Name : ________________ **Lesson 11 Warm Up Quiz** **Class Number:** _____

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. T F Any two matrices that are conformable for matrix multiplication must have the same number of rows.

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2. T F Let $A \in \mathbb{R}^{9\times 6}$ and $X \in \mathbb{R}^{6\times 7}$. Set

$$B = A \cdot X.$$

Then $B(:,3) = A(3,:) \cdot X$

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3. T F Every square matrix is a product of elementary matrices.

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4. T F If $A \in \mathbb{R}^{n\times n}$ is a diagonal matrix with nonzero diagonal entries, and $B \in \mathbb{R}^{m\times n}$, then multiplying B on the right by A scales the rows of B.

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5. T F If A and B are $m \times n$ matrices, then the matrix products AB^T and A^TB are defined.

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6. T F For rectangular matrices A, B, C, with proper dimensions, if $AB = C$ and C has 2 columns, then A has two columns.

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7. T F If $A, B \in \mathbb{R}^{n\times n}$, then $(A - B)(A + B) = A^2 - B^2$.

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8. T F For matrices B, C, D with proper dimensions, if $BC = BD$, then $C = D$.

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9. T F For any $A, B \in \mathbb{R}^{n\times n}$, $AB = BA$.

Multiple Choice For the problems below, circle the correct response for each question.

1. Define the matrix $B \in \mathbb{R}^{4\times 4}$ by the following product:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Let $\mathbf{e}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T$ and $\mathbf{e}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$. Using this definition, we see that $b_{23} = \mathbf{e}_2^T B \mathbf{e}_3$ is given by which of the following:

A. $b_{23} = a_{43}$ B. $b_{23} = a_{24}$ C. $b_{23} = a_{23}$ D. $b_{23} = a_{44}$ E. None of these.

2. Let $A \in \mathbb{R}^{3\times 3}$ be defined as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

where A is nonsingular and $a_{11} \neq 0$. Suppose that we choose $E \in \mathbb{R}^{3\times 3}$ such that the product EA has the following structure:

$$EA = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & * & * \\ 0 & * & * \end{bmatrix}.$$

In this case, the symbol $*$ represents a real numbers. Then, E must be given by which of the following:

A. $E = S_{13}\left(-\dfrac{a_{13}}{a_{11}}\right) \cdot S_{13}\left(-\dfrac{a_{12}}{a_{11}}\right)$

B. $E = S_{31}\left(\dfrac{a_{31}}{a_{11}}\right) \cdot S_{21}\left(\dfrac{a_{12}}{a_{11}}\right)$

C. $E = S_{31}\left(-\dfrac{a_{31}}{a_{11}}\right) \cdot S_{21}\left(-\dfrac{a_{12}}{a_{11}}\right)$

D. $E = S_{31}\left(-\dfrac{a_{11}}{a_{31}}\right) \cdot S_{21}\left(-\dfrac{a_{11}}{a_{21}}\right)$

E. None of these

3. Define the matrix $B \in \mathbb{R}^{4\times 4}$ by the following product:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Using this definition, we see that b_{42} is given by which of the following:

A. $b_{42} = a_{32}$ B. $b_{42} = a_{42}$ C. $b_{42} = a_{24}$ D. $b_{42} = a_{34}$ E. $b_{42} = a_{43}$

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4. Let $A \in \mathbb{R}^{12\times 7}$ and $B \in \mathbb{R}^{12\times 6}$. Suppose $C = B^T A$. What are the dimensions of $C(:,2)$?

A. 7×1 B. 6×1 C. 6×7 D. 7×6 E. 1×6

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5. Which of the following represents the matrix-matrix product: $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 9 \\ 1 & 3.3 & 10.89 \\ 1 & 3.6 & 12.96 \end{bmatrix}$:

A. $\begin{bmatrix} 1 & 3 & 9 \\ 0 & 3.3 & 1.89 \\ 0 & 3.6 & 3.96 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 3 & 9 \\ 0 & 3.3 & 10.89 \\ 0 & 3.6 & 12.96 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 3 & 9 \\ 0 & 3.3 & 1.89 \\ 0 & 0 & 3.96 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 3 & 9 \\ 0 & 0.3 & 10.89 \\ 0 & 0.6 & 12.96 \end{bmatrix}$ E. $\begin{bmatrix} 1 & 3 & 9 \\ 0 & 0.3 & 1.89 \\ 0 & 0.6 & 3.96 \end{bmatrix}$

