
Part 1: Simplify Algebraic Expressions

1. Please simplify by multiplying and combining like terms: $(1 - r) \cdot (1 + r + r^2)$

Solution: We will use distributivity of multiplication over addition to solve this problem.

$$\begin{aligned}(1 - r)(1 + r + r^2) &= 1 \cdot (1 + r + r^2) - r \cdot (1 + r + r^2) \\ &= 1 + r + r^2 - r - r^2 - r^3 \\ &= \boxed{1 - r^3}.\end{aligned}$$

This is the difference of cubes formula that we talked about in class.

2. Please simplify by factoring completely: $4x^3 + 19x^2 - 5x$

Solution: We will follow the procedure for factoring polynomials that we discussed in class.

$$\begin{aligned}4x^3 + 19x^2 - 5x &= x \cdot (4x^2 + 19x - 5), & a = 4, b = 19, c = -5 \\ &= x \cdot (4x^2 + 20x - x - 5), & \text{AC method: } ac = -20, b = +19 \\ &= x \cdot (4x(x + 5) - 1(x + 5)), & \text{Factor by grouping} \\ &= \boxed{x \cdot (4x - 1) \cdot (x + 5)}.\end{aligned}$$

3. Divide and simplify the rational expressions: $\frac{3z + 18}{z - 2} \div \frac{z^2 + 2z - 24}{z - 2}$.

Solution: We will use distributivity of multiplication over addition to solve this problem. Consider:

$$\begin{aligned}\frac{3z + 18}{z - 2} \div \frac{z^2 + 2z - 24}{z - 2} &= \frac{3z + 18}{z - 2} \cdot \frac{z - 2}{z^2 + 2z - 24} \\ &= \frac{3 \cdot (z + 6)}{(z - 2)} \cdot \frac{(z - 2)}{(z - 4) \cdot (z + 6)} \\ &= \frac{3}{(z - 4)} \cdot \frac{(z - 2)}{(z - 2)} \cdot \frac{(z + 6)}{(z + 6)} \\ &= \boxed{\frac{3}{z - 4}}\end{aligned}$$

This equivalence holds as long as $z \neq 2$ and $z \neq -6$.

4. Add and simplify the rational expressions: $\frac{w}{7 + w} + \frac{9w - 35}{w^2 - 49}$

Solution: To subtract fractions with different denominators, we need to find a common denominator. To do so we use the fundamental principle of fractions.

$$\begin{aligned}\frac{w}{w + 7} + \frac{9w - 35}{(w - 7) \cdot (w + 7)} &= \frac{w}{w + 7} \cdot \frac{w - 7}{w - 7} + \frac{9w - 35}{(w - 7) \cdot (w + 7)} \\ &= \frac{w^2 - 7w}{(w - 7) \cdot (w + 7)} + \frac{9w - 35}{(w - 7) \cdot (w + 7)} \\ &= \frac{w^2 - 7w + 9w - 35}{(w - 7) \cdot (w + 7)} \\ &= \frac{w^2 + 2w - 35}{(w - 7) \cdot (w + 7)} \\ &= \frac{(w + 7) \cdot (w - 5)}{(w - 7) \cdot (w + 7)} \\ &= \boxed{\frac{w - 5}{w - 7}}\end{aligned}$$

10 Points

5. Simplify the following radical expression:

$$\sqrt[3]{(w^6 \cdot y^{10})}$$

Solution:

$$\begin{aligned}\sqrt[3]{w^6 \cdot y^{10}} &= (w^6)^{1/3} \cdot \sqrt[3]{y^9 \cdot y} \\ &= w^{6/2} \cdot \sqrt[3]{y^9} \cdot \sqrt[3]{y} \\ &= w^2 \cdot (y^9)^{1/3} \cdot \sqrt[3]{y} \\ &= w^2 \cdot y^{9/3} \cdot \sqrt[3]{y} \\ &= \boxed{w^2 \cdot y^3 \cdot \sqrt[3]{y}}\end{aligned}$$

