

## SECTION 8.1: Rational Expressions and Functions (p. 566 – 577)

- Polynomial expressions (p. 358 - 359)
- Rational expressions (p. 566)
- Rational functions (p. 566)
- Simplified rational expressions (p. 569)
- Caution on canceling (p. 570)

## RECIPROCAL:

For the fraction  $\frac{C}{D}$ , the reciprocal is the multiplicative inverse. To create the reciprocal, we "flip" the fraction upside down  $\frac{D}{C}$  where the numerator becomes the denominator and vice versa.

1. Rational Number: any number that can be expressed as the quotient of two integers

$$\rightarrow \frac{550}{5} \quad \leftarrow \text{Numerator (the number in the top part of a fraction)}$$

$$\quad \quad \quad \leftarrow \text{Denominator (the number in the bottom part of a fraction)}$$

2. Fraction Notation for 1:

$$\frac{A}{A} = 1 \quad \text{as long as the denominator is non-zero}$$

(i.e.  $\frac{A}{A} = 1$  for all non-zero  $A$ )

Remember: Fractions with zero in the denominator are always undefined.

3. Multiplication of Fractions:

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{A \cdot C}{B \cdot D} = \frac{AC}{BD}$$

when multiplying fractions, multiply the numerators together and place product in numerator then multiply denominators together and place product in the denominator

Remember: when multiplying two fractions, multiply the numerators and multiply the denominators

4. Division of Fractions:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{A \cdot D}{B \cdot C} = \frac{AD}{BC}$$

Remember: when dividing a fraction by another fraction, multiply by the reciprocal

when we divide a fraction by another fraction, we multiply the fraction on the left side of division by the RECIPROCAL of the fraction on the right side of the division.

5. Addition of Fractions:

$$\frac{A}{D} + \frac{B}{D} = \frac{A+B}{D}$$

Remember: when we add two fractions, we must have a common denominator

## GOLD STANDARD:

if we add two fractions with a common denominator, we can add the numerators and leave the denominator untouched

6. Addition of Fractions:

$$\frac{A}{D} - \frac{B}{D} = \frac{A-B}{D}$$

## GOLD STANDARD

if we subtract two fractions with a common denominator, we can subtract the ~~denominators~~ numerators and leave the denominator alone

**II. SECTION 8.1: EQUIVALENT FRACTIONS**

Start with the number on the left and use a series of operations to create the **equivalent** expression on the right. Remember, you can change the way a number looks without changing the VALUE by multiplying or dividing by 1 (in any form you want).

Start with:  $5$  End with:  $\frac{30y}{6y}$  ✓

sexy-looking 1: by the fundamental principal of fractions we know  $\frac{6}{6} = 1$  and 1 is the only number we can multiply by and not change the value of our product

$$7. \quad 5 = 5 \cdot 1 = \frac{5}{1} \cdot \frac{6}{6} \cdot \frac{y}{y} = \frac{5 \cdot 6 \cdot y}{1 \cdot 6 \cdot y} = \frac{30y}{6y}$$

We want a 6 and a y in the denominator of our fraction

Start with:  $2$  End with:  $\frac{4x^2}{2x^2}$  ✓

sexy-looking 1: by the fundamental principal of fractions, we know  $\frac{x^2}{x^2} = 1$  as long as  $x \neq 0$

$$8. \quad 2 = 2 \cdot 1 = \frac{2}{1} \cdot \frac{2}{2} \cdot \frac{x^2}{x^2} = \frac{2 \cdot 2 \cdot x^2}{1 \cdot 2 \cdot x^2} = \frac{4x^2}{2x^2}$$

Start with:  $1$  End with:  $\frac{x+5}{5+x}$  ✓

$$9. \quad 1 = 1 \cdot 1 = \frac{1}{1} \cdot \frac{(x+5)}{(x+5)} = \frac{1 \cdot (x+5)}{1 \cdot (x+5)} = \frac{x+5}{5+x}$$

