Answers **Lesson 1 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false. After you've chosen your answer, mark the appropriate space on your Scantron form. Notice that letter A corresponds to true while letter B corresponds to false.

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1. **T** F $\mathbb{Z} \subseteq \mathbb{Q}$

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2. T **F** $0 \in \mathbb{N}$

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3. **T** F $\mathbb{R} \not\subseteq \mathbb{Z}$

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4. **T** F $\sqrt{2} \notin \mathbb{Q}$

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5. **T** F $0 \in \mathbb{Z}$

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6. T **F** The set of nonnegative integers is equal to the set of positive integers.

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7. T **F** $\mathbb{Q} \subseteq \mathbb{N}$

Multiple Choice

For the problems below, circle the correct response for each question.

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1. Let A and B be sets. What does it mean if we say that A is a subset of B?

   A. Some element x in A is also an element of B.

   B. A is an element of B.

   C. Every element x in A is contained in some element y of B.

   D. Every element x in A is also an element of B.

   E. Every element y in B is also an element of A.

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2. For sets A and B, the statement "If $x \in A$, then $x \in B$" is written using which of the following?

   A. $A \leq B$ B. $B \subseteq A$ C. $A = B$ **D. $A \subseteq B$** E. $A \neq B$

Free Response

1. Let $A = \{n \in \mathbb{N} : n - 4 < 10\}$ and $B = \{m \in \mathbb{N} : m^2 \leq 169\}$. Prove that A = B.

Solution: Recall that for a direct proof that $A = B$, we must show that:

I. $A \subseteq B$

II. $B \subseteq A$

Moreover, we recall from lesson 1 that in order to verify the subset relation, we can use the following direct proof method:

Direct Proof of $A \subseteq B$
Proof:
Suppose $x \in A$.

$\vdots$

Therefore, $x \in B$.
By definition, $A \subseteq B$.

Part I: We begin our formal proof by showing $A \subseteq B$:

Proof. Suppose $x \in A$. Then, we can immediately conclude

$$\begin{aligned} x \in A &\implies x \in \mathbb{N} \text{ and } x - 4 < 10 \\ &\implies x \in \mathbb{N} \text{ and } x < 14 \\ &\implies x \in \mathbb{Z} \text{ and } 0 < x < 14 \\ &\implies x \in \{1, 2, 3, 4, ..., 13\} \\ &\implies x^2 \in N \text{ and } 1 < x^2 \leq 169 \\ &\implies x \in B \end{aligned}$$

Thus, we've shown that if $x \in A$, then $x \in B$ and we conclude that $A \subseteq B$ as desired. □

Part II: We continue our formal proof by showing $B \subseteq A$:

Proof. Suppose $x \in B$. Then,

$$\begin{aligned} x \in B &\implies x \in \mathbb{N} \text{ and } x^2 \leq 169 \\ &\implies x \in \mathbb{N} \text{ and } x \leq 13 \\ &\implies x \in \mathbb{N} \text{ and } x < 14 \\ &\implies x \in N \text{ and } x - 4 < 10 \\ &\implies x \in A \end{aligned}$$

We conclude that $B \subseteq A$ since every element of B is also an element of A. □

Given parts I and II above, we know that $A = B$.