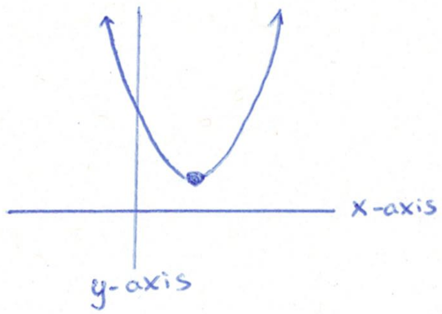
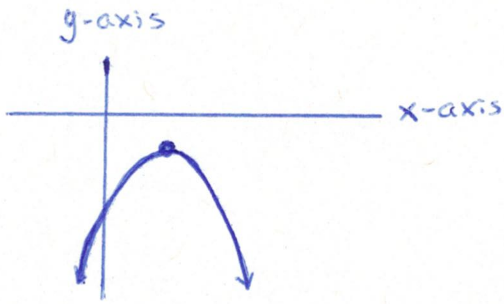
As we see on page 1 of Class23 handout (from today), we have three options for the number of x-intercepts of a parabola:

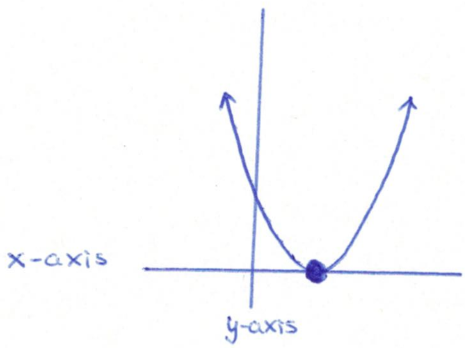
option 1: $\boxed{\text{zero intercepts}}$

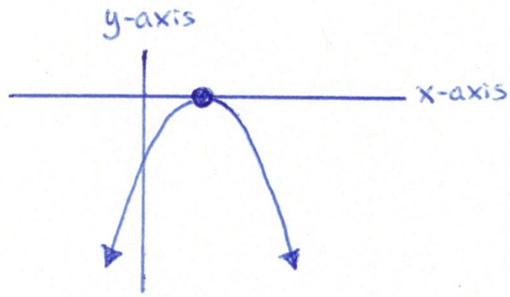




- In this case, our parabola never touches the x-axis
  - ☐ upward facing parabola with vertex above the x-axis
  - ☐ downward facing parabola with vertex below the x-axis

Option 2: $\boxed{\text{.One x-intercept}}$





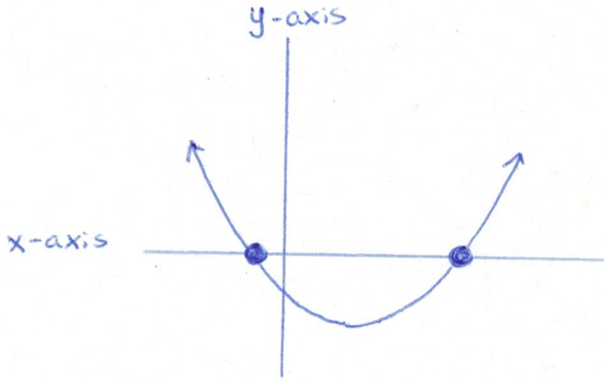
- In this case, our parabola touches the x-axis exactly one tim
  - ☐ upward facing parabbla with vertex on the x-axis
  - ☐ downward facing parabola with vertex on the x-axis

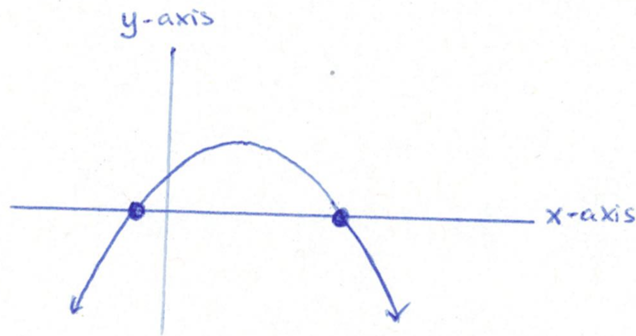
- We see that a quadratic equation

$$\cancel{y}\ ax^2 + bx + c = 0$$

will have one solution if and only if the vertex is the x-intercept (i.e. the vertex sits on the $y=0$ line as shown above).

