## Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

1.	Т	F	Any system of $n$ linear equations in $n$ variables has at most $n$ solutions.			
2.	Т	$\mathbf{F}$	Inconsistent linear systems must have more than one solution.			
3.	Т	F	There exists a real matrix $A$ such that the linear system of equations $A\mathbf{x} = 0$ has exactly two solutions.			
4.	Т	F	Consider the linear systems problem			
			$A\mathbf{x} = \mathbf{b}$			
			where matrix $A \in \mathbb{R}^{m \times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. Then, the two fundamental questions we want to answer when beginning our work on this problem are:			
			i. Existence: Does a solution to this system exist?			
			ii. Uniqueness: Is there a unique solution to this system?			
5.	Т	F	Consider the linear systems problem			
			$A\mathbf{x} = \mathbf{b}$			
			where matrix $A \in \mathbb{R}^{m \times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. If this linear system is inconsistent, there may be an $\mathbf{x} \in \mathbb{R}^n$ such that			
			$\ \mathbf{b} - A\mathbf{x}\ _2 = 0.$			
6.	Т	F	Let $A \in \mathbb{R}^{m \times n}$ . The homogeneous equation $A\mathbf{x} = 0$ is consistent if and only if $\dim(\operatorname{Col}(A)) = n$ .			
7.	Т	F	Let $A, \mathbf{b}$ be given. If $f(\mathbf{x}) = A\mathbf{x}$ , then a solution to the corresponding set of linear system $A\mathbf{x} = \mathbf{b}$ exists if and only if $\mathbf{b}$ is in the range of $f$			

Multiple Choice For the problems below, circle the correct response for each question.

- 1. Suppose that  $L \in \mathbb{R}^{5 \times 5}$  is a general lower triangular system with nonzero lower triangular elements including nonzero diagonal elements. Suppose  $\mathbf{b} \in \mathbb{R}^5$  has all nonzero coefficients. To solve the linear system  $L\mathbf{x} = \mathbf{b}$  using forward substitution requires how many operations (each addition or multiplication between two real numbers counts as one operation).
  - A. 5 B. 10 C. 15 D. 20 E. 25

2. Let  $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & t & 0 \\ 1 & 0 & -1 \end{bmatrix}$ . Find the set of values of t for which the homogeneous system of linear equations

$$A\mathbf{x} = \mathbf{0}$$

has a non-zero solution:

A. t = 1 B.  $t \neq 1$  C. t = 2 D. t = 1 or t = 2 E.  $t \neq 2$ 

3. Stephen Curry plays for the Golden State Warriors as a professional basket ball player in the National Basketball Association (NBA). Suppose we analyze Curry's shooting technique. Below we see a diagram that highlights the typical trajectory of one of Mr. Curry's free-throw shots.



Our data points for this shooting trajectory  $\{(x_i, h_i)\}_{i=1}^3$  represent the observed height  $h_i$  of our basketball when the ball has moved  $x_i$  feet in the horizontal direction for i = 1, 2, 3. From our study of introductory physics, we choose to model the trajectory of the basket ball using the function

$$h(x) = a_0 + a_1 x + a_2 x^2.$$

Find the corresponding linear-systems problem to produce our desired polynomial model.

A. 
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 8 & 64 \\ 1 & 14 & 196 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$$
B. 
$$\begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix} \begin{bmatrix} a_0 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$$
C. 
$$\begin{bmatrix} 1 & 7.25 & 52.5625 \\ 1 & 13.50 & 182.2500 \\ 1 & 10.00 & 100.0000 \end{bmatrix} \begin{bmatrix} a_0 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix}$$
D. 
$$\begin{bmatrix} a_0 & a_1 & a_2 \\ a_0 & a_1 & a_2 \\ a_0 & a_1 & a_2 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$$
E. 
$$\begin{bmatrix} 1 & 0 & 7.25 \\ 1 & 8 & 13.50 \\ 1 & 14 & 10.00 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4. Solve the linear-systems problem design to model the path of Stephen Curry's free throw shot from above. With your solution, determine how high the ball was in the air after traveling x = 5 feet in the horizontal direction. In other words, approximate h(5) using the solution of your linear systems problem. Round your answer to the nearest tenth (round to the nearest one digit to the RIGHT of the decimal place).

A.	$-23.7  {\rm ft}$	B. 0 ft	C. 12.6 ft	D. 12.9 ft	E. 11.9 ft
			00		

5. Consider the following spring-mass system:



Suppose that you are apply masses 1, 2, and 3 to the mass-spring chain illustrated above such that

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.400 \\ 0.200 \end{bmatrix}$$

measured in kg. Assume the acceleration due to earth's gravity is  $g = 9.8m/s^2$ . Also assume that the mass of each spring is zero and that these springs satisfy the ideal version of Hooke's law. Then, which of the following gives the displacement vector

$$\mathbf{u} = \begin{bmatrix} u_1(T) \\ u_2(T) \\ u_3(T) \end{bmatrix}$$

measured in meters at t = T when the system is at equilibrium under the force of gravity on earth.

A. 
$$\begin{bmatrix} -0.196\\ -0.147\\ -0.196 \end{bmatrix}$$
B.  $\begin{bmatrix} 0.098\\ 0.098\\ 0.049 \end{bmatrix}$ C.  $\begin{bmatrix} 0.196\\ 0.245\\ 0.196 \end{bmatrix}$ D.  $\begin{bmatrix} 0.020\\ 0.025\\ 0.020 \end{bmatrix}$ E.  $\begin{bmatrix} 0.098\\ 0.098\\ 0.098 \end{bmatrix}$ 

For Problems 16, consider the following model for a 3-mass, 4-spring chain. Note that positive positions and positive displacements are marked in the downward direction. Assume the acceleration due to earth's gravity is  $g = 9.8m/s^2$ . Also assume that the mass of each spring is zero and that these springs satisfy the ideal version of Hooke's law exactly.



6. Recall that the initial position vector  $\mathbf{x}_0$  and the mass vector  $\mathbf{m}$  store the positions, measured in meters, of each mass at equilibrium when t = 0 and the mass measurements, measured in kg, respectively. Suppose we measure

$$\mathbf{x}_{0} = \begin{bmatrix} x_{1}(0) \\ x_{2}(0) \\ x_{3}(0) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.50 \\ 0.75 \end{bmatrix} \qquad \qquad \mathbf{m} = \begin{bmatrix} m_{1} \\ m_{2} \\ m_{3} \end{bmatrix} = \begin{bmatrix} 0.100 \\ 0.200 \\ 0.100 \end{bmatrix}$$

Which of the following gives the vector  $\mathbf{x}(T) = \begin{bmatrix} x_1(T) & x_2(T) & x_3(T) \end{bmatrix}^T$  as measured in meters, used to store the positions of each mass at equilibrium when when t = T? If necessary, please round your answers to the nearest 3 places after the decimal.

A. 
$$\begin{bmatrix} 0.196\\ 0.392\\ 0.196 \end{bmatrix}$$
B.  $\begin{bmatrix} 0.446\\ 0.892\\ 0.946 \end{bmatrix}$ C.  $\begin{bmatrix} 0.020\\ 0.400\\ 0.200 \end{bmatrix}$ D.  $\begin{bmatrix} 0.270\\ 0.540\\ 0.770 \end{bmatrix}$ E.  $\begin{bmatrix} 0.054\\ 0.108\\ 0.554 \end{bmatrix}$