
Free Response: Solve each of the following problems. Show your work and box your final answer.

$\boxed{2}$ 1. Factor the following polynomial completely: $n^2 - 3n - 40$

Solution: We will follow the procedure for factoring polynomials that we discussed in class.

$$\begin{aligned} & n^2 - 3n - 40, && a = 1, b = -3, c = -40 \\ &= n^2 + 5n - 8n - 40 && \text{AC method: } ac = -40, b = +3 \\ &= n(n+5) - 8(n+5), && \text{Factor by grouping} \\ &= \boxed{(n-8)\cdot(n+5).} \end{aligned}$$

$\boxed{2}$ 2. Solve the quadratic equation below using an algebraic technique:

$$\frac{1}{4}\left(16 - x^2\right) = 0$$

Solution: Recall: the zero product property states if $a \cdot b = 0$, then either $a = 0$ or $b = 0$. We will factor our given polynomial:

$$\begin{aligned} 0 &= \frac{1}{4}\left(16 - x^2\right) \\ &= \frac{1}{4}\left(16 + 0x - x^2\right) \\ &= \frac{1}{4}\left(16 + 4x - 4x - x^2\right) \\ &= \frac{1}{4}\left(4\cdot(4+x) - x\cdot(4+x)\right) \\ &= \frac{1}{4}(4-x)\cdot(4+x) \end{aligned}$$

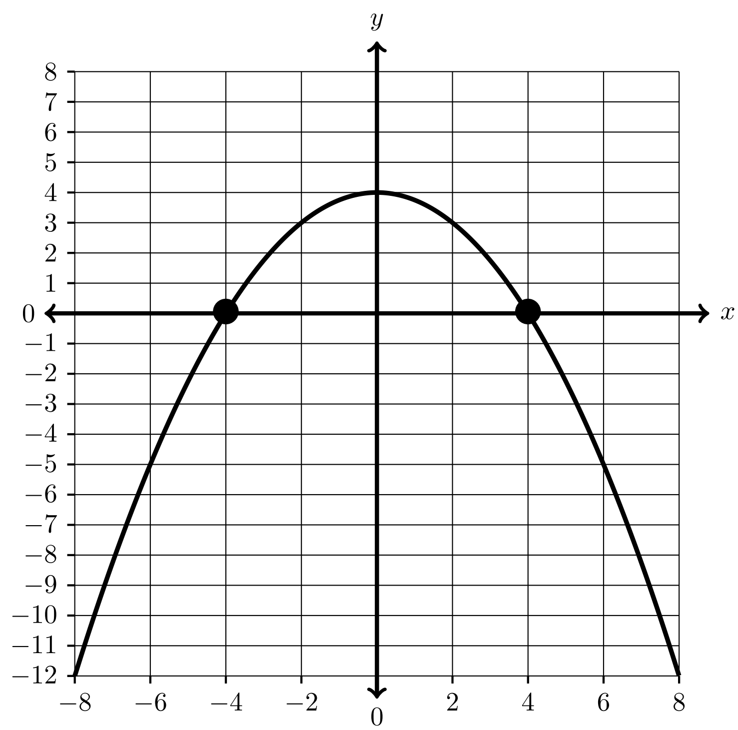
By the zero product property, we know

$$\begin{aligned} (4-x) &= 0 \qquad \text{or} \qquad (4+x) = 0 \\ \boxed{x = 4} & \qquad \text{or} \qquad \boxed{x = -4} \end{aligned}$$

$\boxed{5}$ 3. Solve the quadratic equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Of course, you are welcome to use your calculator. Please specifically identify each point of intersection on your graph. Also, please write each of these points as an ordered pair with an x-coordinate and y-coordinate. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$\frac{1}{4}\left(16 - x^2\right) = 0$$

