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SECTION 6.1: Introduction to Polynomial Factorizations and Equations (p. 432 – 446)

- ☐ Polynomial equation (p. 432)
- ☐ Zeros and Roots (p. 433)
- ☐ The principle of zero products (p. 435)
- ☐ Factoring (p. 436)
- ☐ Tips for factoring (p. 439)
- ☐ Factoring out -1 (p. 440)
- ☐ Find roots using the principle of zero products (p. 441)

Algebraic Technique to solve algebraic equations

To solve an algebraic equation using an algebraic technique, isolate the variable using inverse operations. Remember that the inverse operations we use depend on the type of equation we are trying to solve.

1. Use an algebraic technique to solve the following <u>linear equations</u>: equals sign

solve

algebraic equation

A. $4 \cdot x - 12 = 0$

$\Rightarrow 4x = 12$

$\Rightarrow \frac{4x}{4} = \frac{12}{4}$

$\Rightarrow \boxed{x = 3}$

B. $-\frac{1}{4} \cdot x + \frac{3}{4} = 0$

$\Rightarrow -\frac{x}{4} + \frac{3}{4} = 0$

$\Rightarrow -\frac{x}{4} = -\frac{3}{4}$

$\Rightarrow \frac{4}{1} \cdot \frac{-x}{4} = -\frac{3}{\cancel{4}} \cdot \frac{\cancel{4}}{1}$

$\Rightarrow -x = -3$

$\Rightarrow \boxed{x = 3}$

2. Look at the algebraic techniques you applied above to solve the <u>linear equations</u>. Specifically highlight each time you applied an inverse operation. What patterns do you notice? What inverse operations do you tend to use when solving linear equations.

We used the ideas that:

- addition is the inverse of subtraction and subtraction is the inverse of addition
- multiplication is the inverse of division and division is the inverse of multiplication

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Inverse operations for linear equations

Addition and subtraction are inverse operations.
- Addition annihilates subtraction
- Subtraction annihilates addition

Multiplication and division are inverse operations.
- Multiplication annihilates division
- Division annihilates multiplication

3. Lets look back at our rules of arithmetic. Solve each of the following multiplication problems:

A. $4 \cdot 0 =$ 0

C. $-286 \cdot 0 =$ 0

B. $-13 \cdot 0 =$ 0

D. $x \cdot 0 =$ 0

4. Review your work above. Suppose that you know the product of two numbers a and b is zero. In other words, suppose that you know the following:

$$a \cdot b = 0$$

What can you say about either number a or b? Is it possible that one of these two numbers *nonzero*?

the output of a multiplication of two numbers is zero

If $a \cdot b = 0$, the product of a & b is zero

then one of these numbers must be zero (i.e. $a = 0$ or $b=0$

In other ~~word~~ words if $a \cdot b = 0$ we know that a & b cannot both be nonzero.

Zero product property (inverse operation that annihilates multiplication as long as product is equal zero)

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$

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Zero product property

If $a \cdot b = 0$, then either $a = 0$ or $b = 0$

6. Please rewrite the statement of the zero product property in your own words below. Please feel free to use nontechnical terms. However, make sure you explain the entire zero product property.

We know that the only way to multiply two numbers together to produce an output of zero is if one of the two factors is zero. Thus, if we have two numbers multiplied together equal to zero, we can annihilate the multiplication by setting each factor to zero individually.

7. Use the zero product property to algebraically solve the linear equation: $4 \cdot x - 12 = 0$

$$4x - 12 = 0$$

$$\Rightarrow 4 \cdot (x - 3) = 0$$

$$\Rightarrow 4 \neq 0 \quad \text{or} \quad x - 3 = 0$$

$$\Rightarrow \boxed{x = 3}$$

8. Use the zero product property to algebraically solve the linear equation: $-\frac{1}{4} \cdot x + \frac{3}{4} = 0$

$$-\frac{1}{4} x + \frac{3}{4} = 0 \quad \Rightarrow \quad -\frac{1}{4} \cdot (x - 3) = 0$$

$$\Rightarrow -\frac{1}{4} \neq 0 \quad \text{or} \quad x - 3 = 0$$

$$\Rightarrow \boxed{x = 3}$$

Zero product property: inverse operation for a product equal to zero

The zero product property is an inverse operation applied to multiplication that equals to zero.

- Zero product property annihilates a multiplication. In order to apply the zero product property we need one side of an equation to be written as *product* of two factors and the other side of the equation to be equal to *zero*.

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9. Use the zero product property to algebraically solve the quadratic equation: $4x - x^2 = 0$.

$4x - x^2 = 0 \Rightarrow x \cdot (4 - x) = 0$

$\Rightarrow x = 0$ or $4 - x = 0$

$\Rightarrow \boxed{x = 0}$ or $\boxed{x = 4}$

Graphical Technique to solve an algebraic equation

To find the solution to algebraic equations using a graphical technique, we use the following five step program for salvation:

Step 1:	Graph the function y_1 on the left-hand side of the equals sign.
Step 2:	Graph the function y_2 on the right-hand side of the equals sign.
Step 3:	Find the point(s) of intersection between the graphs of the two functions.
Step 4:	Write each point of intersection as an ordered pair in the form: (x, y)
Step 5:	Set the variable from the original algebraic equation equal to the 1st coordinate of each point of intersection. These "x"-values are the solution(s) to the algebraic equation.