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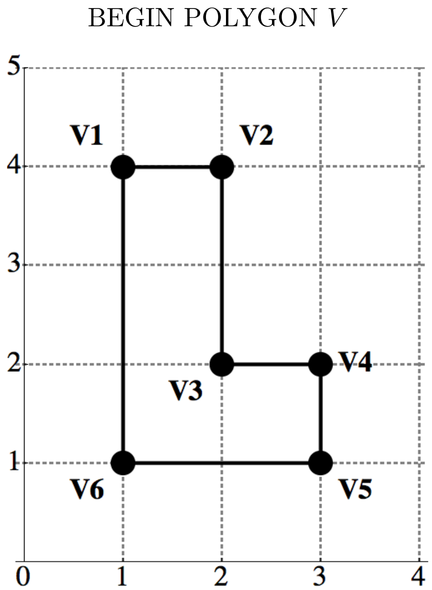
6. Let matrix $P \in \mathbb{R}^{4\times 5}$ be given as follows:

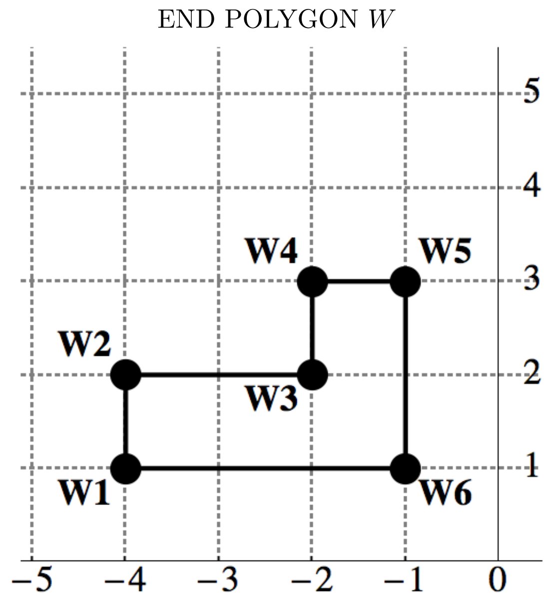
$$\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using this definition, we see that a_{12} is equal to which of the following:

A. $a_{12} = p_{21}$ B. $a_{12} = p_{24}$ C. $a_{12} = p_{42}$ D. $a_{12} = p_{12}$ E. $a_{12} = p_{25}$

For the two problems below, consider the polygons V and W.:





7. Which of the following vertex matrices V encodes the begin polygon above? For this model, assume that the kth column of V encodes vertex Vk, for $k \in \{1, 2, 3, 4, 5, 6\}$:

A. $\begin{bmatrix} 1 & 1 & 2 & 2 & 3 & 3 \\ 1 & 4 & 4 & 2 & 2 & 1 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 & 2 & 3 & 3 & 1 \\ 4 & 4 & 2 & 2 & 1 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 4 & 4 & 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 3 & 3 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 4 & 4 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 & 3 & 3 \end{bmatrix}$ E. $\begin{bmatrix} -4 & -4 & -2 & -2 & -1 & -1 \\ 1 & 2 & 2 & 3 & 3 & 1 \end{bmatrix}$

8. As noted above, let V be the vertex matrix that models the begin polygon and W be the vertex matrix that models the end polygon. Which matrix Q below satisfies equation

$$W = QV$$

A. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ C. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ E. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9. Define matrix A by

$$A = \begin{bmatrix} 2 & 3 & 1 & 4 & 6 \\ 1 & -2 & 3 & 2 & 0 \\ -4 & 1 & 0 & 5 & 7 \\ 6 & -2 & 8 & 0 & -1 \\ -7 & -2 & -1 & 3 & 1 \end{bmatrix}$$

For which of the following matrices E below will the matrix product

$$EA = C$$

not have a zero in the first column?

A. $S_{21}(-0.5)$ B. $S_{31}(2)$ C. $S_{41}(-3)$ D. $S_{51}(3.5)$ E. $S_{41}(3)$

10. Let $A \in \mathbb{R}^{8\times 4}$, $B \in \mathbb{R}^{4\times 7}$, and $C \in \mathbb{R}^{7\times 5}$. Let the matrix D be formed by the product

$$D = (A \cdot B \cdot C)^T$$

What are the dimensions of the matrix $[D(:,4)]^T$?

A. 8×5 B. 5×8 C. 1×5 D. 1×8 E. 5×1

For the next two problems below, consider the wireframe model for a begin polygon V defined by vertex matrix and edge table below.