6. Simplify the radical expression:

$$\frac{\sqrt{98a^2b^9}}{3\sqrt{2a^6b^7}}$$

Solution:

$$\begin{aligned}\frac{\sqrt{98a^2b^9}}{3\sqrt{2a^6b^7}} &= \frac{1}{3} \cdot \frac{\sqrt{98 \cdot a^2 \cdot b^9}}{\sqrt{2 \cdot a^6 \cdot b^7}} \\ &= \frac{1}{3} \cdot \sqrt{\frac{98}{2} \cdot \frac{a^2}{a^6} \cdot \frac{b^9}{b^7}} \\ &= \frac{1}{3} \cdot \sqrt{\frac{49 \cdot b^2}{a^4}} \\ &= \frac{1}{3} \cdot \frac{\sqrt{49} \cdot \sqrt{b^2}}{\sqrt{a^4}} \\ &= \boxed{\frac{7 \cdot |b|}{3a^2}}\end{aligned}$$

10 Points

Part 2: Use Algebraic Techniques to Solve Algebraic Equations

1. $2x - 2 = -5 - x$

Solution:

$$\Rightarrow 3x - 2 = -5$$

$$\Rightarrow 3x = -3$$

$$\Rightarrow \boxed{x = -1}$$

2. $x^2 - 6x = -5$

Solution:

$$\Rightarrow 3x^2 - 3x - 6 = 0$$

$$\Rightarrow 3(x^2 - x - 2) = 0$$

$$\Rightarrow 3(x - 2) \cdot (x + 1) = 0$$

$$\Rightarrow (x - 2) = 0 \text{ OR } (x + 1) = 0$$

$$\Rightarrow \boxed{x = 2} \text{ OR } \boxed{x = -1}$$

3. $2x^2 = 7x - 5$.

Solution:

$$\begin{aligned} \Rightarrow & 2x^2 - 7x + 5 = 0 \\ \Rightarrow & 2x^2 - 2x - 5x + 5 = 0 \\ \Rightarrow & 2x \cdot (x - 1) - 5 \cdot (x - 1) = 0 \\ \Rightarrow & (2x - 5) \cdot (x - 1) = 0 \\ \Rightarrow & (2x - 5) = 0 \text{ OR } (x - 1) = 0 \\ \Rightarrow & \boxed{x = \frac{5}{2}} \text{ OR } \boxed{x = 1} \end{aligned}$$

4. $3 \cdot |x + 2| - 4 = 5$

Solution:

$$\begin{aligned} \Rightarrow & 3 \cdot |x + 2| = 9 \\ \Rightarrow & |x + 2| = 3 \\ \Rightarrow & x + 2 = 3 \text{ OR } x + 2 = -3 \\ \Rightarrow & \boxed{x = 1} \text{ OR } \boxed{x = -5} \end{aligned}$$

12 Points

5. $3 - \frac{5}{x} - \frac{2}{x^2} = 0$

Solution:

$$\implies 3x^2 - 5x - 2 = 0$$

$$\implies 3x^2 - 6x + x - 2 = 0$$

$$\implies (3x + 1) \cdot (x - 2) = 0$$

$$\implies (3x + 1) = 0 \text{ OR } (x - 2) = 0$$

$$\implies \boxed{x = -\frac{1}{3}} \text{ OR } \boxed{x = 2}$$

6. $\frac{x^2 - 5x + 4}{x^2 - 16} = \frac{x - 4}{x}$

Solution:

$$\implies \frac{(x - 4)}{(x - 4)} \cdot \frac{(x - 1)}{(x + 4)} = \frac{(x - 4)}{x}$$

$$\implies 1 \cdot \frac{(x - 1)}{(x + 4)} = \frac{(x - 4)}{x}$$

$$\implies \frac{x}{x} \cdot \frac{(x - 1)}{(x + 4)} = \frac{(x - 4)}{x} \cdot \frac{(x + 4)}{(x + 4)}$$

$$\implies x \cdot (x - 1) = (x - 4) \cdot (x + 4)$$

$$\implies x^2 - x = x^2 - 16$$

$$\implies \boxed{x = 16}$$

12 Points

7. $\sqrt{3x} + 1 = 6$

Solution:

$$\Rightarrow \sqrt{3x} = 6 - 1$$

$$\Rightarrow (\sqrt{3x})^2 = 5^2$$

$$\Rightarrow 3x = 25$$

$$\Rightarrow x = \frac{25}{3} \text{ OR } x = 8.\overline{33}$$

8. $2(x - 4)^3 = -16$

Solution:

