Name : ________________ **Lesson 13 Warm Up Quiz** **Class Number:** _____

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T F For square $A, B \in \mathbb{R}^{n\times n}$, if $AB = BA$ and if A is invertible, then $A^{-1}B = BA^{-1}$.

2. T F If $A, B \in \mathbb{R}^{n\times n}$ invertible, then the product $AB \in \mathbb{R}^{n\times n}$ is also invertible.

3. T F If A and B are square and invertible, then AB is invertible, and $(AB)^{-1} = A^{-1}B^{-1}$.

4. T F If $A \in \mathbb{R}^{n\times n}$ is singular, then the columns of A form a basis for $\mathbb{R}^n$.

5. T F If $A, B \in \mathbb{R}^{n\times n}$ invertible, then the sum $A + B \in \mathbb{R}^{n\times n}$ is also invertible.

6. T F The transpose of a square $n \times n$ shear matrix $S_{ij}(c)$ is the inverse of that matrix. In other words $S_{ji}(c) = (S_{ij}(c))^{-1}$.

7. T F Any square matrix $A \in \mathbb{R}^{n\times n}$ with nonzero diagonals is invertible

8. T F If A is invertible and $c \neq 0$ is a real number, then $(cA)^{-1} = cA^{-1}$.

9. T F Let $A \in \mathbb{R}^{n\times n}$. If $\{E_j\}_{j=1}^k \subseteq \mathbb{R}^{n\times n}$ is a set of elementary matrices such that

$$E_k E_{k-1} \cdots E_2 E_1 A = I_n,$$

then $A^{-1} = E_1^{-1} E_2^{-1} \cdots E_{k-1}^{-1} E_k^{-1}$.

10. T F Let $n \in \mathbb{N}$ and suppost $i, j, k \in \{1, 2, ..., n\}$ with $i \neq j$. Let $P_{ij} \in \mathbb{R}^{n\times n}$ be the permutation matrix that transposes rows i and j. Then

$$\Big(P_{ij} \cdot S_{jk}(c)\Big)^{-1} = S_{kj}(-c).$$

11. T F If a square matrix $A \in \mathbb{R}^{n\times n}$ has a zero on its main diagonal, then it is singular.

12. T F If A is a 3×3 matrix and the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution then A is invertible.

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13. T F Let $n \in \mathbb{N}$ and suppose $i, k \in \{1, 2, ..., n\}$ with $i \neq k$. Then

$$\left((S_{ik}(c))^T\right)^{-1} = S_{ki}(-c).$$

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14. T F For square $A, B \in \mathbb{R}^{n\times n}$, if $AB = BA$ and if A is invertible, then $AB^{-1} = B^{-1}A$.

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15. T F If $A, B \in \mathbb{R}^{n\times n}$ invertible, then the product $AB \in \mathbb{R}^{n\times n}$ is also invertible.

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16. T F If $A \in \mathbb{R}^{3\times 3}$ has three pivot columns, then it is possible to find invertible matrices $E_1, E_2, ..., E_p \in \mathbb{R}^{3\times 3}$ such that

$$E_p E_{p-1} \cdots E_2 E_1 A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Multiple Choice For the problems below, circle the correct response for each question.

1. Let $M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ Then M^{-1} is given by which of the following:

A. $\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$

B. $\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ -0.5 & 0.5 & 0 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & -0.5 & 0.5 \end{bmatrix}$

C. $\begin{bmatrix} -0.5 & 0 & 0 & 0 \\ -0.5 & -0.5 & 0 & 0 \\ 0 & -0.5 & -0.5 & 0 \\ 0 & 0 & -0.5 & -0.5 \end{bmatrix}$

D. $\begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0.5 & -0.5 & 0 & 0 \\ 0 & 0.5 & -0.5 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix}$

E. None of these.

2. Let $M = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 1 \end{bmatrix}$ Then M^{-1} is given by which of the following:

A. $M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 5 & 1 \end{bmatrix}$

B. $M^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 5 & -1 \end{bmatrix}$

C. $M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -3 & 5 & 1 \end{bmatrix}$

D. $M^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 5 & 1 \end{bmatrix}$.

E. None of these.

3. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$. If $B = A^{-1}$, which of the following gives $B(1,:)$?

A. $\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 2 & -1 \end{bmatrix}$ C. $\begin{bmatrix} 2 & -2 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}$

4. Let $B = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 7 & 1 \end{bmatrix}}_{E_3} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & 0 & 1 \end{bmatrix}}_{E_2} \cdot \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}}_{E_1}$ Find B^{-1}:

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 1 & -5 & 7 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 \\ -1 & 5 & -7 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -3 & 2 & -1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $E_3^{-1} \cdot E_2^{-1} \cdot E_1^{-1}$

E. $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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5. Let $M = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$. Then M^{-1} is given by which of the following:

A. $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} -1 & 1 & -2 \\ 0 & -1 & 5 \\ 0 & 0 & -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 & -3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{bmatrix}$

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6. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 5 & 2 \\ 2 & -2 & 0 \end{bmatrix}$. If we let $\mathbf{x}$ be the solution $\mathbf{x}$ to the linear system

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

and calculate $c = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, then the constant c is given by

A. $c = 1$ B. $c = -1$ C. $c = 0$ D. $c = 2$ E. None of these.

7. Let $B = A^{-1}$ where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{bmatrix}$$

Then, which of the following gives $\left(B(1,:)\right)^T$?

A. A^{-1} does not exist

B. $\begin{bmatrix} 3 & 1 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 4 & -5 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ 4 \\ -5 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

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8. Consider the 3×3 matrix A from the problem above. Suppose we use this matrix in the following linear-systems problem

$$\begin{bmatrix} 3 & 1 & -2 \\ -3 & 1 & 0 \\ -6 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$$

If $\mathbf{x}$ is the solution to this linear-system problem, then which of the following gives the value of

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} ?$$

A. -3

B. -2

C. -1

D. 0

E. 2

Free Response

1. Let $\mathbf{w} \in \mathbb{R}^n$ be a vector such that $\mathbf{w}^T\mathbf{w} = 1$. The $n \times n$ matrix

$$H = I_n - 2\mathbf{w}\mathbf{w}^T.$$

is called a **Householder matrix**.

   A. Show that $H = H^T$ (in other words, show that H is symmetric).

   B. How that $H^{-1} = H^T$.

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2. Let $n, i, k \in \mathbb{N}$ such that $1 \leq i \leq n$, $1 \leq k \leq n$ and $i \neq k$. Suppose that $c \in \mathbb{R}$.

   (a) Show $\Big(S_{ik}(c)\Big)^{-1} = S_{ik}\left(-c\right)$

   (b) Show $\Big(D_i(c)\Big)^{-1} = D_i\left(\frac{1}{c}\right)$

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3. Recall that Cramer's formula for the inverse of a 2×2 matrix $A \in \mathbb{R}^2$ is given by

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

where $\det(A) = a_{11}a_{22} - a_{12}a_{21}$.

   (a) Using a sequence of elementary matrices, transform A into I_2. Show each matrix you use.

   (b) Write A^{-1} as a product of the elementary matrices and confirm Cramer's Rule.