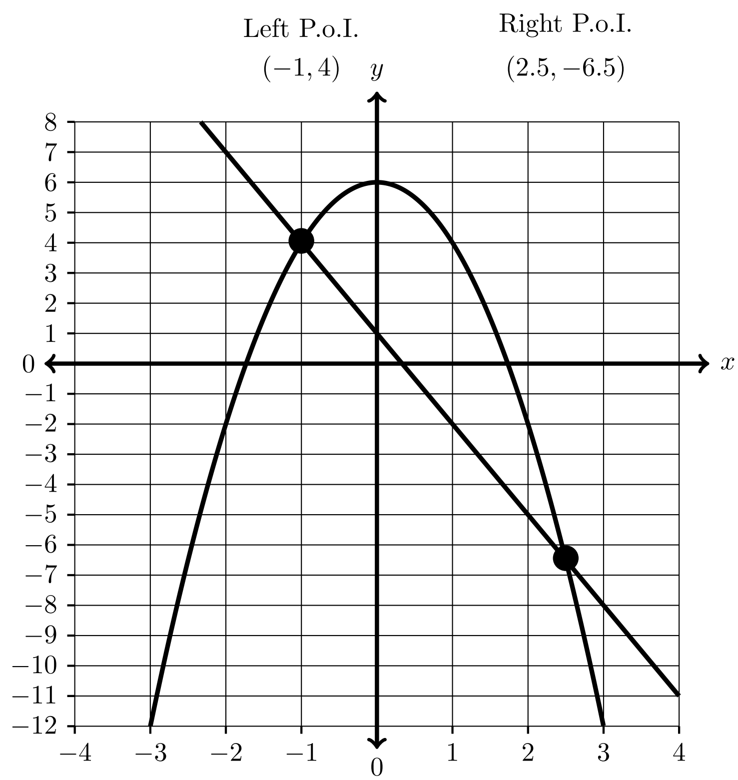
	LHS of Equation	RHS of Equation
x	$y_1 = \frac{1}{4}\left(16 - x^2\right)$	$y_2 = 0$
-8	-12	0
-6	-5	0
-4	0	0
-2	3	0
0	4	0
2	3	0
4	0	0
6	-5	0
8	-12	0



$\boxed{5}$ 4. Solve the quadratic equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Of course, you are welcome to use your calculator. Please specifically identify each point of intersection on your graph. Also, please write each of these points as an ordered pair with an x-coordinate and y-coordinate. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$2x^2 - 12 = x - 2$$

	LHS of Equation	RHS of Equation
x	$y_1 = 1 - 3x$	$y_2 = 6 - 2x^2$
-4	13	-26
-3	10	-12
-2.5	8.5	-6.5
-2	7	-2
-1	4	4
0	1	6
1	-2	4
2	-5	-2
2.5	-6.5	-6.5
3	-8	-12
4	-11	-26



$\boxed{\text{Solutions: } \quad x = -1 \text{ or } x = 2.5}$

3 5. Solve the quadratic equation below using an algebraic technique:

$$2x^2 - 12 = x - 2$$

Solution: We will first get the right-hand side of the equation equal to zero in order to apply the zero product property. Then, we will follow the procedure for factoring polynomials that we discussed in class to write the left-hand side of the equation as a product of two factors.

$$\begin{aligned} 0 &= 2x^2 - 3x - 5, && a = 2, b = -3, c = -5 \\ &= 2x^2 + 2x - 5x - 5 && \text{AC method: } ac = -10, b = -3 \\ &= 2x(x+1) - 5(x+1) && \text{Factor by grouping} \\ &= \boxed{(2x-5)\cdot(x+1).} \end{aligned}$$

By the zero product property, we know

$$(2x - 5) = 0 \qquad \text{or} \qquad (x+1) = 0$$

$$\boxed{x = \frac{5}{2}} \qquad \text{or} \qquad \boxed{x = -1}$$

3 6. In your own words, explain the zero product property. Then, explain how to use the zero product property as an inverse operation to solve quadratic equations.

Solution:

- The zero product property is stated as follows: If $a \cdot b = 0$, then either $a = 0$ or $b = 0$
- In English, we can say: if a product of two numbers is zero, we know that one of the two factors must be equal to zero
- In order to solve quadratic equations use the zero product property, we manipulate the equation to get the right hand side equal to zero. Then, we factor the quadratic expression on the left-hand side of the equals sign to write it as a product of two binomials. In doing so, we have the exact form: $a \cdot b = 0$. We then apply the zero product property to annihilate the multiplication with a zero output and form to linear equations.