Answers **Lesson 10 Warm Up Quiz**

Math 2B: Applied Linear Algebra

True/False For the problems below, circle T if the answer is true and circle F is the answer is false.

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1. **T** F Let $A \in \mathbb{R}^{m\times n}$. If $f(\mathbf{x}) = A\mathbf{x}$, then

$$\text{Rng}(f) = \{\mathbf{b} \in \mathbb{R}^m \,:\, \mathbf{b} = A\mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^n\}.$$

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2. **T** F When multiplying matrix A and vector $\mathbf{x}$, it is important to check that the inner dimensions agree.

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3. **T** F If $f(\mathbf{x}) = A\mathbf{x}$ and $\mathbf{b} \in \text{Rng}(f)$, then $\mathbf{b}$ is linearly dependent on the columns of A.

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4. **T** F If $f(\mathbf{b}) = A^T\mathbf{b}$, then the range of f is the span of the rows of f. In other words, $\text{Rng}(f) = \text{ span } \{[A(i,:)]^T \,:\, i \in \{1, 2, ..., m\}\}$

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5. **T** F When multiplying A^T and $\mathbf{b}$, it is important to check that the inner dimensions agree.

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6. **T** F If $A \in \mathbb{R}^{6\times 5}$, then the function $f(\mathbf{x}) = A\mathbf{x}$ cannot be onto.

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7. **T** F Matrix-vector multiplication allows us to compute a linear combination of vectors efficiently.

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8. **T** F Let $A \in \mathbb{R}^{m\times n}$ and suppose $\mathbf{x} \in \mathbb{R}^n$. $A\mathbf{x} = \sum_{k=1}^{n} x_k A(:,k)$

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9. T **F** Let $A \in \mathbb{R}^{m\times n}$ and suppose $\mathbf{x} \in \mathbb{R}^n$ .If $f(\mathbf{x}) = A\mathbf{x}$, then the codomain of this relation is $\mathbb{R}^n$

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10. **T** F Let $A \in \mathbb{R}^{m\times n}$ and suppose $\mathbf{x} \in \mathbb{R}^n$. If $f(\mathbf{x}) = A\mathbf{x}$, then the range of f is the span of the columns of f. In other words, $\text{Rng}(f) = \text{ span } \{A(:,k) \,:\, k \in \{1, 2, ..., n\}\} \subseteq \mathbb{R}^m$

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11. **T** F If $f(\mathbf{b}) = \mathbf{b}^T A$, then the $\text{Rng}(f) = \{\mathbf{x} \in \mathbb{R}^{1\times n} \,:\, \mathbf{x} = \mathbf{b}^T A \text{ for some } \mathbf{b} \in \mathbb{R}^m\}$

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12. **T** F Let $A \in \mathbb{R}^{m\times n}$. If $f(\mathbf{x}) = A\mathbf{x}$, then the domain space of this relation is $\mathbb{R}^n$

13. T **F** Let $A \in \mathbb{R}^{m\times n}$ and $y \in \mathbb{R}^m$. Then $A^T\mathbf{y} = \sum_{i=1}^{n} y_i \left[A(i,:)\right]^T$

14. **T** F For any $A \in \mathbb{R}^{m\times n}$ and $\mathbf{x} \in \mathbb{R}^n$, $A\mathbf{x} = \sum_{k=1}^{n} x_k A(:,k)$.

15. **T** F Let $A \in \mathbb{R}^{m\times n}$. Define the linear transformation $T : \mathbb{R}^m \to \mathbb{R}^n$ by $T(\mathbf{x}) = A^T\mathbf{x}$. If $\text{Rng}(T) = \mathbb{R}^n$, then $m \geq n$.

16. **T** F When multiplying A and $\mathbf{x}$, it is important to check that the inner dimensions must agree.

17. **T** F If $f(\mathbf{b}) = A^T\mathbf{b}$, then the range of f is the span of the rows of f. In other words, $\text{Rng}(f) = \text{ span } \{[A(i,:)]^T \ : \ i \in \{1, 2, ..., m\}\}$

18. **T** F If $f(\mathbf{x}) = A\mathbf{x}$ and $\mathbf{b} \in \text{Rng}(f)$, then $\mathbf{b}$ is linearly dependent on the columns of A.

19. T **F** If $f(\mathbf{x}) = A\mathbf{x}$, then the codomain of this relation is $\mathbb{R}^n$

20. **T** F If $f(\mathbf{x}) = A\mathbf{x}$, then the range of f is the span of the columns of f. In other words, $\text{Rng}(f) = \text{ span } \{A(:,k) \ : \ k \in \{1, 2, ..., n\}\} \subseteq \mathbb{R}^m$

21. **T** F $A\mathbf{x} = \sum_{k=1}^{n} x_k A(:,k)$

22. **T** F Matrix-vector multiplication allows us to compute a linear combination of vectors efficiently.