Option 3: $\boxed{\text{Two x-intercepts}}$





- In this case, our parabola touches the x-axis two times
  - ☐ upward facing parabola with vertex below the x-axis and left x-intercept to the left of vertex while right x-intercept to the right of vertex
  - ☐ downward facing parabola with vertex above the x-axis and left x-intercept to the left of vertex while right x-intercept to the right of vertex.

All of the previous discussion focused on a graphical method to identify the number of solutions to the quadratic equation

$$ax^2 + bx + c = 0$$

However, as we have seen repeatedly in this class, we can also use algebraic methods to figure out how many ~~roots~~ x-intercepts there are. To this end, we will focus our attention on the quadratic ~~equation~~ formula

$$x = \frac{-b \pm \sqrt{\boxed{b^2 - 4a\cdot c}}}{2a}$$

Let's do a bit of review on the standard form of a quadratic equation to remind ourselves how we use the quadratic formula.

Recall from our previous discussions:

- The standard form of a quadratic equation is given by

$$ax^2 + bx + c = 0 \qquad \boxed{I}$$

- When we solve our quadratic equation $\boxed{I}$, we are finding the 1st-coordinates of the points of intersection between the graphs

$$y_1 = ax^2 + bx + c \qquad \text{and} \qquad y_2 = 0$$

- In other words, our solution(s) to quadratic equation $\boxed{I}$ represent the "x-coordinates" of the x-intercept(s) for the parabola

$$y_1 = ax^2 + bx + c$$

- We can use the quadratic ~~eq~~ formula to find the left and right x-intercepts of our parabola $y_1 = ax^2 + bx + c$ where we know that

1st coordinate of left intercept: $x_1 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

1st coordinate of right intercept: $x_2 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$

With this in mind, let's study the expression containing the square root

$$\sqrt{b^2 - 4ac}$$

We recall from our discussion of square roots

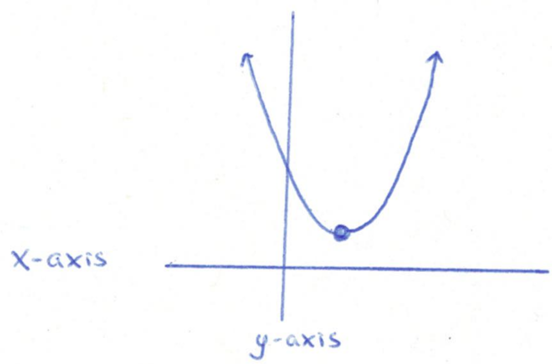
- $\sqrt{b^2 - 4ac}$ exists if the radicand given by $(b^2 - 4ac)$ is a positive number

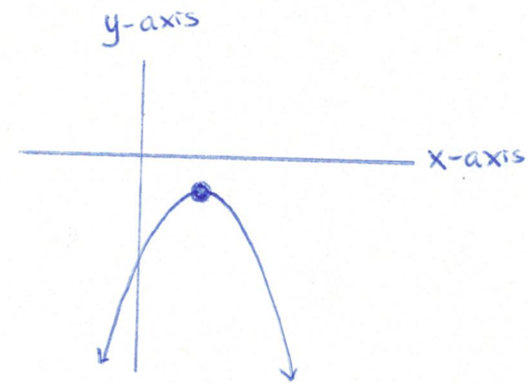
- $\sqrt{b^2 - 4ac} = 0$ if the radicand $(b^2 - 4ac)$ is equal to zero

- $\sqrt{b^2 - 4ac}$ does not exist if the radicand $(b^2 - 4ac)$ is a negative number
  (we "cannot" take a square root of a negative number)

With these observations in hand, let's revisit our three options for the number of x-intercepts of a parabola and classify each case using an algebraic technique.