Addition is commutative: We can write addition in any order we would like  
 $x+5 = 5+x$

Start with:  $-1$  End with:  $\frac{3-x}{x-3}$  ✓

$$10. \quad -1 = -1 \cdot 1 = -\frac{1}{1} \cdot \frac{(x-3)}{(x-3)} = \frac{-1 \cdot (x-3)}{1 \cdot (x-3)} = \frac{-x+3}{x-3}$$

Addition is commutative:  
 The order we write addition in doesn't matter  
 $-x+3 = 3+ -x$

$$= 3 - x$$

## Graphical Techniques

**III. SECTION 8.1: EQUIVALENT FUNCTIONS**

Fill in the tables below, as directed by your instructor. You can use your calculator to do the computations by hand.

11.  $f(x) = \frac{6x-2x^2}{x-3}$

input $x$	output $f(x)$
-1	2
1	-2
3	Error
5	-10
7	-14

12.  $g(x) = -2x$

INPUT $x$	OUTPUT $g(x)$
-1	2
1	-2
3	-6
5	-10
7	-14

Notice that these two tables have identical output values except for input  $x=3$ . This difference arises from the simplification of  $f(x)$  into  $g(x)$ . We can use our work here to conclude that  $x=3$  is not in the domain of  $f(x)$  or  $f(x) \neq g(x)$  if  $x=3$ .

13.  $h(x) = \frac{2x+4}{2x+x^2}$

INPUT $x$	OUTPUT $h(x)$
-4	-0.5
-2	Error
0	Error
2	1
4	0.5

14.  $k(x) = \frac{2}{x}$

INPUT $x$	OUTPUT $k(x)$
-4	-0.5
-2	-1
0	Error
2	1
4	0.5

## 11. Algebraic Method to Simplify our expression

$$\frac{6x - 2x^2}{x - 3} = \frac{2x \cdot (3 - x)}{1 \cdot (x - 3)}$$

$$= \frac{2x}{1} \cdot \underbrace{\frac{(3 - x)}{(x - 3)}}_{-1}$$

$$= 2x \cdot -1$$

$$= \boxed{-2x} \quad \text{as long as} \\ x \neq 3$$

Side Note:

$$\frac{3-x}{x-3} = \frac{-1}{-1} \cdot \frac{(3-x)}{(x-3)}$$

$$= \frac{1}{-1} \cdot \frac{-1 \cdot (3-x)}{(x-3)}$$

$$= \frac{-1}{1} \cdot \frac{(-3+x)}{(x-3)}$$

$$= \frac{-1}{1} \cdot \frac{(x-3)}{(x-3)}$$

$$= -1 \cdot 1$$

$$= -1$$

Notice that  $\frac{(x-3)}{(x-3)} = 1$  as

long as  $x \neq 3$ .

## SECTION 8.2: Multiplication and Division (p. 578 – 584)

 The product of two rational expressions (p. 578)

 The quotient of two rational expressions (p. 579)

Using the rules of fractions that we've studied together, multiply or divide the following rational expressions.

