

## Note on Notation

Recall for ordered pair

$(x, y)$

$x = 1^{\text{st}}$  coordinate which stores input values for this point

$y = 2^{\text{nd}}$  coordinate which stores output values for this point

## Graphical Technique

Class #:

We can also use the vertical line test to check if a given relation is a function. To do so, we draw a vertical line at every point in the domain of our relation. If any of these vertical lines touch ~~more~~ more than one output, the relation is not a function.

Function:

*noun* (Section 7.1 p. 512)

- Nerdy definition: A **function** is a relation where every element in the domain of the relation has exactly one corresponding element in the range of the relation.
- Street definition:

## Vertical line test

We can draw vertical lines at each input point in the domain to test the graph for multiple outputs

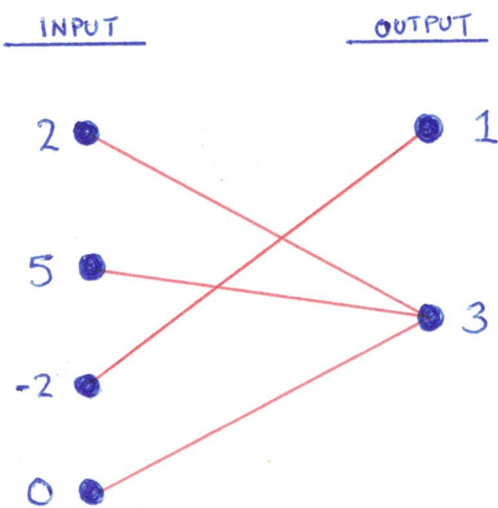
- if each vertical line touches the graph only once, its a function
- if not, the graph does not represent a function

For problems 1 and 2 below, consider the given relationship

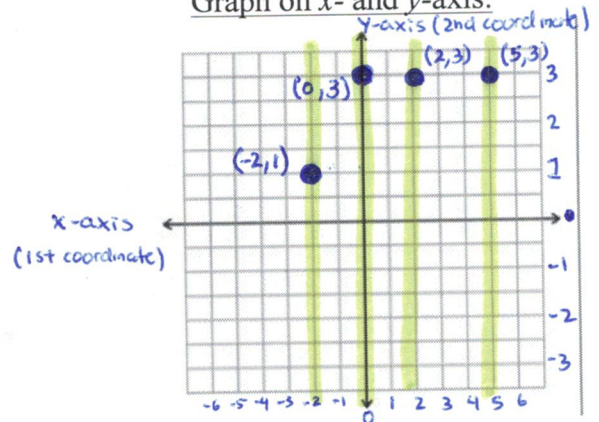
- Draw an input-output diagram.
- Draw the points on an x- and y-axis.
- Determine whether each of the following is a function

1.  $\{(2, 3), (5, 3), (-2, 1), (0, 3)\}$

Draw input-output diagram



Graph on x- and y-axis:



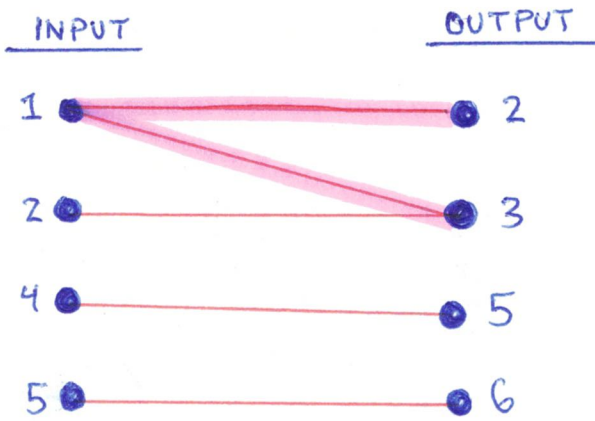
Input-output diagrams are used to visualize relations. We can check if a relation is a function by counting the number of lines coming out of each input element.

- if only one line comes out of each input our relation is a function
- if not the relation is NOT a function

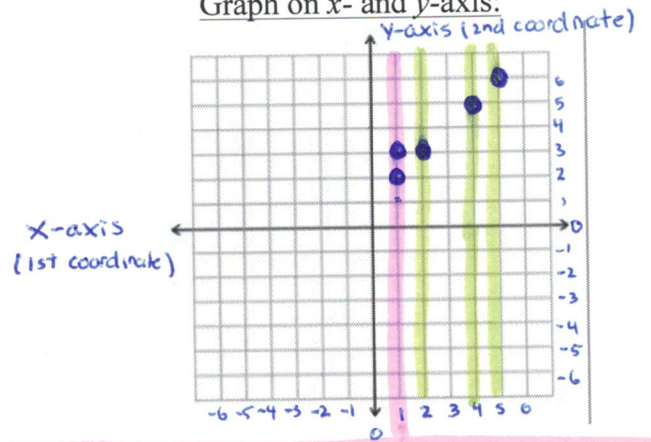
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2.  $\{(1,2), (2,3), (1,3), (4,5), (5,6)\}$

