Name : ________________ **Lesson 18 Warm Up Quiz** **Class Number:** ______

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T F There are three types of elementary row reductions we will use to solve linear systems.

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2. T F Let $A, \mathbf{b}$ be given. If $f(\mathbf{x}) = A\mathbf{x}$, then a solution to the corresponding set of linear system $A\mathbf{x} = \mathbf{b}$ exists if and only if $\mathbf{b}$ is in the codomain of f

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3. T F The homogeneous equation $A\mathbf{x} = \mathbf{0}$ can be inconsistent.

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4. T F If $A \in \mathbb{R}^{m\times n}$ such that $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^m$, then A has m pivot columns.

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5. T F If a system of linear equations has two different solutions, it must have infinitely many solutions.

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6. T F Let $A \in \mathbb{R}^{m\times n}$ and suppose $\mathbf{x} \in \mathbb{R}^n$. Suppose $\mathbf{b} \in \mathbb{R}^m$ is nonzero. Suppose $\mathbf{x}_1^*$ and $\mathbf{x}_2^*$ are solutions to the inhomogeneous system $A\mathbf{x} = \mathbf{b}$. Then any linear combination $c_1\mathbf{x}_1^* + c_2\mathbf{x}_2^*$ is a solution to the linear system $A\mathbf{x} = \mathbf{b}$.

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7. T F Let $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$ be given. The solution set of an inhomogeneous equation $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{x} = \mathbf{x}^* + \mathbf{z}$ where $\mathbf{x}^*$ is a particular solution to the linear system and $\mathbf{z}$ is a solution to the homogeneous system $A\mathbf{x} = \mathbf{0}$.

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8. T F If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

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9. T F If a system of linear equations has no free variables, then is always a unique solution.

10. T F The superposition principle for inhomogeneous systems generalizes the invertible matrix theorem by classifying completely the existence and uniqueness results for a general linear system $A\mathbf{x} = \mathbf{b}$, where $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$ are given, and $\mathbf{x} \in \mathbb{R}^n$ is unknown.

11. T F The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has nonzero solution if and only if the equivalent reduced row echelon form of the matrix equation has at least one free variable.

12. T F Given A and $\mathbf{b}$ of appropriate dimensions, the linear system $A\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ can be written as a linear combination of the vectors $\{A(:,1), A(:,2), ..., A(:,n)\}$.

13. T F If $A \in \mathbb{R}^{m\times n}$ has m linearly independent rows, then the solution to $A\mathbf{x} = \mathbf{b}$ is unique for each $\mathbf{b} \in \mathbb{R}^m$.

14. T F Let $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$ be given. The solution set to a linear system with an augmented matrix

$$\begin{bmatrix} A & \mathbf{b} \end{bmatrix}$$

is identical to the solution set to the linear systems problem $A\mathbf{x} = \mathbf{b}$.

15. T F If $A \in \mathbb{R}^{n\times n}$ has n linearly independent columns, then A is row equivalent to the $n \times n$ identity matrix I_n.

16. T F If $A \in \mathbb{R}^{m\times n}$ has m linearly independent rows, then the function $f(\mathbf{x}) = A\mathbf{x}$ is a one-to-one mapping.

17. T F The general solution to a linear system problem is a description of every element of the solution set to the given linear system.

18. T F Let $A, \mathbf{b}$ be given. If $f(\mathbf{x}) = A\mathbf{x}$, then a solution to the corresponding set of linear system $A\mathbf{x} = \mathbf{b}$ exists if and only if $\mathbf{b}$ is in the range of f

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19. T F Let $A, \mathbf{x}$ be given. If $g(\mathbf{b}) = A^T\mathbf{b}$, then a solution to the corresponding set of linear system $A^T\mathbf{b} = \mathbf{x}$ exists if and only if $\mathbf{x}$ is in the range of g

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20. T F Suppose we have the basis $\{\mathbf{z}_1, ..., \mathbf{z}_d\}$ for $\mathrm{Nul}(A)$ with $d \leq n$. Suppose we have $\mathbf{x}_1^*$ as a particular solution to a given inhomogeneous system $A\mathbf{x} = \mathbf{b}$. Then we can construct any solution to this system as $\mathbf{x} = \mathbf{x}_1^* + \sum_{j=1}^{d} c_j \mathbf{z}_j$.

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21. T F Consider the linear system with m equations and n unknowns given by

$$\begin{aligned} a_{11}x_1 + \ a_{12}x_2 + \cdots + \ a_{1n}x_2 &= \ b_1 \\ a_{21}x_1 + \ a_{22}x_2 + \cdots + \ a_{2n}x_2 &= \ b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_2 &= b_m \end{aligned}$$

where scalars $a_{ik}, b_i \in \mathbb{R}$ are given for all $i \in \{1, 2, ..., m\}$ and $k \in \{1, 2, ..., n\}$ and variables $x_1, x_2, ..., x_n$ are unknown real numbers. The solution set of this linear system is a list of n numbers $y_1, y_2, ..., y_n$ such that if each y_k is substituted for the corresponding x_k, then all m equations will be true.

