}{500} \\ \frac{e_1}{100} \\ \frac{e_2}{100} \\ \frac{e_3}{100} \end{bmatrix} \cdot \begin{bmatrix} 0.10 \\ 0.15 \\ 0.20 \\ 0.20 \\ 0.35 \end{bmatrix} = \frac{h}{200} \cdot 0.10 + \frac{p}{500} \cdot 0.15 + \frac{e_1}{100} \cdot 0.20 + \frac{e_2}{100} \cdot 0.20 + \frac{e_3}{100} \cdot 0.35$$

C. Suppose on the night before the final, you know you've earned the following scores:

Grade Category on Syllabus	Points You Earned
Homework	186
Projects	420
Exam 1	82
Exam 2	90

Assuming you want to get an 85% or above in this class, determine the minimum percent score you will need to earn on the final exam to achieve your goal. Show your work.

Solution: In this case, we are given

$$h = 186, \qquad p = 420, \qquad e_1 = 82, \qquad e_2 = 90$$

and we want to find e_3 such that In order to calculate the final percent score in this class, we consider the following inner product:

$$\left(\frac{186}{200} \cdot 0.10 + \frac{420}{500} \cdot 0.15 + \frac{82}{100} \cdot 0.20 + \frac{90}{100} \cdot 0.20 + \frac{e_3}{100} \cdot 0.35 \right) \geq 0.85.$$

We can isolate e_3 in this inequality to find that

$$e_3 \geq 82.$$

To earn a minimum of 85% in this class, we need to earn a minimum of 82 points on the final exam.