$$\Rightarrow (x - 4)^3 = -8$$

$$\Rightarrow \sqrt[3]{(x - 4)^3} = \sqrt[3]{-8}$$

$$\Rightarrow x - 4 = -2$$

$$\Rightarrow x = 2$$

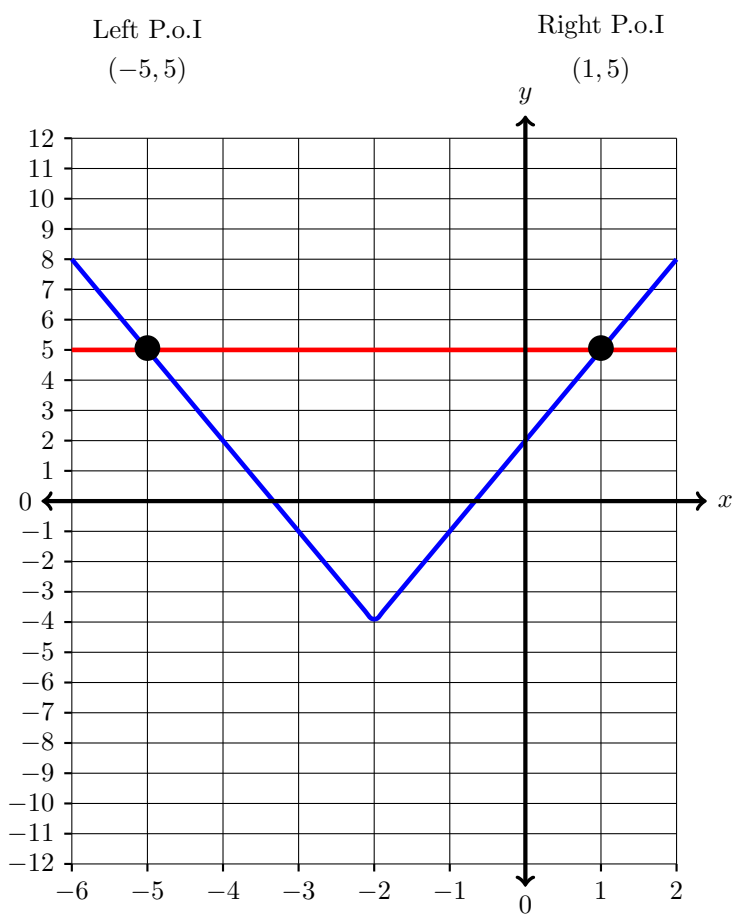
12 Points

Part 3: Use an Graphical Technique to Solve Algebraic Equations

- 10 1. Using a calculator, solve the absolute value equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Please specifically identify each point of intersection on your graph and write each of these points as an ordered pair. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$3 \cdot |x + 2| - 4 = 5$$

	LHS of Equation	RHS of Equation
x	$y_1 = 3 \cdot x + 2 - 4$	$y_2 = 5$
-6	8	5
-5	5	5
-4	2	5
-3	-1	5
-2	-4	5
-1	-1	5
0	2	5
1	5	5
2	8	5