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22. T F Any two linear systems problems with the same solution set must be equivalent linear systems.

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23. T F To solve a linear system, it is sufficient to find a general parametric equation that describes all elements in the solution set of this linear system.

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24. T F All linear systems that have a coefficient matrix with free variables have infinitely many solutions.

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25. T F Let $A, \mathbf{b}$ be given. If $f(\mathbf{x}) = A\mathbf{x}$, then a solution to the corresponding set of linear system $A\mathbf{x} = \mathbf{b}$ exists if and only if $\mathbf{b}$ is in the codomain of f

26. T F If $f(\mathbf{x}) = A\mathbf{x}$ and $\mathbf{b} \in \text{Rng}(f)$, then $\mathbf{b}$ is linearly independent from the columns of A.

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27. T F Let $A, \mathbf{x}$ be given. If $g(\mathbf{b}) = A^T\mathbf{b}$, then a solution to the corresponding set of linear system $A^T\mathbf{b} = \mathbf{x}$ exists if and only if $\mathbf{x}$ is in the range of g

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28. T F If $\mathbf{b} \notin \text{Col}(A)$, then we want to find a solution to $A\mathbf{x} = \mathbf{b}$ using $\text{RREF}(A)$ and the superposition principle.

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29. T F The superposition principle for inhomogeneous systems generalizes the invertible matrix theorem by classifying completely the existence and uniqueness results for a general linear system $A\mathbf{x} = \mathbf{b}$, where $A \in \mathbb{R}^{m\times n}$ and $\mathbf{b} \in \mathbb{R}^m$ are given, and $\mathbf{x} \in \mathbb{R}^n$ is unknown.

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30. T F Solving linear systems using elementary row reductions is equivalent to changing a matrix equation using linear combinations on the rows of that matrix.

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31. T F If $A \in \mathbb{R}^{m\times n}$ such that $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b} \in \mathbb{R}^m$, then A has m pivot columns.

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32. T F Consider the linear systems problem

$$A\mathbf{x} = \mathbf{b}$$

where matrix $A \in \mathbb{R}^{m\times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. If this linear system is inconsistent, there may be an $\mathbf{x} \in \mathbb{R}^n$ such that

$$\|\mathbf{b} - A\mathbf{x}\|_2 = 0.$$

Multiple Choice

For the problems below, circle the correct response for each question.

1. Let

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

By performing elementary row operations on this matrix we see that A is row equivalent to the matrix

$$U = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 0 & 1 & -1 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following represents the solution set S of the linear system $A\mathbf{x} = \mathbf{b}$?

A. $S = \left\{ \mathbf{x} \in \mathbb{R}^4 \, : \, \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 10 \\ 0 \\ 1 \end{bmatrix} \right\}$

B. $S = \left\{ \mathbf{x} \in \mathbb{R}^4 \, : \, \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -17 \\ -10 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $S = \left\{ \mathbf{x} \in \mathbb{R}^4 \, : \, \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 17 \\ 10 \\ 0 \\ 1 \end{bmatrix} \right\}$

D. $A\mathbf{x} \neq \mathbf{b}$ for all $\mathbf{x} \in \mathbb{R}^4$.

E. None of these

2. Suppose $A \in \mathbb{R}^{m \times n}$. Given a nonzero vector $\mathbf{b} \in \mathbb{R}^m$, suppose that you know:

I. Vectors $\mathbf{z}_1, \mathbf{z}_2 \in \mathbb{R}^n$ solve the linear system problem $A\mathbf{x} = \mathbf{0}$

II. Vectors $\mathbf{x}^*, \mathbf{y}^* \in \mathbb{R}^n$ solve the linear system problem $A\mathbf{x} = \mathbf{b}$.

Which of the following is NOT a solution for the linear system problem $A\mathbf{x} = \mathbf{b}$?

A. $\mathbf{x}^* + \mathbf{z}_1$ B. $\mathbf{y}^* + \mathbf{z}_2$ C. $\mathbf{x}^* + \mathbf{y}^*$ D. $3\mathbf{z}_1 + \mathbf{x}^* - 4\mathbf{z}_2$ E. $2\mathbf{x}^* - \mathbf{y}^*$

Free Response

1. Consider the following general linear-systems problem:

$$\underbrace{\begin{bmatrix} -2 & -1 & 6 & -5 & 0 & -3 & 4 & -17 \\ -3 & 2 & 16 & 3 & 7 & 13 & 8 & -7 \\ 1 & 0 & -4 & 1 & 0 & 2 & 2 & -2 \\ 3 & 0 & -12 & 3 & 1 & 9 & 3 & 8 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ 16 \\ 3 \\ 7 \end{bmatrix}}_{\mathbf{b}}$$

   A. How many linearly independent solutions to $A \cdot \mathbf{x} = \mathbf{0}$ are there? Find all linearly independent solutions to this homogeneous equation $A \cdot \mathbf{x} = \mathbf{0}$?

   B. Find the solution set for this GLSP. Is the solution set to the GLSP a subspace of $\mathbb{R}^8$? (Hint: recall the definition of a subspace of a vector space)