Answers **Lesson 12 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T **F** Any system of n linear equations in n variables has at most n solutions.

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2. T **F** Inconsistent linear systems must have more than one solution.

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3. T **F** There exists a real matrix A such that the linear system of equations $A\mathbf{x} = \mathbf{0}$ has exactly two solutions.

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4. **T** F Consider the linear systems problem

$$A\mathbf{x} = \mathbf{b}$$

where matrix $A \in \mathbb{R}^{m\times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. Then, the two fundamental questions we want to answer when beginning our work on this problem are:

i. Existence: Does a solution to this system exist?

ii. Uniqueness: Is there a unique solution to this system?

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5. T **F** Consider the linear systems problem

$$A\mathbf{x} = \mathbf{b}$$

where matrix $A \in \mathbb{R}^{m\times n}$ and vector $\mathbf{b} \in \mathbb{R}^m$ are given and vector $\mathbf{x} \in \mathbb{R}^n$ is unknown and desired. If this linear system is inconsistent, there may be an $\mathbf{x} \in \mathbb{R}^n$ such that

$$\|\mathbf{b} - A\mathbf{x}\|_2 = 0.$$

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6. T **F** Let $A \in \mathbb{R}^{m\times n}$. The homogeneous equation $A\mathbf{x} = \mathbf{0}$ is consistent if and only if $\dim(\mathrm{Col}(A)) = n$.

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7. **T** F Let $A, \mathbf{b}$ be given. If $f(\mathbf{x}) = A\mathbf{x}$, then a solution to the corresponding set of linear system $A\mathbf{x} = \mathbf{b}$ exists if and only if $\mathbf{b}$ is in the range of f

Multiple Choice

For the problems below, circle the correct response for each question.

1. Suppose that $L \in \mathbb{R}^{5\times 5}$ is a general lower triangular system with nonzero lower triangular elements including nonzero diagonal elements. Suppose $\mathbf{b} \in \mathbb{R}^5$ has all nonzero coefficients. To solve the linear system $L\mathbf{x} = \mathbf{b}$ using forward substitution requires how many operations (each addition or multiplication between two real numbers counts as one operation).

   A. 5 **B. 10** C. 15 D. 20 E. 25

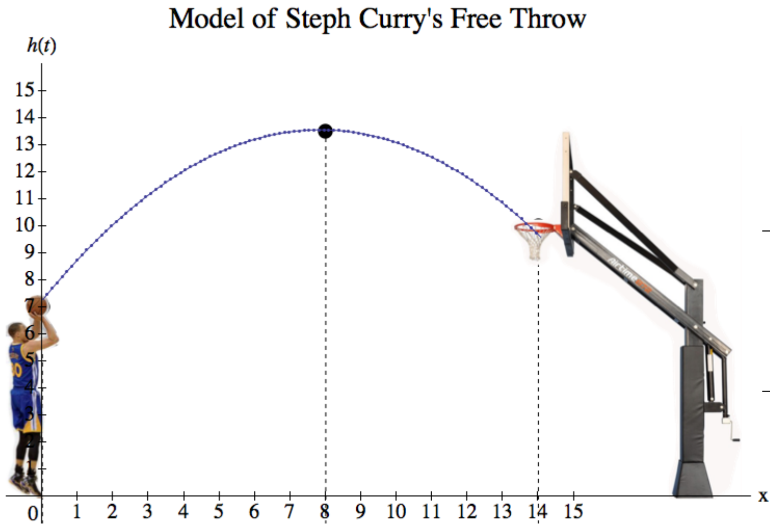
2. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & t & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Find the set of values of t for which the homogeneous system of linear equations

$$A\mathbf{x} = \mathbf{0}$$

   has a non-zero solution:

   A. $t = 1$ B. $t \neq 1$ C. $t = 2$ D. $t = 1$ or $t = 2$ **E.** $t \neq 2$

3. Stephen Curry plays for the Golden State Warriors as a professional basket ball player in the National Basketball Association (NBA). Suppose we analyze Curry's shooting technique. Below we see a diagram that highlights the typical trajectory of one of Mr. Curry's free-throw shots.



