Answers **Lesson 14 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	**T**	F	For square matrices A, B , if $AB = I$, then A is invertible.
2.	T	**F**	For matrices A, B with proper dimensions, If $AB = I_n$, then A is invertible.
3.	T	**F**	Every square matrix is a product of elementary matrices.
4.	**T**	F	Every invertible matrix is a product of elementary matrices.
5.	**T**	F	If A is a 3×3 matrix with three pivot positions, then for some $t \in \mathbb{N}$ there exist elementary matrices $E_1, E_2, ..., E_t \in \mathbb{R}^3$ such that $E_t \cdots E_2 \cdot E_1 \cdot A = I_3$.
6.	T	**F**	Any square matrix with nonzero diagonal entries must be invertible.

Free Response

1. Show that the matrix $A \in \mathbb{R}^{5\times 5}$ given by

$$A = \begin{bmatrix} 0 & a & 0 & 0 & 0 \\ b & 0 & c & 0 & 0 \\ 0 & d & 0 & e & 0 \\ 0 & 0 & f & 0 & g \\ 0 & 0 & 0 & h & 0 \end{bmatrix}$$

in not invertible for any nonzero values $a, b, c, d, e, f, g, h \in \mathbb{R}$. Explain your solution.

Solution: We know that $A \in \mathbb{R}^{5\times 5}$ is invertible if and only if the columns of A are linearly independent. Consider the columns

$$A(:,1) = \begin{bmatrix} 0 \\ b \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad A(:,3) = \begin{bmatrix} 0 \\ c \\ 0 \\ d \\ 0 \end{bmatrix}, \qquad A(:,5) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ g \\ 0 \end{bmatrix}$$

By assumption we know $b \neq 0$ and $g \neq 0$. We claim that these is a nonzero linear combinations of these three column that produces a zero output. To this end, define scalar weights

$$\alpha_1 = -\frac{c}{b}, \qquad \alpha_2 = -\frac{f}{g}$$

Consider the linear combination

$$\alpha_1 A(:,1) + A(:,3) + \alpha_2 A(:,5) = \left(-\frac{c}{b}\right) \cdot \begin{bmatrix} 0 \\ b \\ 0 \\ 0 \\ 0 \end{bmatrix} + 1 \cdot \begin{bmatrix} 0 \\ c \\ 0 \\ d \\ 0 \end{bmatrix} + \left(-\frac{f}{g}\right) \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ g \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus we see that the vector

$$\mathbf{x} = \begin{bmatrix} \alpha_1 \\ 0 \\ 1 \\ 0 \\ \alpha_2 \end{bmatrix}$$

satisfies the equation $A \cdot \mathbf{x} = \mathbf{0}$. This immediately implies that the columns of A must be linearly dependent and thus A is not invertible.