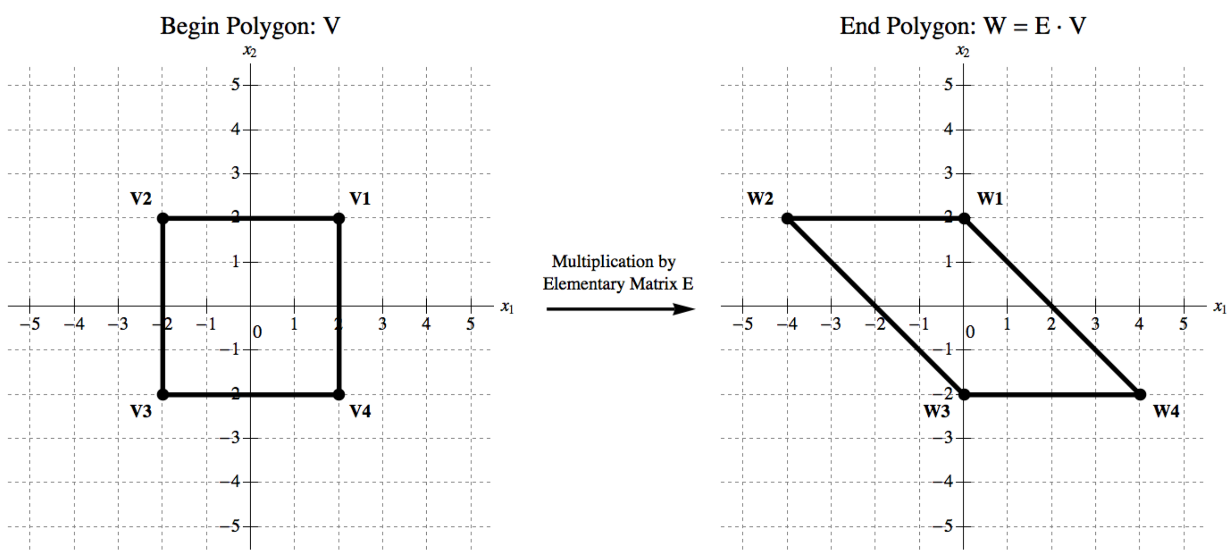
$$V = \begin{bmatrix} 2 & -2 & -2 & 2 \\ 2 & 2 & -2 & -2 \end{bmatrix}$$

Edge #	Start Vertex	End Vertex
1	1	2
2	2	3
3	3	4
4	4	1

Let W be a wireframe model for an end polygon given by

$$W = \begin{bmatrix} 0 & -4 & 0 & 4 \\ 2 & 2 & -2 & -2 \end{bmatrix}$$

Assume W formed by multiplying V by some matrix $E \in \mathbb{R}^{2\times 2}$ with $W = E \cdot V$. Also, assume that the edge tables of V and W are identical. Under these assumptions, the wireframe model for both V and W are given below.



11. Choose the matrix E used to produce W in this situation:

A. $S_{21}(-2)$ B. $S_{12}(-2)$ C. $S_{21}(-1)$ D. $S_{12}(1)$ E. $S_{12}(-1)$

12. Find the length of edge 4 from the wireframe model for the end polygon $W = E \cdot V$ in the problem above.

A. 2 B. $\sqrt{20}$ C. 4 D. $4\sqrt{2}$ E. 0

13. Let matrix $B \in \mathbb{R}^{4\times 4}$ be given as follows:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

In symbols, we can write

$$B = S_{41}(-2) \cdot A \cdot D_4(3)$$

Using this definition, we see that b_{44} is equal to which of the following:

A. $-6a_{44}$ B. $-2a_{14} + 3a_{44}$ C. $-6a_{14} + 3a_{44}$ D. $3a_{14} - 6a_{44}$ E. $3a_{14} - 2a_{44}$

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14. Let $n \in \mathbb{N}$ with $n \geq 3$. Suppose that we define the matrix

$$B = I_n + c_1\, \mathbf{e}_2\, \mathbf{e}_1^T - c_2\, \mathbf{e}_3\, \mathbf{e}_1^T$$

where $\mathbf{e}_k = I_n(:, k)$. Which of the following is equivalent to B^{-1}?

A. $S_{21}(c_1) \cdot S_{31}(-c_2)$ B. $S_{31}(c_2) - S_{21}(c_1)$ C. $S_{31}(c_2) \cdot S_{21}(-c_1)$

D. $S_{21}\left(\dfrac{1}{c_1}\right) \cdot S_{31}\left(\dfrac{-1}{c_2}\right)$ E. $S_{12}(c_1) \cdot S_{13}(-c_2)$

Free Response

1. Write the definition for the column-partition version of matrix-matrix multiplication.

2. Let $A \in \mathbb{R}^{4\times 4}$. Multiply A on the right by a matrix X to achieve each of the operations below. In each case, specifically state the entry-by-entry definition of the matrix X used to accomplish these operations.

   A. Double column 1

   B. Interchange columns 1 and 4

   C. Add 2 times column 2 to column 3

   D. Delete column 4 (so that the column dimension is reduced by 1)

3. Write the definition for the row-partition version of matrix-matrix multiplication.

4. Let $X \in \mathbb{R}^{4\times 4}$. Multiply X on the left by a matrix A to achieve each of the operations below. In each case, specifically state the entry-by-entry definition of the matrix A used to accomplish these operations.

   A. Halve row 3

   B. Add row 2 to row 4

   C. Swap rows 1 and 2

   D. Subtract row 1 from each of the other rows