Option 1: $\boxed{\text{Zero Intercepts}}$





- this situation occurs if there is a negative number inside the square root symbol in quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

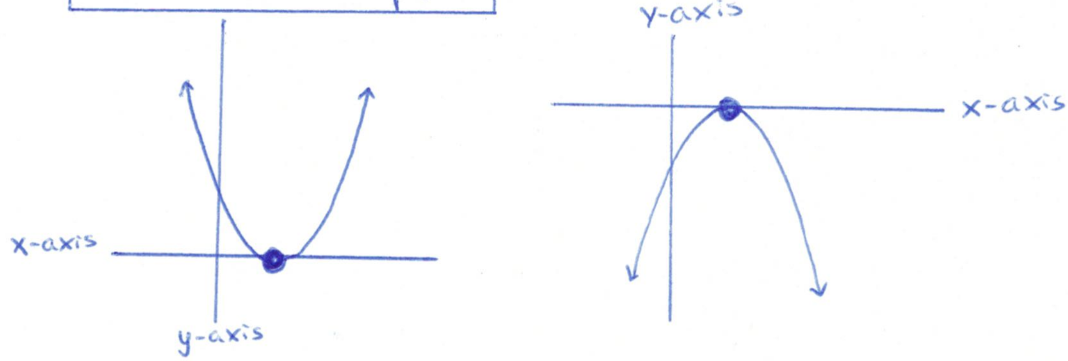
$$\Longleftrightarrow \quad b^2 - 4ac < 0$$

In other words, we say that our quadratic equation

$$ax^2 + bx + c = 0$$

has no solution (doesn't have x-intercepts) if the discriminant $(b^2 - 4ac)$ is negative.

Option 2: $\boxed{\text{One Intercept}}$



- this situation occurs if there is a zero inside the square root of quadratic formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

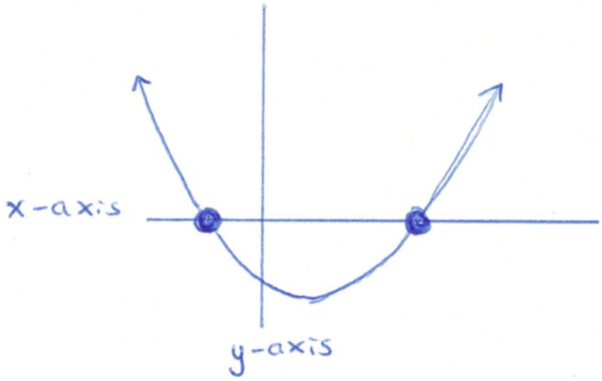
$$\Longleftrightarrow \qquad b^2 - 4ac = 0$$

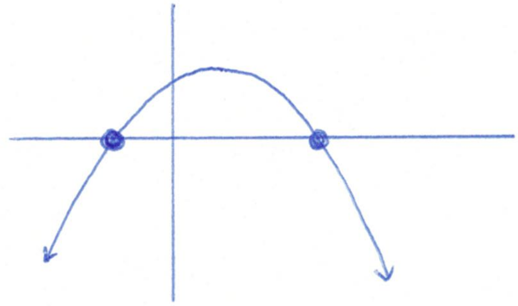
- In other words, we say our quadratic equation

$$ax^2 + bx + c = 0$$

has ONE solution (has one x-intercept) if the discriminant $b^2 - 4ac = 0$.

Option 3: $\boxed{\text{Two Intercepts}}$





- This situation occurs if there is a positive number inside the square root symbol of quadratic formula

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Longleftrightarrow \quad b^2 - 4ac > 0$$

Remark: □ If (b^2-4ac) is a positive perfect square, we say our two solutions are rational numbers

□ If (b^2-4ac) is a positive number ~~but~~ but not a perfect square, our two solutions are called irrational

- In other words, we say our equation $ax^2+bx+c=0$ has TWO solutions (two x-intercepts) if the discriminant $b^2-4ac>0$ is positive.

