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[5] 1. Multiply and simplify completely: $\dfrac{3z+18}{z-2} \cdot \dfrac{z-2}{z^2+2z-24}$.

Solution: We will use distributivity of multiplication over addition to solve this problem. Consider:

$$\begin{aligned}
\frac{3z+18}{z-2} \cdot \frac{z-2}{z^2+2z-24} &= \frac{3\cdot(z+6)}{(z-2)} \cdot \frac{(z-2)}{(z-4)\cdot(z+6)} \\
&= \frac{3}{(z-4)} \cdot \frac{(z-2)}{(z-2)} \cdot \frac{(z+6)}{(z+6)} \\
&= \boxed{\frac{3}{z-4}}
\end{aligned}$$

This equivalence holds as long as $z \neq 2$ and $z \neq -6$. Here we used the fundamental principle of fractions to conclude

$$\frac{(z-2)}{(z-2)} = 1 \text{ if } z \neq 2 \qquad \text{and} \qquad \frac{(z+6)}{(z+6)} = 1 \text{ if } z \neq -6.$$

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[5] 2. Add and simplify completely: $\dfrac{d^2}{d-2} + \dfrac{4}{2-d}$

Solution: To subtract fractions with different denominators, we need to find a common denominator. Recall that $2-d = -1\cdot(d-2)$:

$$\begin{aligned}
\frac{d^2}{d-2} + \frac{4}{2-d} &= \frac{d^2}{d-2} + \frac{-1}{-1} \cdot \frac{4}{2-d} \\
&= \frac{d^2}{d-2} + \frac{4}{d-2} \\
&= \frac{d^2-4}{d-2} \\
&= \frac{(d-2)\cdot(d+2)}{d-2} \\
&= \boxed{d+2}
\end{aligned}$$

Here, we assumed that $d \neq 2$ since $\dfrac{d-2}{d-2} = 1$ as long as $d-2 \neq 0$.

5 3. Solve. Make sure to check for extraneous solutions. If no solution exists, state this:

$$\frac{x}{x+1} + \frac{5}{x} = \frac{1}{x^2+x}$$

Solution: To subtract fractions with different denominators, we need to find a common denominator. To do so we use the fundamental principle of fractions.

$$\Longrightarrow \quad \frac{x}{x+1} \cdot \frac{x}{x} + \frac{5}{x} \cdot \frac{x+1}{x+1} = \frac{1}{x^2+x}$$

$$\Longrightarrow \quad \frac{x^2+5x+5}{x \cdot (x+1)} = \frac{1}{x^2+x}$$

$$\Longrightarrow \quad x^2+5x+5 = 1$$

$$\Longrightarrow \quad x^2+5x+4 = 0$$

$$\Longrightarrow \quad (x+4) \cdot (x+1) = 0$$

$$\Longrightarrow \quad \boxed{x = -4} \text{ and } x \neq -1$$

5 4. In your own words, explain the inverse operation for rational equations. Then, explain how to use this inverse to solve rational equations. (Hint: see problem 3 above.)

Solution:

- We've studied two inverse operations for rational equations:

    1. If $\frac{A}{D} = \frac{B}{D}$, then $A = B$.
    2. If $\frac{A}{B} = D$, then $A = BD$.

- We can describe inverse 1 in English as follows: if we have a rational equation and one side of the equation has denominator B, we can eliminate this denominator by multiplying both sides of the equation by B.
- We can describe inverse 2 in English as follows: if we have a rational equation where equal fractions have identical denominators, then we eliminate these denominators and set the numerators equal to each other.
- In order to solve rational equation using either of these inverses, we manipulate the expressions on each side of the equals sign to get the entire expression combined into one fraction with a single denominator. Then, depending on which inverse is applicable, we annihilate denominators using the appropriate technique and solve the equation that results.