Draw input-output diagram



Graph on x- and y-axis:

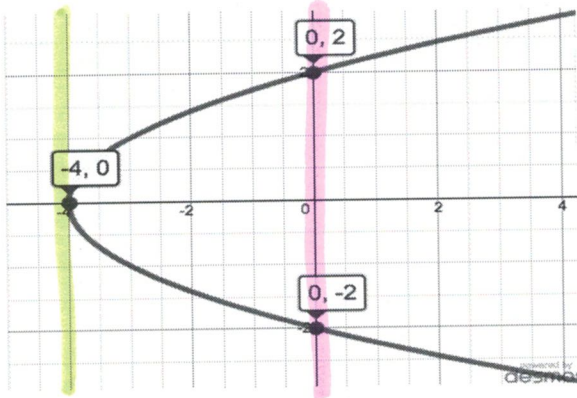


Notice the vertical line  $x=1$  intersects TWO points from our relation (NOT a function)

SECTION 7.1: Determining if a Relation Represents a Function (p. 512)

For problems 3, 4, 5, and 6, observe the given graph. Using this graph, determine whether the relationship displayed represents a function or not. Justify your answer.

3.



The relation  $x = y^2 - 4$  is NOT a function. We see that the graph of this relation fails the vertical line test. In particular, the vertical line

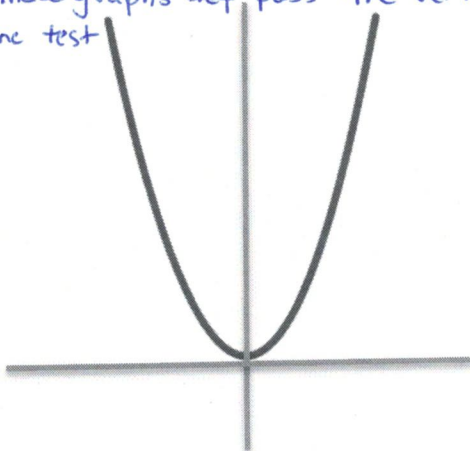
$$x = 0$$

touches the graph at two points  $(0, 2)$  and  $(0, -2)$  confirming that this relation is NOT a function.

4.

Parabola

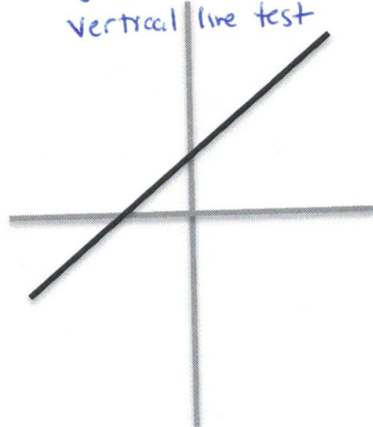
Parabolas "are functions" because these graphs ~~are~~ pass the vertical line test



5.

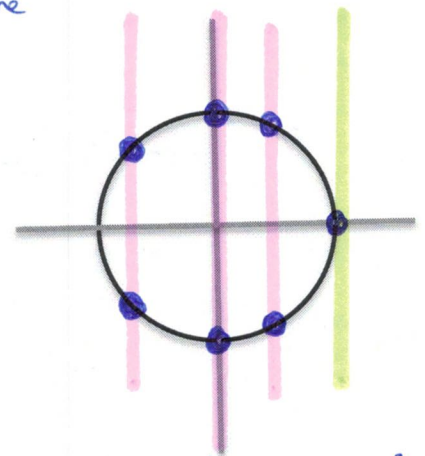
Line

Lines are functions. The graphs of lines pass the vertical line test



6.

Circle



A circle "is not a function" The graph of a circle fails the vertical line test.

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Algebraic Technique

When evaluating functions at a given input our first step is to replace variable with input value using parenthesis, then simplify

SECTION 7.1: Finding Function Values (p. 515)

For problems 7 and 8 below, evaluate the given function at the given points.