Name:______________________________ Class #:______

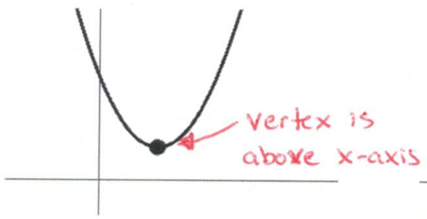
SECTION 10.3: The Discriminant (p. 760 – 765)

- ☐ Quadratic formula for the solution of a quadratic equation in standard form (p. 754)

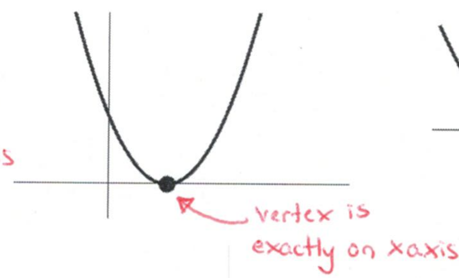
$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{OR} \quad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

- ☐ The discriminant: $b^2 - 4ac = 0$ (p. 761)
- ☐ Three scenarios for x-intercepts of parabola (p. 743)
    - No x-intercepts: no real solution to equation $ax^2 + bx + c = 0$
    - One x-intercept: One solution to equation $ax^2 + bx + c = 0$
    - Two x-intercepts: Two solution to equation $ax^2 + bx + c = 0$
        - Rational Solutions
        - Irrational Solutions
- ☐ Classification of solutions of quadratic equation using discriminant (p. 761)

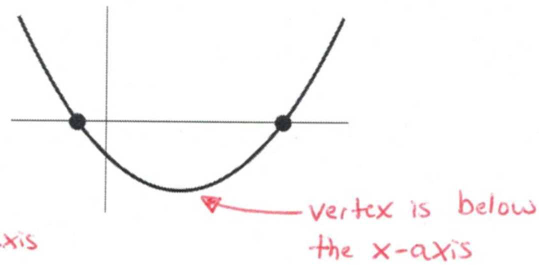
UPWARD FACING PARABOLA



Upward facing parabola
with NO x–intercept

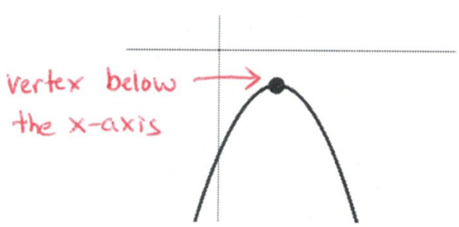


Upward facing parabola
with ONE x–intercept

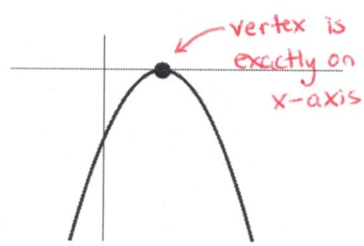


Upward facing parabola
with TWO x–intercept

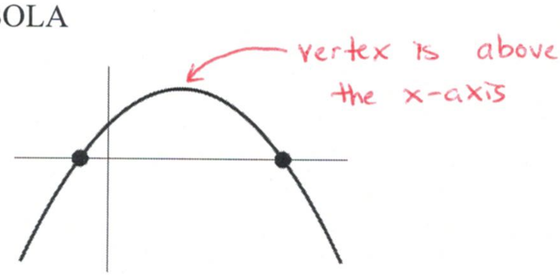
DOWNWARD FACING PARABOLA



Downward facing parabola
with NO x–intercept



Downward facing parabola
with ONE x–intercept



Downward facing parabola
with TWO x–intercept

Graphically, we can use the figures above to figure out how many solutions exist to the quadratic equation $ax^2 + bx + c = 0$.

