SECTION 7.4: Absolute-Value Equations and Inequalities (p. 547 – 558)
  ☐ Absolute value (p. 547)
  ☐ The absolute-value principle for equations (p. 548)
  ☐ Algebraic and graphical approaches to solve absolute value equations (p. 549)
  ☐ The absolute-value principle for equations with TWO absolute values (p. 551)
  ☐ Principles for solving absolute-value problems (p. 552)

For problems 1 through 7 below, answer the following questions in groups. You do not need to show your work. Think about it and write down your thoughts. Write down anything you think might help another student understand how you came up with your answers.

1. Nico takes the absolute value of a number, and gets 5. What number(s) did he use?
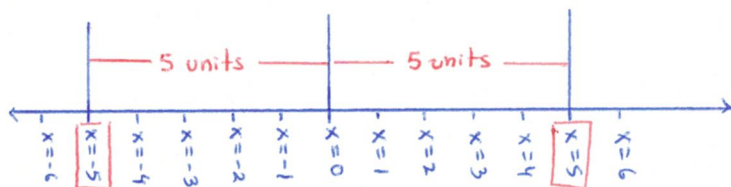
2. Nanci takes the absolute value of a number, and gets 0. What number(s) did she use?

   Nanci's number must be zero
   since zero is 0 units away from
   itself.

3. Jayshawn takes the absolute value of a number to get -8. What number(s) did he use?

   No solution

4. Mavic doubles the absolute value of a number to get 18. What number(s) did he use?

   9 or -9

5. Sam takes the absolute value of a number then subtracts 5 from the result to get -1. What number(s) did she use?

   4 or -4

6. Jacyln adds 1 to a number then takes the absolute value of the result and gets 3. What number(s) did she use?

   2 or -4

7. Char says that she can multiply the absolute value of a number by -4 and the result will be 20. What number(s) could she be thinking of?

   No solution

SECTION 7.4: WRITING EQUATIONS INVOLVING ABSOLUTE VALUES

The descriptions from the previous problems are below. Use algebraic symbols to translate the problems you just solved into equations. If you had to think through more than one step to arrive at your answers, try to write the steps using algebraic symbols as well.

8. The absolute value of Nico' number is 5.

Equation: $|n| = 5$

Solution(s): $n = 5$ OR $n = -5$

9. The absolute value of Nanci's number is 0.

Equation $|n| = 0$

Solution(s): $n = 0$

10. The absolute value of Jayshawn's number is -8.

Equation: $|j| = -8$

Solution(s): No solution
(absolute value output must be nonnegative)

11 Mavic doubles the absolute value of the number to get 18.

Equation: $2 \cdot |m| = 18$

Solution(s): $m = 9$ or $m = -9$

12. Sam takes the absolute value, then subtracts 5 from the result to get -1.

Equation: $|s| - 5 = -1$

Solution(s): $s = 4$ or $s = -4$

13. When Jacyln adds 1 to a number, then takes the absolute value of the result, she gets 3.

Equation: $|j + 1| = 3$

Solution(s): $j = 2$ or $j = -4$

14. Char multiplies the absolute value of a number by -4 and the result will be 20. What number(s) could she be thinking of?

Equation: $-4 \cdot |c| = 20$

Solution(s): No solution

SECTION 7.4: Determine the [hidden] equation, and then solve the equation.

[Sticky note:] Graphical Method
See work on reverse side for more about the solution to #15 Using a graphical method

