$\qquad$
$\qquad$

## Math 2B: Applied Linear Algebra

True/False For the problems below, circle $T$ if the answer is true and circle $F$ is the answer is false.

1. T F Suppose $V_{1}$ and $V_{2}$ are sets. Then any function is a subset of a cross product of sets in the form $V_{1} \times V_{2}$
2. $\mathrm{T} \quad \mathrm{F} \quad$ Any relation from $V_{1}$ to $V_{2}$ is also a function $f: V_{1} \rightarrow V_{2}$.
3. $\mathrm{T} \quad \mathrm{F} \quad$ A function $f: D \rightarrow V_{2}$ is a subset of $D \times V_{2}$
4. $\mathrm{T} \quad \mathrm{F} \quad$ If $f(\mathbf{x})=A \mathbf{x}$, then the $\operatorname{Rng}(f)=\left\{\mathbf{b} \in \mathbb{R}^{m}: \mathbf{b}=A \mathbf{x}\right.$ for some $\left.\mathbf{x} \in \mathbb{R}^{n}\right\}$
5. T F If $f(\mathbf{x})=\mathbf{x}^{T}$, then $\operatorname{Dom}(f)=\mathbb{R}^{n}$
6. T F Any function is a subset of a cross product of sets
7. $\mathrm{T} \quad \mathrm{F}$ All functions are relations.

Multiple Choice For the problems below, circle the correct response for each question.

1. Which of the following represents the 8 -bit binary representation of the number 207 ?
A. 11110011
B. 11010001
C. 10001011
D. 10111111
E. 11001111
2. Define each of the following sets:
$C([0,1])=\{f(x) \mid$ function $f:[0,1] \rightarrow \mathbb{R}$ is a continuous function on interval $[0,1]$.
$C^{(1)}([0,1])=\left\{f(x) \mid\right.$ function $f:[0,1] \rightarrow \mathbb{R}$ has a continuous first derivative $f^{\prime}(x)$ on interval $[0,1]$. \}
In other words, $C([0,1])$ is the set of continuous functions $f:[0,1] \rightarrow \mathbb{R}$ while $C^{(1)}([0,1])$ is the set of functions $f:[0,1] \rightarrow \mathbb{R}$ with continuous first derivatives. From Math 1A (Single-Variable, Differential Calculus), we know that if $f(x)$ is differentiable on $[0,1]$, then $f$ is continuous on $[0,1]$. Identify the set theoretic formulation of this theorem:
A. $C([0,1]) \subseteq C^{(1)}([0,1])$
B. $C([0,1])=C^{(1)}([0,1])$
C. $C([0,1]) \cap C^{(1)}([0,1])=\emptyset$

$$
\text { D. } C^{(1)}([0,1]) \subseteq C([0,1]) \quad \text { E. } C([0,1]) \cup C^{(1)}([0,1])=C^{(1)}([0,1])
$$

3. Let 10110110 be an 8 -bit binary integer. What is the decimal representation of this number?
A. 109
B. 364
C. 218
D. 80880880
E. 182
4. Let $E \subseteq \mathbb{R} \times \mathbb{R}$ be given by

$$
E=\left\{(x, y): \frac{x^{2}}{1024}+\frac{y^{2}}{729}<1\right\}
$$

Which of the following cannot be true about the relation $E$ ?
A. $\operatorname{Dom}(E)=[-32,32]$
B. $\operatorname{Dom}(E)=(-32,32)$
C. $\operatorname{Rng}(E)=(-27,27)$
D. E is not a function
E. Codomain $(E)=\mathbb{R}$
5. Suppose that we define ellipse $E=\left\{(x, y): \frac{x^{2}}{25}+\frac{y^{2}}{9}=1\right\}$. Find the range, $\operatorname{Rng}(E)$, of this relation.
A. $(-5,5)$
B. $[-5,5]$
C. $[-3,3]$
D. $(-3,3)$
E. $\mathbb{R}$

## Free Response

1. Let $A$ and $B$ be sets. Write the formal set-theoretic definition of a relation from $A$ to $B$. Give two examples of relations that exist in your daily life.
2. Let $A$ and $B$ be sets. Suppose $R$ is a relation from $A$ to $B$. Write the formal set-theoretic definition of each of the following:
i. Domain space of $R$
ii. Domain of $R$
iii. Domain of $R$
iv. Range of $R$
3. Give an example of a relation in which the domain space is not equal to the domain.
4. Give an example of a relation in which the codomain is not equal to the range.
5. State the formal, set-theoretic definition of a mathematical function.
6. Convert the following unsigned decimal numbers into the corresponding unsigned binary representation:
i. 47
ii. 127
iii. 128
7. Convert each of the following unsigned binary numbers into unsigned decimal numbers:
i. 1001101
ii. 10111
iii. 101011011
8. Describe the 2-bit gray-scale color map as a function. Define the domain space and codomain of this function. Describe how this relates to digital images.
