Name : ________________ **Lesson 17 Warm Up Quiz** **Class Number**: ______

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T F The non-pivot columns of a matrix are always linearly dependent on the other columns of that matrix.

2. T F If matrices $A, B \in \mathbb{R}^{m\times n}$ are row equivalent, they have the same reduced echelon form.

3. T F By interchanging two rows of a matrix (i.e. by multiplying by transposition matrices), we can change the location of the pivot positions in the RREF form of the matrix.

4. T F The equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\mathbf{x} = \mathbf{0}$ if and only if there are no free variables.

5. T F An elementary matrix must be square and invertible.

6. T F The column index of the pivot columns of a matrix can be changed using row operations on the matrix.

7. T F Any $n \times n$ elementary matrix (dilation, shear, permutation) has at least n nonzero entries and at most $n+1$ nonzero entries.

8. T F Sometimes the linear dependence relationships between columns of a matrix will be affected by elementary row operations on that matrix.

9. T F Every matrix is row equivalent to a unique matrix in echelon form.

10. T F Given a matrix $B \in \mathbb{R}^{m\times n}$ in echelon form, a basis for the $\mathrm{Col}(B)$ can be generated using the pivot columns of B (the columns with a single nonzero entry).

11. T F If A is a 3×3 matrix with three pivot positions, there exist elementary matrices $E_1, E_2, ..., E_t$ such that $E_t \cdots E_2 E_1 A = I_3$.

12. T F A free variable in a linear system corresponds to a non-pivot column in the coefficient matrix of the linear system.

13.	T	F	The echelon form of a matrix is unique.
14.	T	F	A basic variable in a linear system corresponds to a pivot column in the coefficient matrix of the linear system.
15.	T	F	Solving linear systems using elementary row reductions is equivalent to changing a matrix equation using linear combinations on the rows of that matrix.
16.	T	F	Suppose $A, B \in \mathbb{R}^{m\times n}$ for some $m, n \in \mathbb{N}$. If $A(i,:) = B(i,:)$ for some $i \in \{1, 2, ..., m\}$ then A must be row equivalent to B.
17.	T	F	Performing row operations on matrix $A \in \mathbb{R}^{m\times n}$ via multiplication by a sequence of elementary matrices $E_1, E_2, ..., E_t \in \mathbb{R}^{m\times m}$ can change the linear dependence relationships between the columns of A.
18.	T	F	If $A \in \mathbb{R}^{m\times n}$ is row equivalent to a matrix $U \in \mathbb{R}^{m\times n}$ in echelon form, and if the matrix U has k nonzero rows, then the dimension of the solution space for $A\mathbf{x} = \mathbf{0}$ is $m - k$.
19.	T	F	If two matrices are row equivalent, then they have the same number of rows.
20.	T	F	If the matrix equation $A\mathbf{x} = \mathbf{b}$ is transformed into matrix equation $C\mathbf{x} = \mathbf{d}$ via elementary row operations, then the solutions sets of both equations are identical.
21.	T	F	If $A \in \mathbb{R}^{m\times n}$ is row equivalent to $B \in \mathbb{R}^{m\times n}$ and the columns of A span $\mathbb{R}^m$ then so do the columns of B.
22.	T	F	Every elementary row operation is reversible.
23.	T	F	Two matrices are row equivalent if they have the same number of rows.
24.	T	F	We only use row reduction techniques on augmented matrices associated with some linear system.

25. T F Consider the linear systems problem

$$A\mathbf{x} = \mathbf{b}$$

where matrix $A \in \mathbb{R}^{m\times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. Suppose we apply elementary row operations on our linear system $A\mathbf{x} = \mathbf{b}$ to produce a new linear system $C\mathbf{x} = \mathbf{d}$. Then, the solution sets to these two linear systems are identical.

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26. T F The row reduced echelon form of a matrix is not unique. In other words, in some cases, a matrix may be row reduced to more than one matrix in reduced row echelon form using a different sequence of row operations.

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27. T F There are two steps to reducing a matrix to reduced row echelon form including:

    Step 1: Forward Phase- the reduction of the matrix to row echelon form
    Step 2: Backward Phase- the reduction of the row echelon form of the matrix into reduced row echelon form.

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28. T F There are three conditions that every matrix in row echelon form must satisfy.

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29. T F If B is an echelon form of A, then the pivot columns of B form a basis for the column space of A.

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30. T F There are three types of elementary row reductions we will use to solve linear systems.

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31. T F Every matrix is row equivalent to a unique matrix in row echelon form.

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32. T F Every matrix is row equivalent to a unique matrix in reduced row-echelon form.

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33. T F Suppose we are given

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$$

Then, there exists an $\mathbf{x} \in \mathbb{R}^2$ such that $\|A \cdot \mathbf{x} - \mathbf{b}\|_2 = 0$

Multiple Choice

For the problems below, circle the correct response for each question.