15. $|2b + 1| + 2 = 7$

$\Rightarrow \quad |2b+1| + 2 - 2 = 7 - 2$

$\Rightarrow \quad |2b+1| + 0 = 5$

$\Rightarrow \quad |2b+1| = 5$

$\Rightarrow \quad 2b + 1 = 5 \qquad \text{OR} \qquad 2b + 1 = -5$

$\Rightarrow \quad 2b = 4 \qquad \text{OR} \qquad 2b = -6$

$\Rightarrow \quad \boxed{b = 2} \qquad \text{OR} \qquad \boxed{b = -3}$

16. $3\,|8 + w| = 42$

$\Rightarrow \quad \dfrac{3}{3} \cdot \dfrac{|w+8|}{1} = \dfrac{42}{3}$

$\Rightarrow \quad |w + 8| = 14$

$\Rightarrow \quad w + 8 = 14 \qquad \text{or} \qquad w + 8 = -14$

$\Rightarrow \quad \boxed{w = 6} \qquad \text{or} \qquad \boxed{w = -22}$

17. $-3\,|x + 8| = 12$

$\Rightarrow \quad \dfrac{-3}{-3} \cdot \dfrac{|x+8|}{1} = \dfrac{12}{-3}$

$\Rightarrow \quad |x + 8| = -4$

No solution: recall that the absolute value function $f(x) = |x|$ must have nonnegative output values. In other words, for any input value x, we know $|x| \geq 0$. Thus, there is no value for x s.t.

$$|x+8| < 0$$

and we ~~conclude~~ conclude there is no solution.

18. $-4\,|2m - 15| + 9 = -11$

$\Rightarrow \quad -4 \cdot |2m - 15| = -20$

$\Rightarrow \quad \dfrac{-4}{-4} \cdot \dfrac{|2m - 15|}{1} = \dfrac{-20}{-4}$

$\Rightarrow \quad |2m - 15| = 5$

$\Rightarrow \quad 2m - 15 = 5 \qquad \text{OR} \qquad 2m - 15 = -5$

$\Rightarrow \quad 2m = 20 \qquad \text{OR} \qquad 2m = 10$
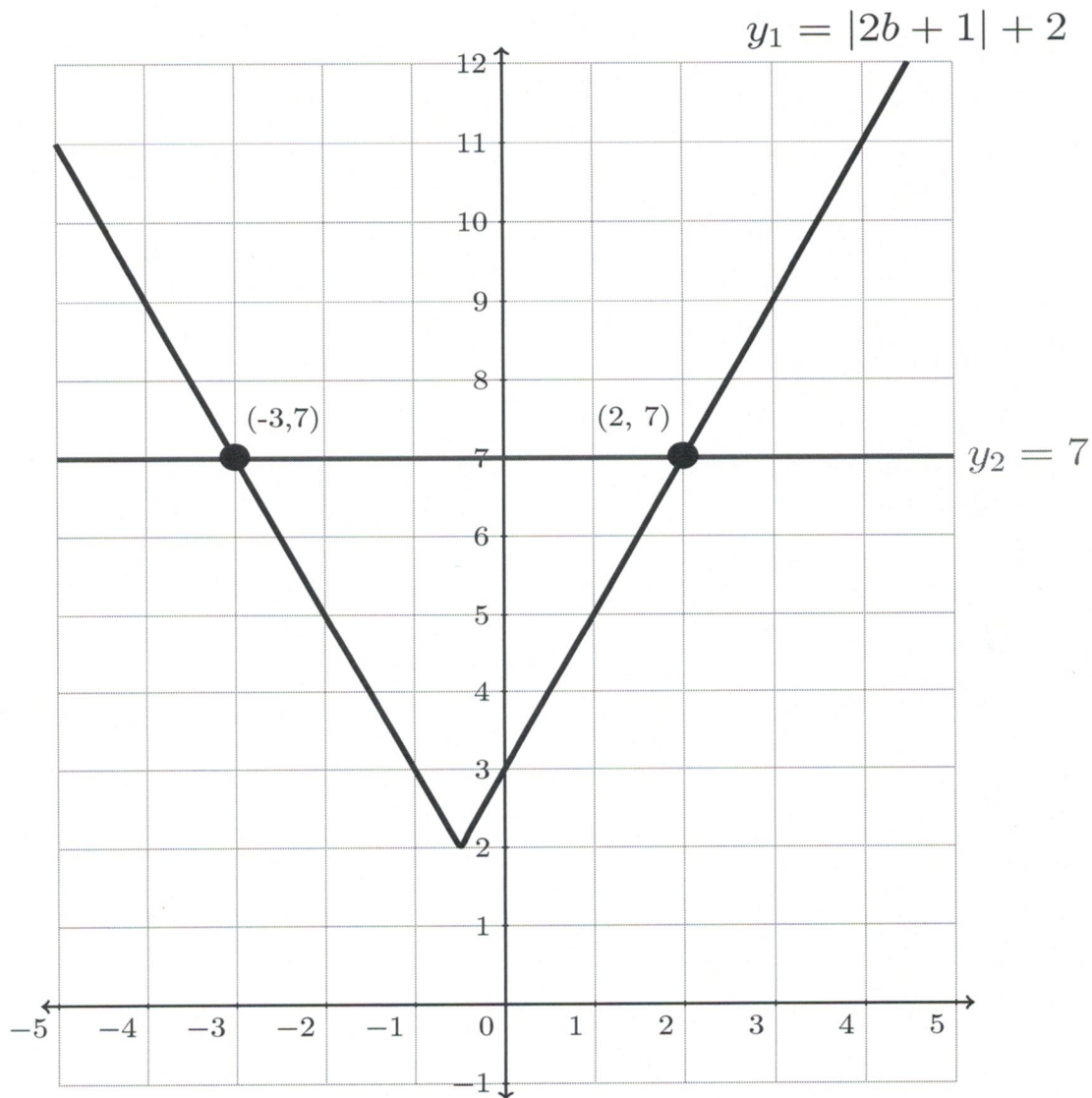
$\Rightarrow \quad \boxed{m = 10} \qquad \text{OR} \qquad \boxed{m = 5}$