23. T **F** Let $A \in \mathbb{R}^{m\times n}$ and $\mathbf{x} \in \mathbb{R}^n$. Then the matrix-vector product $A \cdot \mathbf{x}$ represents a linear combination of the rows of A with scalar multiples defined by the entries of $\mathbf{x}$.

Multiple Choice

For the problems below, circle the correct response for each question.

1. Let $A \in \mathbb{R}^{m\times n}$ be given and suppose $\mathbf{x} \in \mathbb{R}^n$. If $f(\mathbf{x}) = A\mathbf{x}$, which of the following statements is false:

   A. $\mathbf{0} \in \text{Rng}(f)$.

   B. The codomain of $f(\mathbf{x})$ is $\mathbb{R}^m$.

   C. The domain of $f(x)$ is $\mathbb{R}^n$.

   D. $\text{Rng}(f) = \{$set of all solutions to the linear-systems problem.$\}$.

   E. $\mathbf{b} \in \text{Rng}(f)$ if and only if $\mathbf{b}$ is a linear combinations of the columns of A.

2. Suppose that $A \in \mathbb{R}^{20\times 5}$ and $\mathbf{x} \in \mathbb{R}^5$. How many total operations on real numbers are necessary to solve the matrix-vector multiplication problem $A\mathbf{x} = \mathbf{b}$? Remember that each multiplication between two real numbers counts as one operation and each addition between two real numbers counts as one operation.

   A. 100 **B. 180** C. 90 D. 25 E. 200

3. Let $\mathbf{x} \in \mathbb{R}^2$ be a nonzero vector. Suppose

$$Q = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

   and let $\mathbf{y} = Q\mathbf{x}$. For what angle θ is $\mathbf{x}^T\mathbf{y} = 0$

   A. 0 B. $\frac{\pi}{6}$ C. $\frac{5\pi}{4}$ D. $\frac{4\pi}{3}$ **E.** $\frac{6\pi}{4}$

4. Define three vectors in $\mathbb{R}^4$ as

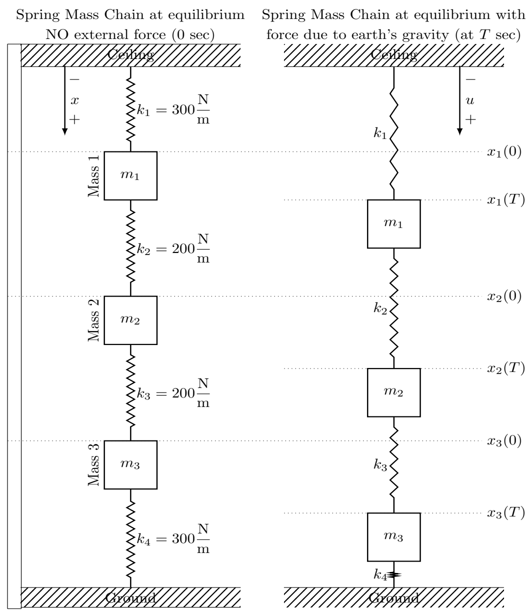
$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 10 \\ 15 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 0 \\ 25 \\ 100 \\ 225 \end{bmatrix}, \qquad \mathbf{a}_4 = \begin{bmatrix} 5 \\ -45 \\ -45 \\ 5 \end{bmatrix}$$

   Please confirm that $\mathbf{a}_4 = 5 \cdot \mathbf{a}_1 - 15 \cdot \mathbf{a}_2 + 1 \cdot \mathbf{a}_3$. Then, choose the vector $\mathbf{x} \in \mathbb{R}^4$ such that

$$\left[\ \mathbf{a}_1 \mid \mathbf{a}_2 \mid \mathbf{a}_3 \mid \mathbf{a}_4\ \right] \cdot \mathbf{x} = \mathbf{0}$$

   A. $\begin{bmatrix} 45 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ B. $\begin{bmatrix} 5 \\ -15 \\ 1 \\ 1 \end{bmatrix}$ C. $\begin{bmatrix} -5 \\ 15 \\ -1 \\ -1 \end{bmatrix}$ **D.** $\begin{bmatrix} 5 \\ -15 \\ 1 \\ -1 \end{bmatrix}$ E. The product will never be zero

For the following two problems, consider the following spring-mass system



5. Consider the mass-spring chain from the diagram above. Recall the model for the mass spring chain is given by $M\ddot{\mathbf{u}}(t) + K\mathbf{u}(t) = \mathbf{F}_e(t)$. Identify the stiffness matrix K for the given values of k_i?