$$\begin{aligned}
 1. \quad \frac{16}{25} \cdot \frac{35}{12} &= \frac{4 \cdot 4}{5 \cdot 5} \cdot \frac{5 \cdot 7}{3 \cdot 4} \\
 &= \frac{4 \cdot 5 \cdot 4 \cdot 7}{4 \cdot 5 \cdot 5 \cdot 3} \\
 &= 1 \cdot 1 \cdot \frac{4 \cdot 7}{5 \cdot 3} \\
 &= \boxed{\frac{28}{15}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \frac{2}{3} \div \frac{14}{15} &= \frac{2}{3} \cdot \frac{15}{14} \\
 &= \frac{2}{3} \cdot \frac{3 \cdot 5}{2 \cdot 7} \\
 &= \frac{2}{2} \cdot \frac{3}{3} \cdot \frac{5}{7} \\
 &= 1 \cdot 1 \cdot \frac{5}{7} = \boxed{\frac{5}{7}}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \frac{x-1}{x+2} \cdot \frac{x^2+2x}{3-3x} &= \frac{(x-1)}{(x+2)} \cdot \frac{x \cdot (x+2)}{3(1-x)} \\
 &= \frac{(x+2)}{(x+2)} \cdot \frac{(x-1)}{(1-x)} \cdot \frac{x}{3} \\
 &= \frac{1}{1} \cdot \frac{-1}{1} \cdot \frac{x}{3} = \boxed{\frac{-x}{3}} \\
 &\quad \text{(if } x \neq -2) \quad \text{(if } x \neq 1) \quad \text{if } x \neq 1, -2
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \frac{x}{5} \div \frac{7x}{25} &= \frac{x}{5} \cdot \frac{25}{7x} \\
 &= \frac{x}{5} \cdot \frac{5 \cdot 5}{7 \cdot x} \\
 &= \frac{5}{5} \cdot \frac{x}{x} \cdot \frac{5}{7} \\
 &= 1 \cdot 1 \cdot \frac{5}{7} = \boxed{\frac{5}{7}} \quad \text{if } x \neq 0 \\
 &\quad \text{(if } x \neq 0)
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (x+2) \cdot \frac{(x-1)}{(x+2)} &= \frac{(x+2)}{1} \cdot \frac{(x-1)}{(x+2)} \\
 &= \frac{(x+2)}{(x+2)} \cdot \frac{(x-1)}{1} \\
 &= \frac{1}{1} \cdot (x-1) \\
 &\quad \text{(if } x \neq -2)
 \end{aligned}$$

$$6. \quad \frac{10t+20}{2t^2-3t+1} \cdot \frac{t^2-1}{5t+10} = \quad \text{(see next page for details)}$$

$$\begin{aligned}
 &= \frac{10 \cdot (t+2)}{(2t-1) \cdot (t-1)} \cdot \frac{(t-1)(t+1)}{5 \cdot (t+2)} \\
 &= \frac{10}{5} \cdot \frac{(t-1)}{(t-1)} \cdot \frac{(t+2)}{(t+2)} \cdot \frac{(t+1)}{(2t-1)} \\
 &= 2 \cdot 1 \cdot 1 \cdot \frac{(t+1)}{2t-1} \\
 &\quad \text{(if } t \neq 1) \quad \text{(if } t \neq -2)
 \end{aligned}$$

$$= \boxed{(x-1)} \quad \text{if } x \neq -2$$

$$= \boxed{2(t+1)} \quad \text{if } t \neq 1, -2$$

6. Consider the denominator

$$2t^2 - 3t + 1$$

$$a=2, b=-3, c=1$$

$$= 2t^2 - 2t - t + 1$$

$$= 2t \cdot (t-1) - 1 \cdot (t-1)$$

$$= (2t-1) \cdot (t-1)$$

The AC Method

Multiply

$$\begin{array}{ccc} & +2 & \\ -2 & \times & -1 \\ & -3 & \end{array}$$

Add

$$-3t = -2t - t$$

Consider the numerator  $t^2 - 1$ . Recall the difference of squares

$$A^2 - B^2 = (A - B) \cdot (A + B)$$

$$t^2 - 1 = (t - 1) \cdot (t + 1)$$

$$\text{"A=t" "B=1"}$$

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$$7. (w^2 - 3w - 10) \cdot \frac{w+4}{w^2-10w+25} = \left( \begin{array}{l} \text{see next} \\ \text{page for} \\ \text{details} \end{array} \right)$$

$$= \frac{w^2 - 3w - 10}{1} \cdot \frac{(w+4)}{(w-5)(w-5)}$$

$$= \frac{(w-5) \cdot (w+2)}{1} \cdot \frac{(w+4)}{(w-5)^2}$$

$$= \frac{(w-5)}{(w-5)} \cdot \frac{(w+2) \cdot (w+4)}{(w-5)}$$

$$= \underset{w \neq 5}{1} \cdot \frac{(w+2) \cdot (w+4)}{(w-5)} = \boxed{\frac{(w+2) \cdot (w+4)}{(w-5)}}$$