Distance x from release position (ft)	Height h of basketball above ground (ft)
0	7.25
8	13.50
14	10.00

Our data points for this shooting trajectory $\{(x_i, h_i)\}_{i=1}^{3}$ represent the observed height h_i of our basketball when the ball has moved x_i feet in the horizontal direction for $i = 1, 2, 3$. From our study of introductory physics, we choose to model the trajectory of the basket ball using the function

$$h(x) = a_0 + a_1 x + a_2 x^2.$$

Find the corresponding linear-systems problem to produce our desired polynomial model.

A. $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 8 & 64 \\ 1 & 14 & 196 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 7.25 & 52.5625 \\ 1 & 13.50 & 182.2500 \\ 1 & 10.00 & 100.0000 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix}$

D. $\begin{bmatrix} a_0 & a_1 & a_2 \\ a_0 & a_1 & a_2 \\ a_0 & a_1 & a_2 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix} = \begin{bmatrix} 7.25 \\ 13.50 \\ 10.00 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 7.25 \\ 1 & 8 & 13.50 \\ 1 & 14 & 10.00 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4. Solve the linear-systems problem design to model the path of Stephen Curry's free throw shot from above. With your solution, determine how high the ball was in the air after traveling $x = 5$ feet in the horizontal direction. In other words, approximate $h(5)$ using the solution of your linear systems problem. Round your answer to the nearest tenth (round to the nearest one digit to the RIGHT of the decimal place).

A. -23.7 ft

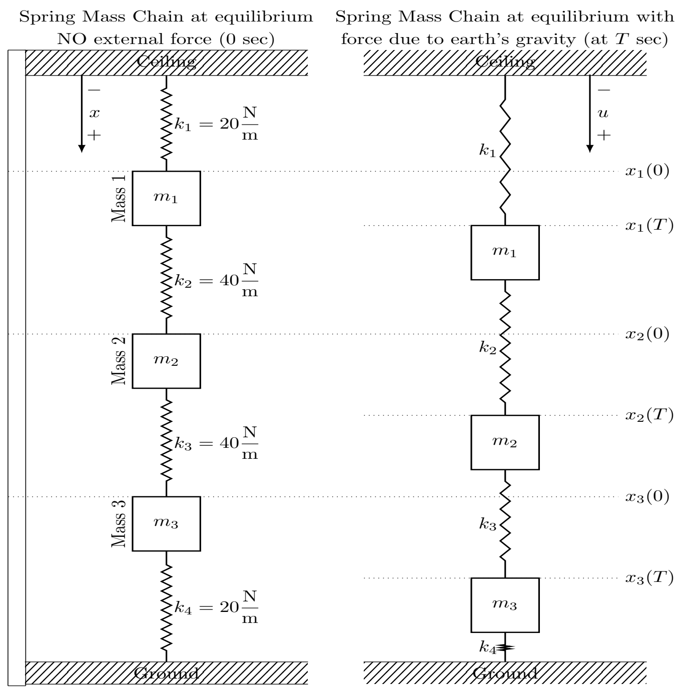
B. 0 ft

C. 12.6 ft

D. 12.9 ft

E. 11.9 ft

5. Consider the following spring-mass system:



Suppose that you are apply masses 1, 2, and 3 to the mass-spring chain illustrated above such that

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.400 \\ 0.200 \end{bmatrix}$$