1. Let the matrix $A \in \mathbb{R}^{3\times 5}$ be given by

$$A = \begin{bmatrix} 1 & -1 & 4 & 3 & 5 \\ 0 & 1 & -2 & -4 & 6 \\ 0 & 0 & 0 & 1 & -9 \end{bmatrix}.$$

Let $T(\mathbf{x}) = A\mathbf{x}$. Which of the following is true:

A. The codomain of T is $\mathbb{R}^5$.

B. The range of T is the same as the codomain.

C. T is one-to-one.

D. T is a bijection.

E. None of these.

For the next four problems, assume that the matrix $A \in \mathbb{R}^{4\times 6}$ is given by

$$A = \begin{bmatrix} 1 & 2 & -5 & -2 & 6 & 14 \\ 0 & 0 & -2 & -2 & 7 & 12 \\ 2 & 4 & -5 & 1 & -5 & -1 \\ 0 & 0 & 4 & 4 & -14 & -24 \end{bmatrix}$$

2. Find RREF(A):

A. $\begin{bmatrix} 1 & 2 & -5 & -2 & 6 & 14 \\ 2 & 4 & -5 & 1 & -5 & -1 \\ 0 & 0 & -2 & -2 & 7 & 12 \\ 0 & 0 & 4 & 4 & -14 & -24 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 0 & 2 & 3 & 7 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 2 & -2.5 & 0.5 & -2.5 & -0.5 \\ 0 & 0 & 1 & 1 & -3.5 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. Which of the following vectors in NOT a solution to $A\mathbf{x} = \mathbf{0}$?

A. $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} 3 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ C. $\begin{bmatrix} 7 \\ 0 \\ 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$ D. $\begin{bmatrix} -4 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 9 \\ 0 \\ 3 \\ -3 \\ 0 \\ 0 \end{bmatrix}$ E. $\begin{bmatrix} 6 \\ 0 \\ 2 \\ -2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 0 \\ -1 \\ 0 \\ -2 \\ -1 \end{bmatrix}$

4. Which of the following sets of vectors are linearly dependent? Choose all that apply.

A. $\{A(:,1), A(:,3), A(:5)\}$ B. $\{A(:,2), A(:,3), A(:6)\}$ C. $\{A(:,1), A(:,3), A(:4)\}$

D. $\{A(:,1), A(:,4), A(:5)\}$ E. $\{A(:,2), A(:,4), A(:6)\}$

5. Find $\dim\big(\mathrm{Nul}(A)\big) + \dim\big(\mathrm{Nul}(A^T)\big)$:

A. 1 B. 2 C. 3 D. 4 E. 5

6. Which of the following must be true? Choose all that apply.

A. $\mathrm{rank}\left(A^T\right) = 3$ B. $\mathrm{Col}\left(A^T\right) \subseteq \mathbb{R}^6$ C. $\left(AA^T\right)^{-1}$ exists

D. $\left(A^T A\right)^{-1}$ exists E. $\mathrm{Col}(A) = \mathbb{R}^3$

For the next three problems, assume that the matrix $A \in \mathbb{R}^{4\times 7}$ is given by

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 0 & 0 & 1 \\ 2 & 6 & -5 & -2 & -1 & -2 & 1 \\ 0 & 0 & 5 & 10 & 5 & 10 & 5 \\ 2 & 6 & 0 & 8 & 4 & 12 & 8 \end{bmatrix}$$

7. Find RREF(A):

A. $\begin{bmatrix} 1 & 3 & -2.5 & -1 & -.5 & -1 & 0.5 \\ 0 & 0 & 1 & 2 & 1 & 2 & 1.0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 1.0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0.0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -3 & 0 & 4 & 2 & 0 & -1.0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0.0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 0 & 0 & 3 & 4 & 2 & 1.0 \\ 0 & 1 & 0 & 0 & 2 & 1 & 0.0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

8. How many linearly independent solutions are there to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$:

A. 1 B. 3 C. 4 D. 5 E. 7

9. Which of the following is NOT a solution for the linear-systems problem $A\mathbf{x} = \mathbf{0}$?

A. $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ B. $\begin{bmatrix} 4 \\ 0 \\ 2 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ C. $\begin{bmatrix} -2 \\ 0 \\ -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ D. $\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -2 \end{bmatrix}$ E. $\begin{bmatrix} 1.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.5 \\ 1.0 \end{bmatrix}$

Free Response

1. Let $A \in \mathbb{R}^{m \times n}$. Recall the definition of the $\text{Nul}(A)$ and $\text{Col}(A)$.

2. Consider the following matrix:

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$$

   A. Transform A into $U = \text{RREF}(A)$ using elementary row operations. Show your steps.

   B. Prove that the linearly dependence relations between the columns of U are identical to the linear dependence relations on the columns of A.

   C. Using information found in U, specifically identify the linearly independent columns of A.

   D. Using information found in U, specifically identify the linearly dependent columns of A. Then, for each linearly dependent column of A, write this column as a linear combination of the previous columns.

3. Transform the general linear-systems problem:

$$\underbrace{\begin{bmatrix} 1 & 3 & 1 & 3 & 3 & 5 \\ 2 & 6 & 0 & 4 & 4 & 0 \\ 1 & 3 & 3 & 5 & 5 & 15 \\ 2 & 6 & 0 & 4 & 7 & 9 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} -3 \\ -4 \\ -5 \\ -13 \end{bmatrix}}_{\mathbf{b}}$$

   into an equivalent system $U \cdot \mathbf{x} = \mathbf{y}$ where $U = \text{RREF}(A)$.

4. What is our strategy to solve the general linear-systems problem? Compare and contrast this strategy with the technique we used to solve the square linear-systems problem.