15. Let's solve this problem using our graphical method. To do so, we graph the expressions on either side of the equals sign separately by setting

$$y_1 = |2b + 1| + 2, \qquad\qquad\qquad y_2 = 7$$

We remember that the solution of our equation are the "x-coordinates" of the points of intersection.

$$y_1 = |2b + 1| + 2$$

Using the graph below (our on our graphing calculator), we see the two points of intersection are

$(-3, 7)$ corresponding to solution $\boxed{b = -3}$

$(2, 7)$ corresponding to solution $\boxed{b = 2}$

OPTIONAL CHALLENGE PROBLEM

Graphical Method
See work on reverse
side for a solution to
#19 using a graphical
method

Determine the number of soluti_____ e the equation.

19. $|3x + 6| = |6x - 12|$

$\Rightarrow$ $3x + 6 = 6x - 12$ OR $3x + 6 = -(6x - 12)$

$\Rightarrow$ $3x + 18 = 6x$ OR $\Rightarrow$ $3x + 6 = -6x + 12$

$\Rightarrow$ $18 = 3x$ $\Rightarrow$ $3x = -6x + 6$

 $\Rightarrow$ $9x = 6$

$\Rightarrow$ $\dfrac{18}{3} = \dfrac{3x}{3}$ $\Rightarrow$ $\dfrac{9x}{9} = \dfrac{6}{9}$

$\Rightarrow$ $\boxed{x = 6}$ $\Rightarrow$ $\boxed{x = \dfrac{2}{3}}$

20. $-2\,|4y - 12| - 5 = 4|y + 1| + 15$

$\Rightarrow$ $-2 \cdot |4y - 12| = 4 \cdot |y + 1| + 20$

$\Rightarrow$ $\dfrac{-2}{-2} \cdot |4y - 12| = \dfrac{4 \cdot |y + 1| + 20}{-2}$

$\Rightarrow$ $|4y - 12| = -2 \cdot |y + 1| - 10$

$\boxed{\text{No solution.}}$ Consider the LHS of this equation given by $|4y - 12|$.
The output to the LHS will always be greater than or equal to zero.
Compare this to the RHS $-2 \cdot |y + 1| - 10$. The output to the
right hand side will always be less than or equal to -10. There is
no way the two outputs can be the same. Thus we conclude
there is no solution to this equation.

