Free Response: Solve each of the following problems. Show your work and box your final answer.

$\boxed{3}$ 1. Solve the quadratic equation below using an algebraic technique: $2x^2 + 2x - 10 = 11 - x^2$

Solution: We will first get the left-hand side of the equation equal to zero in order to apply the zero product property. Then, we will follow the procedure for factoring polynomials that we discussed in class to write the left-hand side of the equation as a product of two factors.

$$\begin{aligned} 0 &= 3x^2 + 2x - 21, && a = 3, b = 2, c = -21 \\ &= 3x^2 + 9x - 7x - 21 && \text{AC method: } ac = -63, b = 2 \\ &= 3x(x+3) - 7(x+3) && \text{Factor by grouping} \\ &= (3x-7)\cdot(x+3). \end{aligned}$$

By the zero product property, we know

$$(3x - 7) = 0 \qquad \text{or} \qquad (x+3) = 0$$

$$\boxed{x = \frac{7}{3}} \qquad \text{or} \qquad \boxed{x = -3}$$

$\boxed{3}$ 2. In your own words, explain the zero product property. Then, explain how to use the zero product property as an inverse operation to solve quadratic equations.

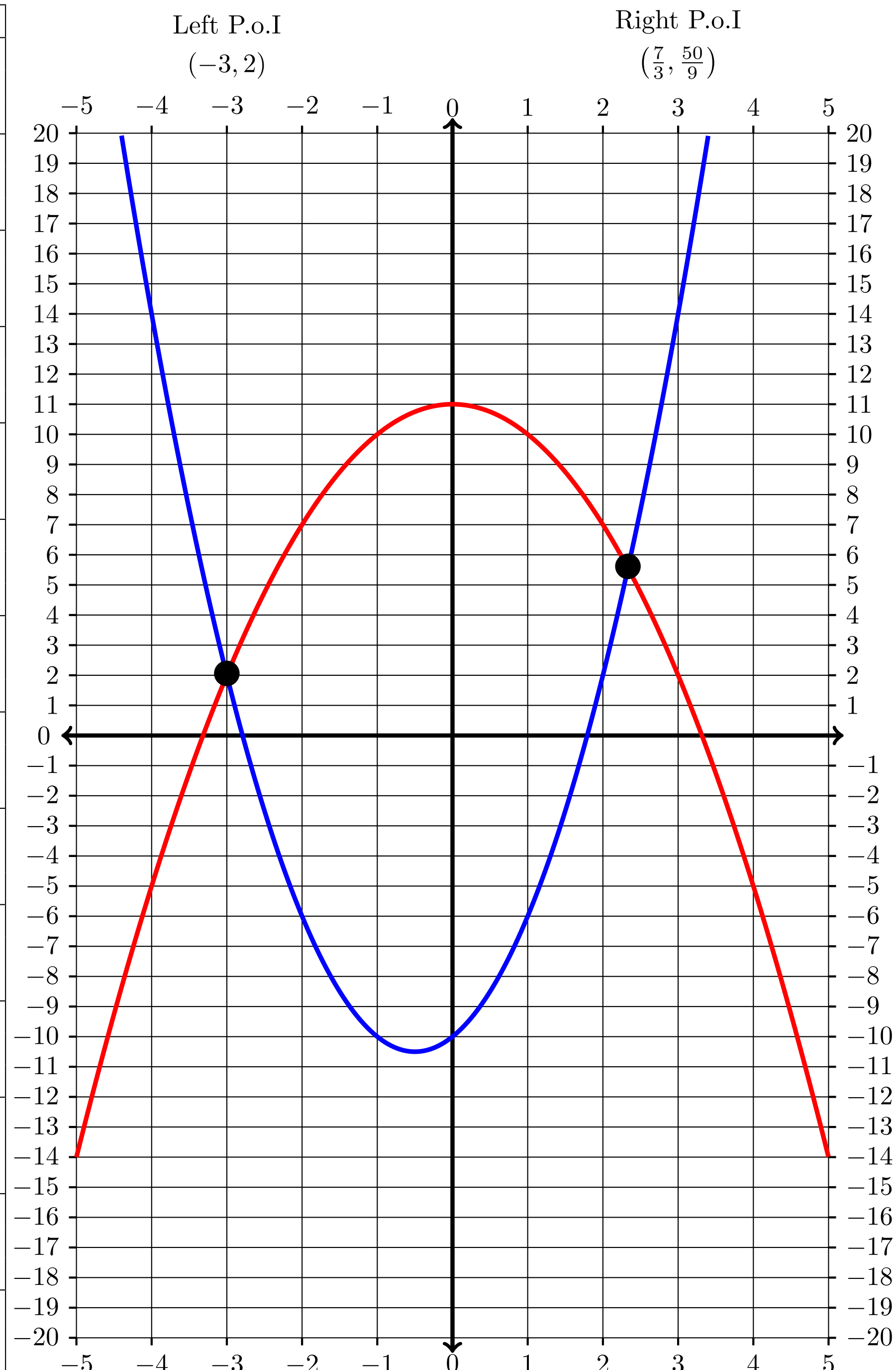
Solution:

- The zero product property is stated as follows: If $a \cdot b = 0$, then either $a = 0$ or $b = 0$
- In English, we can say: if a product of two numbers is zero, we know that one of the two factors must be equal to zero
- In order to solve quadratic equations use the zero product property, we manipulate the equation to get the right hand side equal to zero. Then, we factor the quadratic expression on the left-hand side of the equals sign to write it as a product of two binomials. In doing so, we have the exact form: $a \cdot b = 0$. We then apply the zero product property to annihilate the multiplication with a zero output and form to linear equations.