$$8. \frac{x^2-2x-3}{x^2-4} \div \frac{3-x}{x+2} =$$

$$= \frac{(x-3) \cdot (x+1)}{(x-2) \cdot (x+2)} \cdot \frac{(x+2)}{-1 \cdot (x-3)}$$

$$= \frac{1}{-1} \cdot \frac{(x-3)}{(x-3)} \cdot \frac{(x+2)}{(x+2)} \cdot \frac{\cancel{(x+1)}}{\cancel{-(x+2)}} \cdot \frac{(x+1)}{(x-2)}$$

$$= -1 \cdot \underset{(x \neq 3)}{1} \cdot \underset{(x \neq -2)}{1} \cdot \frac{\cancel{(x+1)}}{\cancel{-(x+2)}} \cdot \frac{(x+1)}{(x-2)}$$

$$= \boxed{-1 \cdot \frac{(x+1)}{(x-2)} = \frac{-x-1}{x-2}}$$

OPTIONAL CHALLENGE PROBLEMS

$$9. \frac{(a^2+3a+2)}{a^2-4} \div \frac{5a^2+10a}{a-2} =$$

$$= \frac{(a+2) \cdot (a+1)}{(a-2)(a+2)} \div \frac{5a \cdot (a+2)}{(a-2)}$$

$$= \frac{(a+2) \cdot (a+1)}{(a-2)(a+2)} \cdot \frac{(a-2)}{5a \cdot (a+2)}$$

$$= \frac{(a+2)}{(a+2)} \cdot \frac{(a-2)}{(a-2)} \cdot \frac{(a+1)}{5a(a+2)} = \boxed{\frac{a+1}{5a(a+2)}}$$

$$10. \frac{10b+20}{b} \cdot \frac{b^2}{b+2} =$$

$$= \frac{10(b+2)}{b} \cdot \frac{b^2}{(b+2)}$$

$$= \frac{b}{b} \cdot \frac{(b+2)}{(b+2)} \cdot \frac{10 \cdot b}{1}$$

$$= \underset{(b \neq 0)}{1} \cdot \underset{(b \neq -2)}{1} \cdot 10 \cdot b$$

$$= \boxed{10b} \text{ if } b \neq 0, -2$$

$$11. \frac{(y+1)}{y^2-1} \div \frac{(y+1)}{(y^2-2y+1)} =$$

$$= \frac{(y+1)}{(y-1)(y+1)} \cdot \frac{(y-1) \cdot (y-1)}{(y+1)}$$

$$= \frac{(y+1)}{(y+1)} \cdot \frac{(y-1)}{(y-1)} \cdot \frac{(y-1)}{(y+1)}$$

$$= \underset{y \neq -1}{1} \cdot \underset{y \neq 1}{1} \cdot \frac{y-1}{y+1} = \boxed{\frac{(y-1)}{(y+1)}}$$

7. Consider the numerator

$$a=1, b=-3, c=-10$$

$$w^2 - 3w - 10$$

$$= w^2 + 2w - 5w - 10$$

$$= w(w+2) - 5(w+2)$$

$$= (w-5) \cdot (w+2)$$

The AC Method

Multiply

$$\begin{array}{r} -10 \\ +2 \quad -5 \\ -3 \\ \text{Add} \end{array}$$

$$-3w = 2w - 5w$$

Consider the denominator

$$a=1, b=-10, c=25$$

$$w^2 - 10w + 25$$

$$= w^2 - 5w - 5w + 25$$

$$= w \cdot (w-5) - 5 \cdot (w-5)$$

$$= (w-5) \cdot (w-5)$$

The AC Method

Multiply

$$\begin{array}{r} +25 \\ -5 \quad -5 \\ -10 \\ \text{Add} \end{array}$$

$$-10w = -5w - 5w$$