19. Let's solve this problem using our graphical method. To do so, we graph the expressions on either side of the equals sign separately by setting

$$y_1 = |3x + 6| \qquad\qquad y_2 = |6x - 12|$$

We remember that the solution of our equation are the "x-coordinates" of the points of intersection.
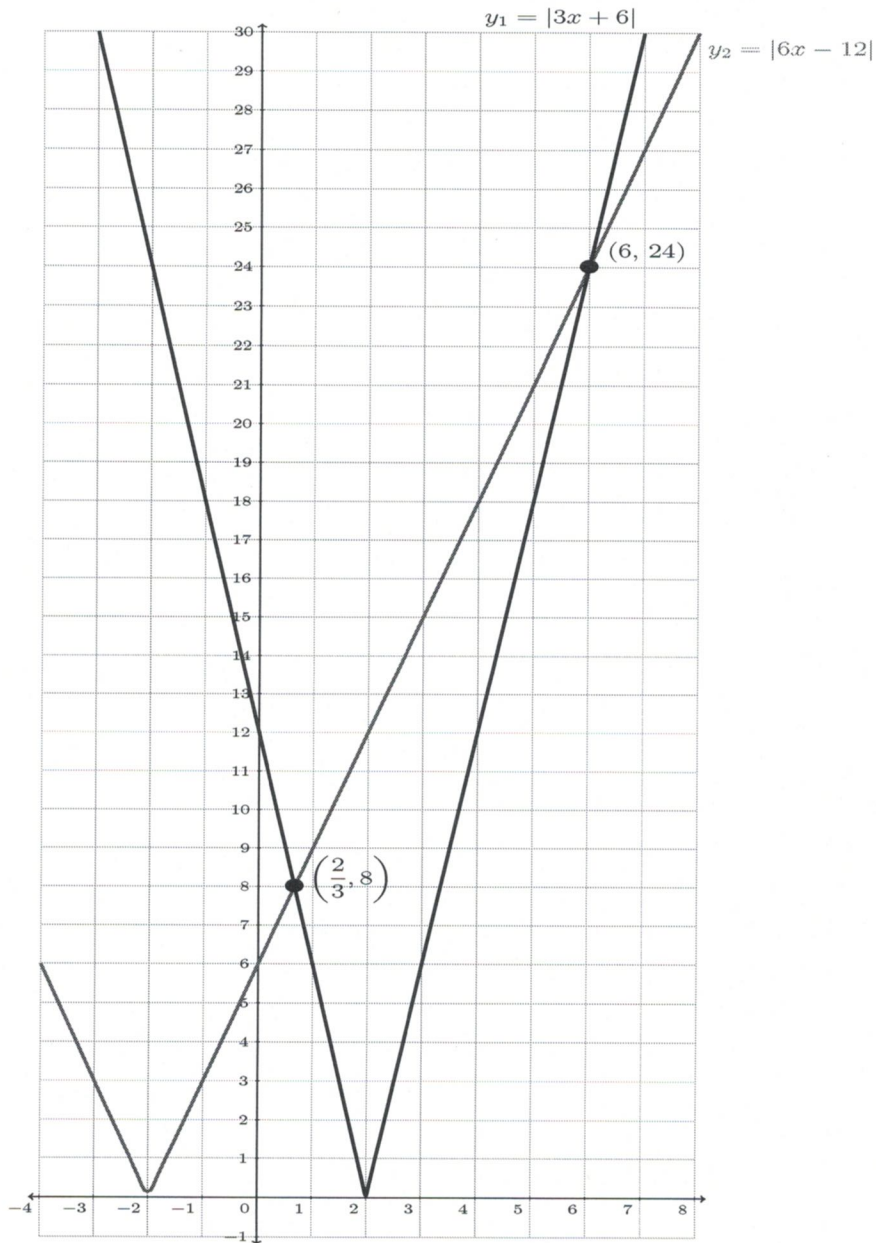
Using the graph below (our on our graphing calculator), we see the two points of intersection are

$$\left(\frac{2}{3}, 8\right) \text{ corresponding to solution } \boxed{x = \frac{2}{3}}$$

$$(6, 24) \text{ corresponding to solution } \boxed{x = 6}$$