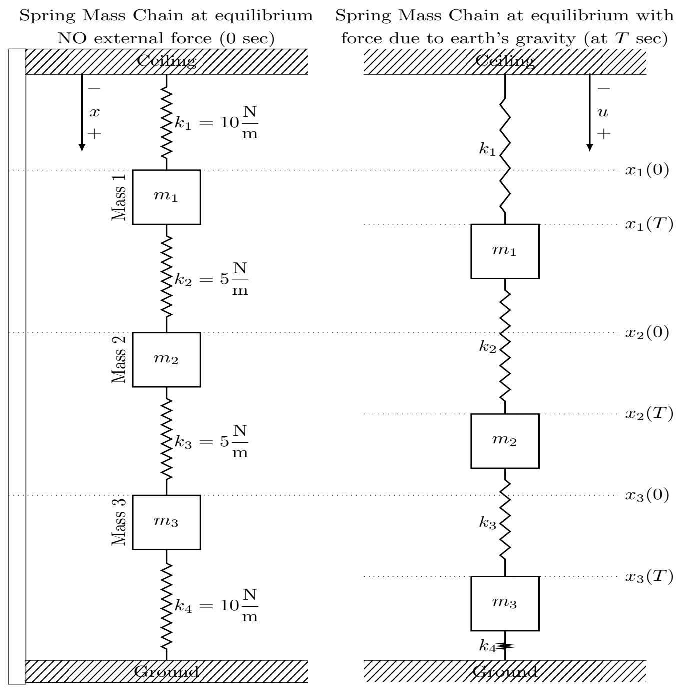
measured in kg. Assume the acceleration due to earth's gravity is $g = 9.8m/s^2$. Also assume that the mass of each spring is zero and that these springs satisfy the ideal version of Hooke's law. Then, which of the following gives the displacement vector

$$\mathbf{u} = \begin{bmatrix} u_1(T) \\ u_2(T) \\ u_3(T) \end{bmatrix}$$

measured in meters at $t = T$ when the system is at equilibrium under the force of gravity on earth.

A. $\begin{bmatrix} -0.196 \\ -0.147 \\ -0.196 \end{bmatrix}$ B. $\begin{bmatrix} 0.098 \\ 0.098 \\ 0.049 \end{bmatrix}$ **C.** $\begin{bmatrix} 0.196 \\ 0.245 \\ 0.196 \end{bmatrix}$ D. $\begin{bmatrix} 0.020 \\ 0.025 \\ 0.020 \end{bmatrix}$ E. $\begin{bmatrix} 0.098 \\ 0.098 \\ 0.098 \end{bmatrix}$

For Problems 16, consider the following model for a 3-mass, 4-spring chain. Note that positive positions and positive displacements are marked in the downward direction. Assume the acceleration due to earth's gravity is $g = 9.8m/s^2$. Also assume that the mass of each spring is zero and that these springs satisfy the ideal version of Hooke's law exactly.



6. Recall that the initial position vector $\mathbf{x}_0$ and the mass vector $\mathbf{m}$ store the positions, measured in meters, of each mass at equilibrium when $t = 0$ and the mass measurements, measured in kg, respectively. Suppose we measure

$$\mathbf{x}_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.50 \\ 0.75 \end{bmatrix} \qquad \mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 0.100 \\ 0.200 \\ 0.100 \end{bmatrix}$$

Which of the following gives the vector $\mathbf{x}(T) = \begin{bmatrix} x_1(T) & x_2(T) & x_3(T) \end{bmatrix}^T$ as measured in meters, used to store the positions of each mass at equilibrium when when $t = T$? If necessary, please round your answers to the nearest 3 places after the decimal.

A. $\begin{bmatrix} 0.196 \\ 0.392 \\ 0.196 \end{bmatrix}$ **B.** $\begin{bmatrix} 0.446 \\ 0.892 \\ 0.946 \end{bmatrix}$ C. $\begin{bmatrix} 0.020 \\ 0.400 \\ 0.200 \end{bmatrix}$ D. $\begin{bmatrix} 0.270 \\ 0.540 \\ 0.770 \end{bmatrix}$ E. $\begin{bmatrix} 0.054 \\ 0.108 \\ 0.554 \end{bmatrix}$