A. $\begin{bmatrix} 500 & -200 & 0 \\ -200 & 400 & -200 \\ 0 & -200 & 500 \end{bmatrix}$ B. $\begin{bmatrix} 300 & -200 & 0 \\ -200 & 200 & -200 \\ 0 & -200 & 300 \end{bmatrix}$ C. $\begin{bmatrix} 500 & -200 & -200 \\ -200 & 400 & -200 \\ -200 & -200 & 500 \end{bmatrix}$

D. $\begin{bmatrix} -500 & 200 & 0 \\ 200 & -400 & 200 \\ 0 & 200 & -500 \end{bmatrix}$ E. $\begin{bmatrix} 300 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 300 \end{bmatrix}$

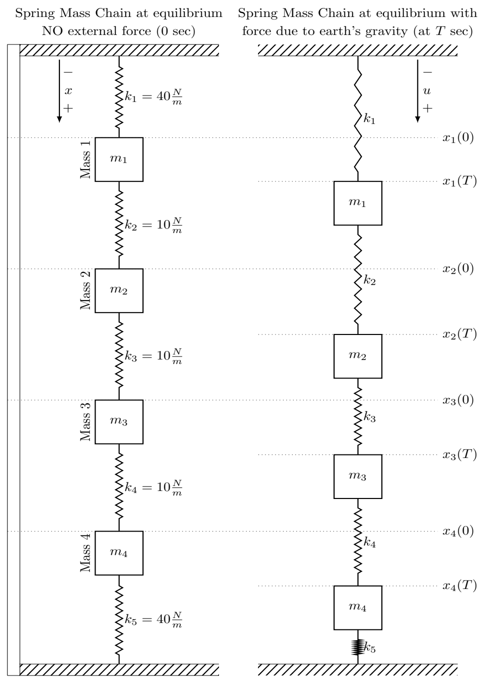
6. Suppose that you are given the displacement vector when $t = T$ at equilibrium to find

$$\mathbf{u} = \begin{bmatrix} u_1(T) \\ u_2(T) \\ u_3(T) \end{bmatrix} = \begin{bmatrix} 0.98 \\ 1.96 \\ 0.98 \end{bmatrix}$$

measured in meters. Then, which of the following gives the mass vector $\mathbf{m} = \begin{bmatrix} m_1 & m_2 & m_3 \end{bmatrix}^T$ as measured in kg? Assume the acceleration due to earth's gravity is $g = 9.8m/s^2$. Also assume that the mass of each spring is zero and that these springs satisfy Hooke's law exactly.

A. $\begin{bmatrix} 10 \\ 40 \\ 10 \end{bmatrix}$ B. $\begin{bmatrix} 9.8 \\ 39.2 \\ 9.8 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ D. $\begin{bmatrix} 0.1 \\ 0.4 \\ 0.1 \end{bmatrix}$ E. $\begin{bmatrix} 29.4 \\ 39.2 \\ 29.4 \end{bmatrix}$

For the next two problems below, consider the following 4-mass, 5-spring chain presented below.



7. Recall that the initial position vector $\mathbf{x}_0$ and the final position vector $\mathbf{x}(T)$ store the positions, measured in meters, of each mass at equilibrium when $t = 0$ and when $t = T$ respectively. Suppose we measure

$$\mathbf{x}_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 0.200 \\ 0.400 \\ 0.600 \\ 0.800 \end{bmatrix} \qquad \mathbf{x}(T) = \begin{bmatrix} x_1(T) \\ x_2(T) \\ x_3(T) \\ x_4(T) \end{bmatrix} = \begin{bmatrix} 0.249 \\ 0.498 \\ 0.698 \\ 0.849 \end{bmatrix}$$

where each entry is given in meters. Using this information, which of the following vectors gives the force vector $\mathbf{f}_s$ that encodes the forces stored in each spring in this system?

A. $\begin{bmatrix} 1.960 \\ 0.490 \\ 0.000 \\ -0.490 \\ -1.960 \end{bmatrix}$ B. $\begin{bmatrix} 1.960 \\ 0.980 \\ 0.980 \\ 1.960 \end{bmatrix}$ C. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.000 \\ 0.490 \\ 1.470 \end{bmatrix}$ D. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.490 \\ 1.470 \end{bmatrix}$ E. $\begin{bmatrix} 0.049 \\ 0.049 \\ 0.000 \\ -0.049 \\ -0.049 \end{bmatrix}$

8. Under the same assumptions as the problem above, which of the following gives the mass vector

$$\mathbf{m} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix}^T$$

measured in kg, used to produce this position data?

A. $\begin{bmatrix} 0.200 \\ 0.100 \\ 0.100 \\ 0.200 \end{bmatrix}$ B. $\begin{bmatrix} 1.470 \\ 0.490 \\ 0.490 \\ 1.470 \end{bmatrix}$ C. $\begin{bmatrix} 0.250 \\ 0.200 \\ 0.200 \\ 0.250 \end{bmatrix}$ D. $\begin{bmatrix} 2.450 \\ 1.960 \\ 1.960 \\ 2.450 \end{bmatrix}$ **E.** $\begin{bmatrix} 0.150 \\ 0.050 \\ 0.050 \\ 0.150 \end{bmatrix}$