$\boxed{5}$ 3. Using a calculator, solve the quadratic equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Please specifically identify each point of intersection on your graph and write each of these points as an ordered pair. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$2x^2 + 2x - 10 = 11 - x^2$$

	LHS	RHS
x	$2x^2 + 2x - 10$	$11 - x^2$
-5	30	-14
-4	14	-5
-3	2	2
$-\frac{7}{3}$	$-3.77\overline{7}$	$5.55\overline{5}$
-2	-6	7
-1.5	-8.5	8.75
-1	-10	10
0	-10	11
1	-6	10
2	2	7
$\frac{7}{3}$	$5.55\overline{5}$	$5.55\overline{5}$
3	14	2
4	30	-5
5	50	-14



Solutions: $x = -3$ or $x = \dfrac{7}{3} \approx 2.33\overline{3}$

$\boxed{5}$ 4. Using a calculator, solve the absolute value equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Please specifically identify each point of intersection on your graph and write each of these points as an ordered pair. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$4|x+1| - 6 = 6$$

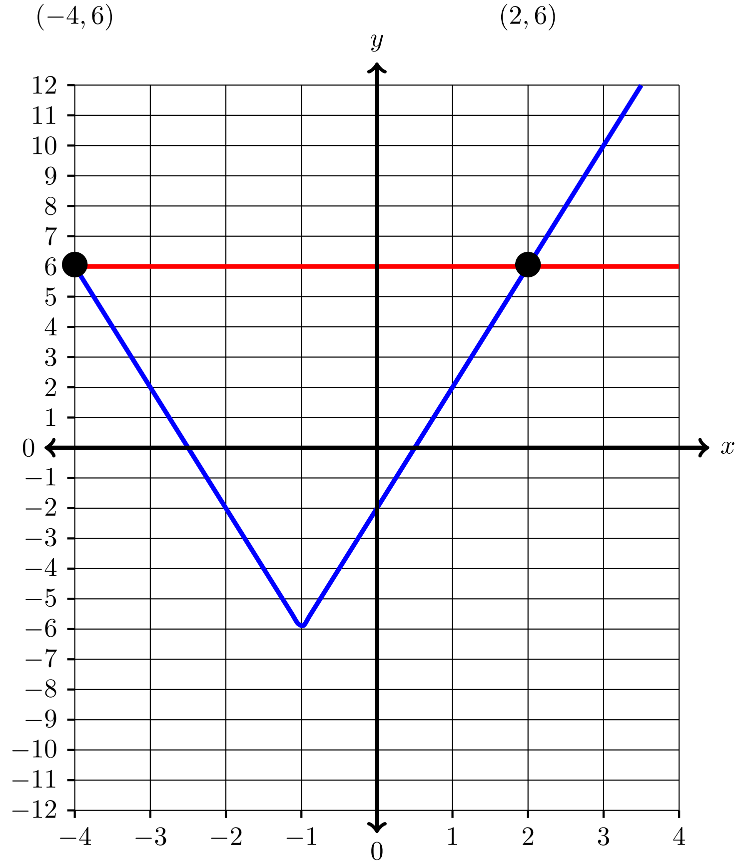
	LHS of Equation	RHS of Equation
x	$y_1 = 4\|x+1\| - 6$	$y_2 = 6$
-4	6	6
-3	2	6
-2	-2	6
-1.5	-4	6
-1	-6	6
0	-2	6
1	2	6
1.5	4	6
2	6	6
3	10	6
4	14	6

Left P.o.I

$(-4, 6)$

Right P.o.I

$(2, 6)$



Solutions: $x = -4$ or $x = 2$

$\boxed{2}$ 5. Solve the absolute value equation below using an algebraic technique:

$$4|x+1| - 6 = 6$$

Solution: To solve this absolute value problem, we use our inverse operation:

$$\begin{aligned} &\Longrightarrow & 4 \cdot |x+1| = 12 \\ &\Longrightarrow & |x+1| = 3 \\ &\Longrightarrow & x+1 = -3 \qquad \text{OR} \qquad x+1 = 3 \end{aligned}$$

Thus we see there are two solutions including $\boxed{x = -4}$ and $\boxed{x = 2.}$

$\boxed{2}$ 6. In your own words, explain the inverse operation for absolute value equations. Then, explain how to use this inverse to solve absolute values equations.

Solution:

- The inverse of an absolute value is stated as follows: If $|\,x\,| = c$ for nonnegative constant c, then either $x = -c$ or $x = +c$. If $|\,x\,| = c$ where $c < 0$, there is no solution to such an equation.
- In English, we can say: get a "pure" absolute value on the left-hand side and a nonnegative constant on the right-hand side. Then, set the thing inside the absolute value bars equal to the positive and negative value on the right-hand side.
- In order to solve absolute value equations use this, we manipulate the equation to get a "pure" absolute value expression on the left-hand side of the equation. Such and expression has no additional operations outside of the absolute value bars. We also check that the right-hand side is nonnegative. Then, we set the expression inside the absolute value bars equal to the positive and negative of the right-hand side (which is the inverse).