10. Use a graphical technique to solve the equation:

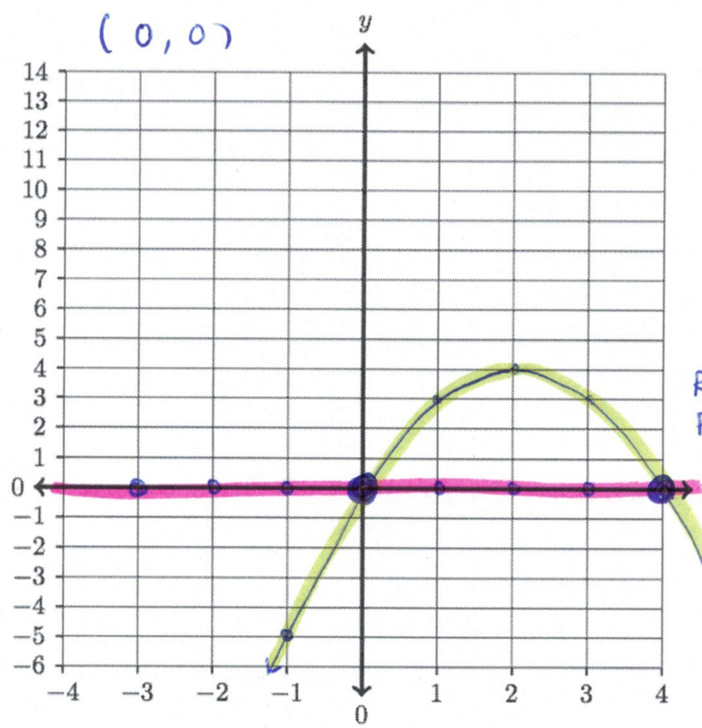
$$4x - x^2 = 0.$$

Make sure to demonstrate all five steps of this process. Of course, you are welcome to use your calculator. Please specifically identify each point of intersection on your graph. Also, please write each of these points as an ordered pair with an *x*-coordinate and *y*-coordinate. Use this information to find a solution to this algebraic equation.

	LHS of Equals Sign	*RHS of Equals Sign*
x	$y_1 = 4x - x^2$	$y_2 = 0$
-1	-5	0
0	0	0
1	3	0
2	4	0
3	3	0
4	0	0
5	-5	0

Left P.O.I

(0, 0)



Right P.O.I

(4, 0)

Solutions: $\boxed{x = 0}$ or $\boxed{x = 4}$

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We are now going to expand on our work from problem 9 above to develop more general an algebraic technique to solve quadratic equations[1]. In this algebraic technique, we will use the zero product property as our inverse operation. In order to apply the zero product property we need to do two things:

Thing 1: Get one side of an equation equal to zero

Thing 2: Factor the other side of the equation and write it as a *product* of two factors

As we will see, factoring is the inverse operation of multiplication of polynomials. Let's warm up with some review of how to multiply two polynomials together.

SECTION 5.5: Multiplication of Polynomials (p. 380 – 387)

- ☐ Distributive law: $a \cdot (b \pm c) = a \cdot b \pm a \cdot c$
- ☐ The product of two polynomials (p. 383)

Expand each of the following products using distributivity (or FOIL).

10. $(x+3)(x-5)$

$(x+3)\cdot(x-5) = x\cdot(x-5) + 3\cdot(x-5)$

$= x^2 - 5x + 3x - 15$

$= \boxed{x^2 - 2x - 15}$

11. $(y+4)(y-7)$

$(y+4)\cdot(y-7) = y\cdot(y-7) + 4\cdot(y-7)$

$= y^2 - 7y + 4y - 28$

$= \boxed{y^2 - 3y - 28}$

[1] Recall from week 1, quadratic equations involve squared terms. One fun way to remember this is to use you language skills: the word *cuadro* means square in Spanish. Thus, *quadra*tic equations involve squares

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SECTION 6.2: Trinomials of the type $x^2 + bx + c$ (p. 446 – 455)

- □ FOIL: First, outer, inner, last (p. 446)
- □ Factor $x^2 + bx + c$ when c is positive (p. 447)
- □ Factor $x^2 + bx + c$ when c is negative (p. 449)
- □ To factor $x^2 + bx + c$ (p. 450)
- □ Algebraic and graphical approach to solving $x^2 + bx + c = 0$ (p. 450-451)

Solve each of the following quadratic equations using the zero product property. Remember, to apply the zero product property as an inverse operation to solve a quadratic equation, we need to do two things:

Thing 1: Get one side of an equation equal to zero

Thing 2: Factor the other side of the equation and write it as a *product* of two factors

12. $x^2 - 36 = -5x$

$x^2 - 36 = -5x \Rightarrow x^2 + 5x - 36 = 0$

$a = 1 \quad b = 5 \quad c = -36$

Multiply

-36

$+9 \quad -4$

$+5$

Add

$\Rightarrow \underbrace{x^2 + 9x} \underbrace{- 4x - 36} = 0$

$\Rightarrow 5x = 9x - 4x$

$\Rightarrow x \cdot (x+9) - 4(x+9) = 0$

$\Rightarrow (x-4) \cdot (x+9) = 0$

$\Rightarrow x - 4 = 0$ or $x + 9 = 0 \Rightarrow \boxed{x = 4}$ or $\boxed{x = -}$

13. $t^2 - 3t = 28$

$a = 1 \quad b = -3 \quad c = -28$

$t^2 - 3t = 28 \Rightarrow t^2 - 3t - 28 = 0$

Multiply

-28

$+7 \quad -4$

-3

Add

$\Rightarrow \underbrace{t^2 + 7t} \underbrace{- 4t - 28} = 0$

$\Rightarrow t \cdot (t+7) - 4 \cdot (t+7) = 0$

$\Rightarrow -3t = 7t - 4t$

$\Rightarrow (t-4) \cdot (t+7) = 0$

$\Rightarrow t - 4 = 0$ or $t + 7 = 0 \Rightarrow \boxed{t = 4}$ or $\boxed{t = -7}$