7. Let  $f(x) = \frac{x^2 - 25}{5 - x}$ .

a. Find  $f(2)$

input  $x = 2$

$$f(2) = \frac{(2)^2 - 25}{5 - (2)}$$

$$= \frac{4 - 25}{5 - 2}$$

$$= \frac{-21}{3} = \boxed{-7}$$

b. Find  $f(5)$

input  $x = 5$

$$f(5) = \frac{(5)^2 - 25}{5 - (5)}$$

$$= \frac{5^2 - 25}{5 - 5}$$

$$= \frac{25 - 25}{0}$$

$$= \frac{0}{0}$$

undefined

(Not possible. we cannot divide by zero)

See reverse

## Note on Notation

Function notation is given by

$$y = f(x) \quad \text{"f of x"}$$

- Input is inside parenthesis
- output is the expression that results when evaluating  $f$  at given input
- parenthesis do not represent multiplication

## WARNING: Division by 0

Note that division is connected to multiplication

$$\bullet \quad x = \frac{10}{2} \Rightarrow 2 \cdot x = 10$$

if we multiply the quotient by the denominator of our fraction, we should get the numerator

Given this relation, consider

$$\bullet \quad x = \frac{10}{0} \Rightarrow 0 \cdot x = 10$$

When dividing any nonzero number by zero, we are trying to find a quotient that will multiply to zero to produce our nonzero numerator. This is not possible.

We can similarly consider

$$\bullet \quad x = \frac{0}{0} \Rightarrow 0 \cdot x = 0$$

There is no single, well defined  $x$  that satisfies this equation.

and thus we say  $\frac{0}{0}$

does not exist

8. Let  $h(t) = -t - t^2$ . Find

a. Find  $h(2)$

input  $t=2$

$$h(2) = -(2) - (2)^2$$

$$= -2 - (2^2)$$

$$= -2 - 4$$

$$= \boxed{-6}$$

b. Find  $h(-3)$

input  $t=-3$

$$h(-3) = -(-3) - (-3)^2$$

$$= +3 - (-3 \cdot -3)$$

$$= 3 - 9$$

$$= \boxed{-6}$$

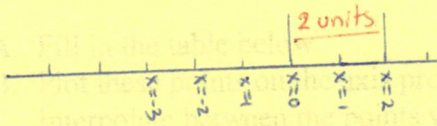
WARNING

$$(-3)^2 \neq -3^2$$

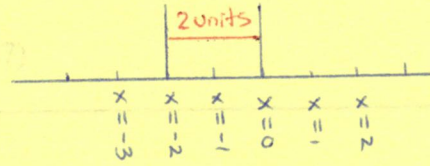
# Algebraic Technique

We can define the absolute value function graphically using the real number line as the distance away from zero.

$|2|$ : gives the "distance" between  $x=2$  and  $x=0$

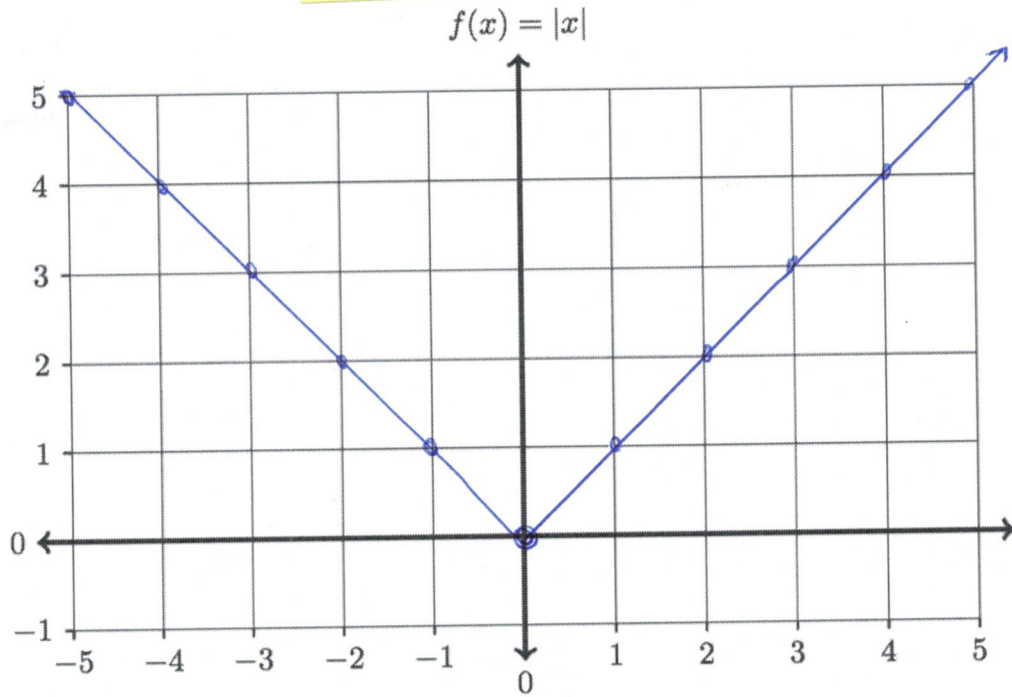


$|-2|$ : gives the "distance" between  $x=-2$  and  $x=0$



$\Rightarrow |-2| = 2$

$x$	$f(x) =  x $
-5	5
-4	4
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3
4	4
5	5