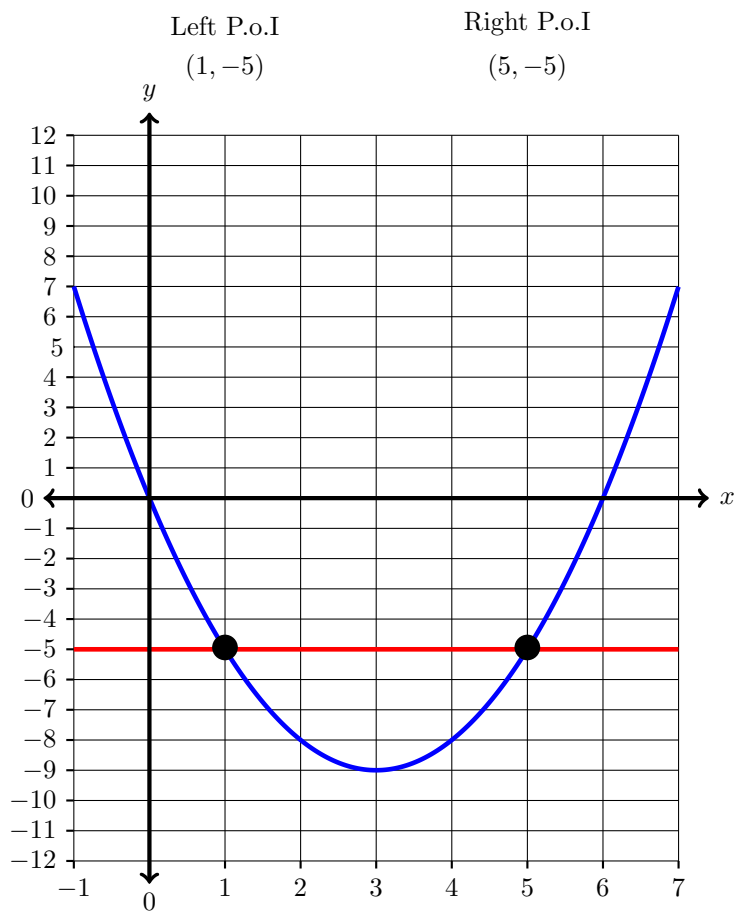


Solutions: $x = -5$ or $x = 1$

- 10 1. Using a calculator, solve the absolute value equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Please specifically identify each point of intersection on your graph and write each of these points as an ordered pair. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$x^2 - 6x = -5$$

	LHS of Equation	RHS of Equation
x	$y_1 = x^2 - 6x$	$y_2 = -5$
-1	7	-5
0	0	-5
1	-5	-5
2	-8	-5
3	-9	-5
4	-8	-5
5	-5	-5
6	0	-5
7	7	-5

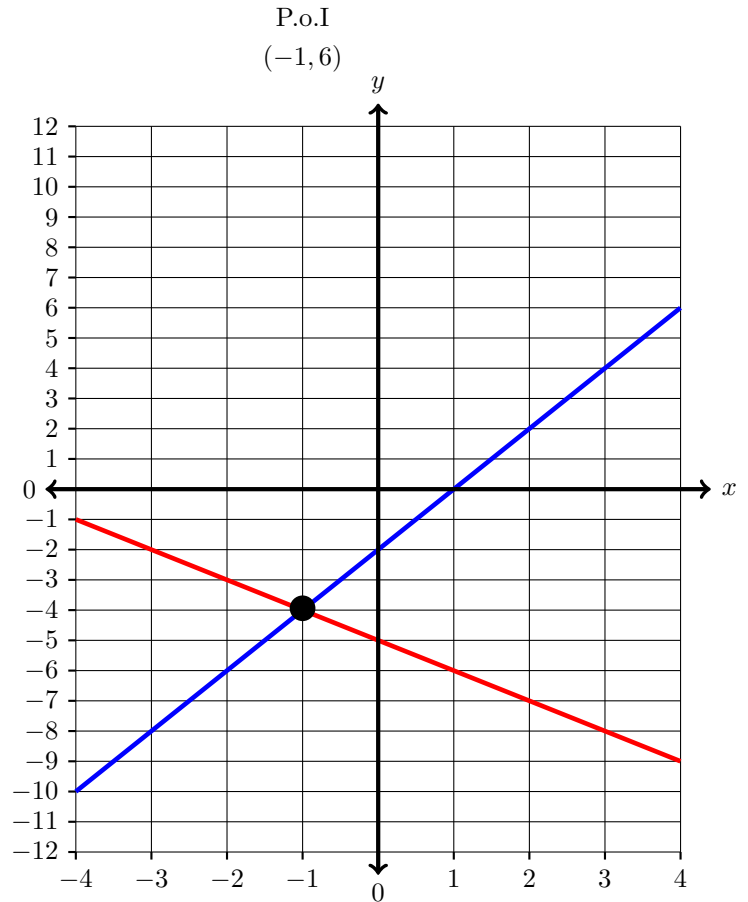


Solutions: $x = 1$ or $x = 5$

- 10 1. Using a calculator, solve the absolute value equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Please specifically identify each point of intersection on your graph and write each of these points as an ordered pair. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

$$2x - 2 = -5 - x$$

	LHS of Equation	RHS of Equation
x	$y_1 = 2x - 2$	$y_2 = -5 - x$
-4	-10	-1
-3	-8	-2
-2	-6	-3
-1	-4	-4
0	-2	-5
1	0	-6
2	2	-7
3	4	-8
4	6	-9



Solution: $x = -1$

10 Points