Definitions: Define each of the following vocabulary words. (5 points each)

1. Algebraic technique for solving algebraic equations

2. Graphical technique for solving algebraic equations

Evaluate each of the following. Show your work. Box your final answer. (5 points each)

3. $-4^2 + 12 \div 3 \cdot 2$

Solution: We will solve this problem using the order of operations: $-4^{2} + 12 \div 3 \cdot 2 = -1 \cdot (4)^{2} + (12 \div 3) \cdot 2$

$$= -16 + 4 \cdot 2$$

$$= -8.$$

 $\frac{5}{2} + \frac{3}{2} \cdot \frac{1}{2}$

4.

Solution: Let's use the properties of opera- tions on fractions to simplify this expression:				
$\frac{5}{2} + \frac{3}{2} \cdot \frac{1}{2} = \frac{5}{2} + \frac{3 \cdot 1}{2 \cdot 2}$				
$=rac{5}{2}+rac{3}{4}$				
$=\frac{10}{4}+\frac{3}{4}$				
$=\boxed{\frac{13}{4}=3.25}$				

Factor completely. Show your work. Box your final answer. (5 points each)

5.
$$2x^4 + x^3 - 3x^2$$
 6. $4t^2 - 81$

Solution:

$$2x^{4} + x^{3} - 3x^{2} = x^{2} \cdot (2x^{2} + x - 3)$$

$$= x^{2} \cdot (2x^{2} + 3x - 2x - 3)$$

$$= x^{2} \cdot (x \cdot (2x + 3) - 1 \cdot (2x + 3))$$

$$= \boxed{x^{2} \cdot (2x + 3) \cdot (x - 1)}$$

Solution: $4t^{2} - 81 = (4t^{2} + 18t - 18t - 81)$ $= (2t \cdot (2t + 9) - 9 \cdot (2t + 9))$ $= (2t - 9) \cdot (2t + 9)$

Solve each of the following linear equations. Show your work. Box your final answer. (5 points each)

8.

7.
$$4b + 9 + 2 = 17 + 6b$$
Solution:

$$\Rightarrow 4b + 11 = 17 + 6b$$

$$\Rightarrow 11 = 17 + 2b$$

$$\Rightarrow -6 = 2b$$

$$\Rightarrow b = -3.$$

$$2 \cdot (1 - 3h) - 5 \cdot (2h + 3) = -21$$

Solution:

$$\implies 2-6h-10h-15 = -21$$

$$\implies -h(6+10) + (2-15) = -21$$

$$\implies -16h = -8$$

$$\implies h = \frac{8}{16} = \frac{1}{2} = 0.5$$

Using the zero product property, solve each of the following quadratic equations.. Show your work. Box your final answer. (5 points each)

9.
$$x^2 - 2x - 11 = 1 - x$$
 10. $8x = x^2$

Solution: $\Rightarrow x^{2} - x - 12 = 0$ $\Rightarrow x^{2} - 4x + 3x - 12 = 0$ $\Rightarrow x \cdot (x - 4) + 3 \cdot (x - 4) = 0$ $\Rightarrow (x + 3) \cdot (x - 4) = 0$ $\Rightarrow x + 3 = 0 \text{ or } x - 4 = 0$ $\Rightarrow x + 3 = 0 \text{ or } x - 4 = 0$



Solve each of the following absolute value equations. Show your work. Box your final answer. (5 points each)

11.
$$4-2|x-3| = -2$$

Solution: $\implies -2|x-3| = -6$ $\implies |x-3| = 3$ $\implies x-3 = -3 \text{ or } x-3 = 3$ $\implies x = 0 \text{ or } x = 6.$

12.
$$-2|x-2| - 2 = 14$$

Solution:	
\Rightarrow	-2 x-2 = 16
\Rightarrow	x-2 = -8
\Rightarrow	NO SOLUTION

13. (10 points) Solve the absolute value equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Of course, you are welcome to use your calculator. Please specifically identify each point of intersection on your graph. Also, please write each of these points as an ordered pair with an x-coordinate and y-coordinate. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

	LHS of Equation	RHS of Equation
x	$y_1 = 4 - 2 x - 3 $	$y_2 = -2$
-1	-4	-2
0	-2	-2
1	0	-2
2	2	-2
3	4	-2
4	2	-2
5	0	-2
6	-2	-2
7	-4	-2

$$4 - 2|x - 3| = -2$$



14. (10 points) Solve the quadratic equation below using a graphical technique. Make sure to demonstrate all five steps of this process. Of course, you are welcome to use your calculator. Please specifically identify each point of intersection on your graph. Also, please write each of these points as an ordered pair with an x-coordinate and y-coordinate. Make sure to finish step 5 and use this information to explicitly state the solution(s) to this algebraic equation:

	LHS of Equation	RHS of Equation			
x	$y_1 = x^2 - 2x - 11$	$y_2 = 1 - x$			
-4	13	5	$\begin{bmatrix} \text{Left P.o.I} & \text{Right P.o.I} \\ (-3,4) & y & (4,-3) \end{bmatrix}$		
-3	4	4	$ \begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$		
-2	-3	3			
-1	-8	2			
0	-11	1			
1	-12	0			
2	-11	-1			
3	-8	-2			
4	-3	-3			
5	4	-4	Solutions: $x = -3$ or $x = 4$		
6	13	-5			

$$x^2 - 2x - 11 = 1 - x$$

For the questions below, please be as specific as possible in your descriptions. Use full sentences and describe your thoughts in detail. Please be sure to address each item in the question prompt. (5 points each)

15. In your own words, explain the zero product property. Explain how factoring quadratic expressions is related to the zero product property. Then, explain how to use the zero product property as an inverse operation to solve quadratic equations.

16. In your own words, explain the inverse operation for absolute value equations. Then, explain how to use this inverse to solve absolute values equations.

17. (10 points total) Use the following graph to answer the questions below:



A. Match each of the functions in the graph above with its corresponding equation. Write f(x), g(x) or h(x) in the spaces below. (3 points each)

$$\underline{h(x)} = 3.5 - x - \frac{1}{2}x^2 \qquad \underline{g(x)} = -3 - \frac{1}{4}x \qquad \underline{f(x)} = \left|\frac{x+1}{2}\right| + 1$$

- B. Find the point(s) on the graph above corresponding to solution(s) of the equation h(x) = 0. Put a dot on each point and **label each point with the letter "A"**. (1 points)
- C. Using part A above, approximate the x-values for which h(x) = 0: (2 points) (-3.8,0) and (1.8,0)
- D. Find the point(s) on the graph at which g(x) = -4. Put a dot on each point and label each point with the letter "B". (1 points)
- E. For what x-values(s) is g(x) = -4? (2 points) _____ x = 4
- F. Find the point(s) on the graph above corresponding to solution(s) of the equation

$$3.5 - x - \frac{1}{2}x^2 = \left|\frac{x+1}{2}\right| + 1$$

Put a dot on each point and label each point with the letter "C". (1 points)

G. (2 points) Using part F, find the x-value(s) for which $3.5-x-\frac{1}{2}x^2 = \left|\frac{x+1}{2}\right|+1$: <u>x = -3 or x = 1</u>