10. What is the x-intercept of this graph?

The x-intercept is at point  $(0,0)$  with  $x=0$ .

11. Why does the graph of the absolute value function never go below the line  $y = 0$ ?

The absolute value must be non negative.

## Calculator Technique

To use a TI calculator to graph an absolute value function, push the MATH button, scroll to NUM list and choose "abs"

Name: \_\_\_\_\_

### Calculator Technique

We can look at the graph  
OR we can use the "TABLE"  
button combined with the  
"TBL SET" button on our  
calculator to find points  
on our graph

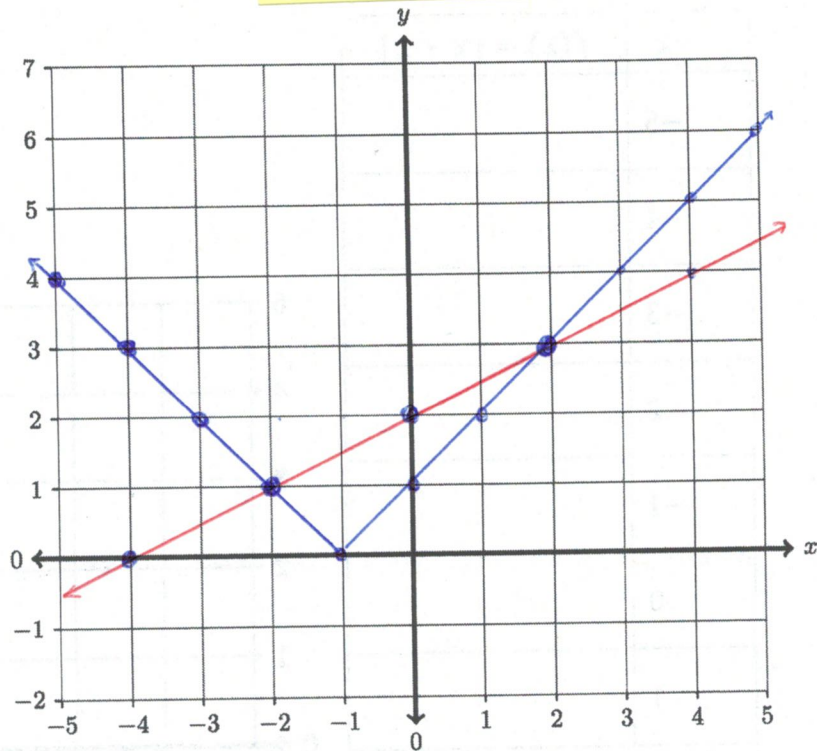
### Graphical Technique to solve algebraic equations

To find the solution to algebraic equations using a graphical technique

- Step 1: Identify and graph the function on the left-hand side
- Step 2: Identify the function on the right-hand side of the equation and graph the function on the same axis you used in step 1.
- Step 3: Find the point(s) of intersection between the graphs
- Step 4: Identify the 1<sup>st</sup> coordinate (the "x" value) of the intersection point(s)
- Step 5: The 1<sup>st</sup> coordinate(s) represent the solution(s) to the equation

2. Use a graphical technique to solve the equation  $|x + 1| = 2 + \frac{1}{2}x$

x	$y_1 =  x + 1 $	$y_2 = 2 + \frac{1}{2}x$
-5	4	-1/2
-4	3	0
-3	2	1/2
-2	1	1
-1	0	3/2
0	1	2
1	2	5/2
2	3	3
3	4	7/2
4	5	4
5	6	9/2



3. Using your table and graph above, identify the solution(s) for equation:  $|x + 1| = 2 + \frac{1}{2}x$

Recall: the solution to our equation occurs where the graphs intersect

Left point of intersection:  $(-2, 1) \Rightarrow \boxed{x = -2}$

Right point of intersection:  $(2, 3) \Rightarrow \boxed{x = 2}$

4. Explain, in your own words, how to use the graphical technique to solve algebraic equations.

Rewrite the graphical technique from above in your own language.