Name : ________________ **Lesson 15 Warm Up Quiz** **Class Number:** ______

Math 2B: Applied Linear Algebra

True/False

For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T F Backward substitution is used to solve lower-triangular, linear systems.

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2. T F The diagonal elements of upper triangular U from the LU factorization of A are unique.

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3. T F Let $A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 7 & 7 \\ 6 & 18 & 22 \end{bmatrix}$. Notice that

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 4 \end{bmatrix}.$$

Let $\mathbf{x} \in \mathbb{R}^3$ be the vector such that $A\mathbf{x} = \begin{bmatrix} 6 \\ 24 \\ 70 \end{bmatrix}$. Then $x_1 + x_2 + x_3 = 3$.

Multiple Choice

For the problems below, circle the correct response for each question.

1. Suppose we are given a matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 5 & -2 \\ 2 & -2 & 0 \end{bmatrix}$.

   Find the matrix $L \in \mathbb{R}^{3\times 3}$ from the LU factorization of A.

   A. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$
   B. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$
   C. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$
   D. $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix}$
   E. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$

2. The LU Factorization of a given 3×3 matrix is $A = \begin{bmatrix} 2 & 6 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.5 & -1 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 6 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4.5 \end{bmatrix}}_{U}$.

   Use the LU factorization of A combined with forward and backward substitution to find the solution to the linear systems problem

$$\begin{bmatrix} 2 & 6 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -9 \end{bmatrix}$$

   Which of the following gives $-x_1 + x_2 + x_3$?

   A. -2
   B. -1
   C. 0
   D. 1
   E. 2

3. Below we are given the LU Factorization of the matrix A and vector $\mathbf{b}$:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 6 & -4 & 2 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & -5 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 6 \\ 0 \end{bmatrix}$$

Use this LU Factorization to solve the linear system $A\mathbf{x} = \mathbf{b}$ by solving the two linear systems

$$L\mathbf{y} = \mathbf{b} \qquad \text{and} \qquad U\mathbf{x} = \mathbf{y}$$

Then find $\mathbf{y}^T\mathbf{x}$:

A. -22 B. 22 C. -16 D. -18 E. 18

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4. Consider the following matrix equation

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -5 & 1 \end{bmatrix}}_{L_3} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}}_{L_2} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}}_{L_1} \cdot \underbrace{\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 3 & 1 \\ 3 & 0 & 2 & 2 \\ 1 & 4 & 0 & 1 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & -35 \end{bmatrix}}_{U}$$

Find the lower-triangular matrix $L \in \mathbb{R}^{4\times 4}$ from the LU factorization of the matrix A:

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 3 & 1 & 0 \\ 1 & -3 & 5 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -3 & -3 & 1 & 0 \\ -1 & 3 & -5 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -3 & 1 & 0 \\ -22 & 18 & -5 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 9 & 3 & 1 & 0 \\ 40 & 12 & 5 & 1 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 2 & -1 & -1 & 0 \\ 2 & -3 & 5 & -35 \end{bmatrix}$

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5. Consider the 3×3 matrices given by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

As discussed in class, we can multiply the matrix A by a sequence of three elementary matrices E_1, E_2, E_3 to produce the upper-triangular matrix $U \in \mathbb{R}^{3\times 3}$ with

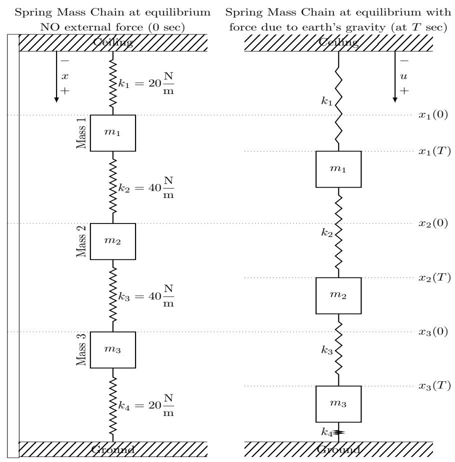
$$E_3 \cdot E_2 \cdot E_1 \cdot A = U.$$

Which of the following matrices is <u>NOT</u> one of the elementary matrices E_i we used to accomplish this transformation?

A. $\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -9 & 0 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1.5 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

Free Response

For problems 1 - 2 below, consider the following mass-spring chain:



1. Recall from our previous discussion, this system results in a linear systems problem

$$\underbrace{\begin{bmatrix} 60 & -40 & 0 \\ -40 & 80 & -40 \\ 0 & -40 & 60 \end{bmatrix}}_{K} \cdot \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}}_{\mathbf{u}} = 9.8 \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{\mathbf{m}}$$

   Find the LU Factorization of matrix K by multiplying K on the left by a sequence of elementary matrices.

2. Use the LU factorization method we have discussed in class to find the displacement vector $\mathbf{u} \in \mathbb{R}^3$ measured in meters at $t = T$ when the system is at equilibrium under the force of gravity on earth in each of the following cases:

   A. $\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.400 \\ 0.200 \end{bmatrix}$ B. $\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 1.0 \\ 0.5 \end{bmatrix}$

For problems 3 - 4, consider the nonsingular linear-systems problem

$$\underbrace{\begin{bmatrix} 0 & 2 & 1 \\ 2 & 6 & 1 \\ 1 & 1 & 4 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 0 \\ -2 \\ 8 \end{bmatrix}}_{\mathbf{b}}$$

3. Calculate the LU factorization of A from problem 1 above.

4. Solve the original linear-systems problem above using the LU Factorization you found in problem 3.