Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	Т	F	Suppose V_1 and V_2 are sets. Then any function is a subset of a cross product of sets in the form $V_1\times V_2$
2.	Т	F	Any relation from V_1 to V_2 is also a function $f: V_1 \to V_2$.
3.	Т	F	A function $f: D \to V_2$ is a subset of $D \times V_2$
4.	Т	F	If $f(\mathbf{x}) = A\mathbf{x}$, then the $\operatorname{Rng}(f) = \{\mathbf{b} \in \mathbb{R}^m : \mathbf{b} = A\mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^n\}$
5.	Т	F	If $f(\mathbf{x}) = \mathbf{x}^T$, then $\text{Dom}(f) = \mathbb{R}^n$
6.	Т	F	Any function is a subset of a cross product of sets
7.	Т	F	All functions are relations.

Multiple Choice For the problems below, circle the correct response for each question.

1. Which of the following represents the 8-bit binary representation of the number 207?A. 11110011B. 11010001C. 10001011D. 10111111E. 11001111

2. Define each of the following sets:

 $C([0,1]) = \{f(x) \mid \text{function } f: [0,1] \to \mathbb{R} \text{ is a continuous function on interval } [0,1]. \}$

 $C^{(1)}([0,1]) = \{f(x) \mid \text{function } f: [0,1] \to \mathbb{R} \text{ has a continuous first derivative } f'(x) \text{ on interval } [0,1]. \}$

In other words, C([0,1]) is the set of continuous functions $f:[0,1] \to \mathbb{R}$ while $C^{(1)}([0,1])$ is the set of functions $f:[0,1] \to \mathbb{R}$ with continuous first derivatives. From Math 1A (Single-Variable, Differential Calculus), we know that if f(x) is differentiable on [0,1], then f is continuous on [0,1]. Identify the set theoretic formulation of this theorem: A. $C([0,1]) \subseteq C^{(1)}([0,1])$ B. $C([0,1]) = C^{(1)}([0,1])$ C. $C([0,1]) \cap C^{(1)}([0,1]) = \emptyset$

D.
$$C^{(1)}([0,1]) \subseteq C([0,1])$$
 E. $C([0,1]) \cup C^{(1)}([0,1]) = C^{(1)}([0,1])$

3. Let 10110110 be an 8-bit binary integer. What is the decimal representation of this number?

A. 109	B. 364	C. 218	D. 80880880	E. 182
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4. Let $E \subseteq \mathbb{R} \times \mathbb{R}$ be given by

$$E = \left\{ (x,y) \, : \, \frac{x^2}{1024} + \frac{y^2}{729} < 1 \right\}$$

Which of the following cannot be true about the relation E?

- A. Dom(E) = [-32, 32] B. Dom(E) = (-32, 32) C. Rng(E) = (-27, 27)
 - D. E is not a function

E. Codomain $(E) = \mathbb{R}$

5. Suppose that we define ellipse $E = \left\{ (x, y) : \frac{x^2}{25} + \frac{y^2}{9} = 1 \right\}$. Find the range, Rng(E), of this relation. A. (-5,5) B. [-5,5] C. [-3,3] D. (-3,3) E. \mathbb{R}

Free Response

- 1. Let A and B be sets. Write the formal set-theoretic definition of a relation from A to B. Give two examples of relations that exist in your daily life.
- 2. Let A and B be sets. Suppose R is a relation from A to B. Write the formal set-theoretic definition of each of the following:

i. Domain space of R	ii. Domain of R	iii. Domain of R	iv. Range of R
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3. Give an example of a relation in which the domain space is not equal to the domain.

4. Give an example of a relation in which the codomain is not equal to the range.

5. State the formal, set-theoretic definition of a mathematical function.

6. Convert the following unsigned decimal numbers into the corresponding unsigned binary representation:
i. 47
ii. 127
iii. 128
7. Convert each of the following unsigned binary numbers into unsigned decimal numbers:

ii. 10111

i. 1001101

iii. 101011011

8. Describe the 2-bit gray-scale color map as a function. Define the domain space and codomain of this function. Describe how this relates to digital images.