Name: _______________ Class #: _______

1A. Solve the quadratic equation below using the quadratic formula. Be sure to specifically identify the discriminant of the quadratic formula.

$$x^2 = 4x - 4$$

$a = 1$, $b = -4$, $c = 4$

$x^2 = 4x - 4 \Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$

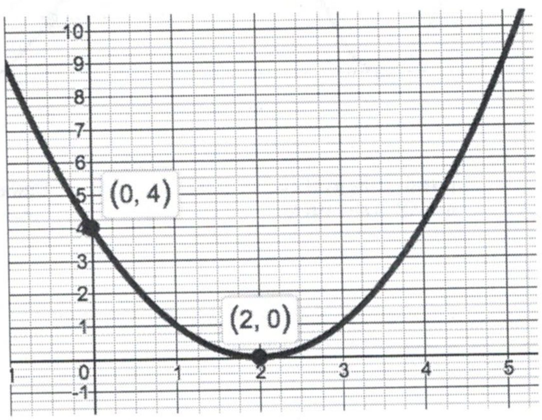
add / subtra
zero

$\Rightarrow x = \dfrac{4 \pm \sqrt{0}}{2}$

$\Rightarrow x = \dfrac{4}{2}$

$\Rightarrow \boxed{x = 2}$

Consider the graph of the quadratic function $y_1 = x^2 - 4x + 4$ given below.



1B. How many x-intercepts does the quadratic function $y_1 = x^2 - 4x + 4$ have?

This parabola has exactly one x-intercept at the point $(2, 0)$ where $x = 2$.

1C. Look at the discriminant from part 1A and the quadratic formula, why does your answer to 1B make sense?

The discriminant of this equation is $b^2 - 4ac = 0$.

Name:________________________ Class #:______

2A. Solve the quadratic equation below using the quadratic formula. Be sure to specifically identify the discriminant of the quadratic formula.

$$2p^2 = 12 - 5p$$

$a=2$, $b=5$, $c=-12$

$\Rightarrow 2p^2 + 5p - 12 = 0$

$\Rightarrow p = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow p = \dfrac{-5 \pm \sqrt{5^2 - 4\cdot 2\cdot(-12)}}{2\cdot 2}$

$\Rightarrow p = \dfrac{-5 \pm \sqrt{25+96}}{4}$

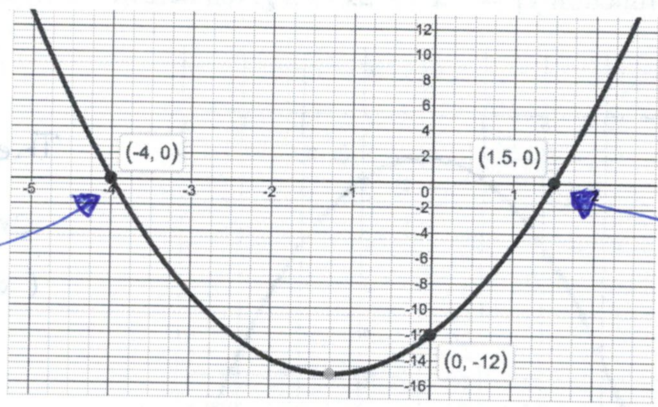
$\Rightarrow p = \dfrac{-5 \pm \sqrt{121}}{4} = \dfrac{5 \pm 11}{4}$

$\Rightarrow p_1 = \dfrac{-5-11}{4}$ and $p_2 = \dfrac{-5+11}{4}$

$\Rightarrow \boxed{p_1 = -4}$ and $\boxed{p_2 = \frac{3}{2} = 1.5}$

Consider the graph of the quadratic function $y_1 = 2x^2 + 5x - 12$ given below.

Left P.O.I $(-4,0)$



Right P.O.I $(1.5, 0)$

2B. How many x-intercepts does the quadratic function $y_1 = 2x^2 + 5x - 12$ have?

This parabola has two points of intersection as seen above.

2C. Look at the discriminant from part 2A and the quadratic formula. Why does your answer to 2B make sense?

The discriminant in this case is $b^2 - 4ac = 121 > 0$

Notice -5 ± 11 produces two solutions.

Name:________________ Class #:________

3. Solve the quadratic equation below using the quadratic formula. Be sure to specifically identify the discriminant of the quadratic formula.

$$-t^2 = 2t + 3$$

$a=1,\ b=2,\ c=3$

$\Rightarrow t^2 + 2t + 3 = 0$

$\Rightarrow t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

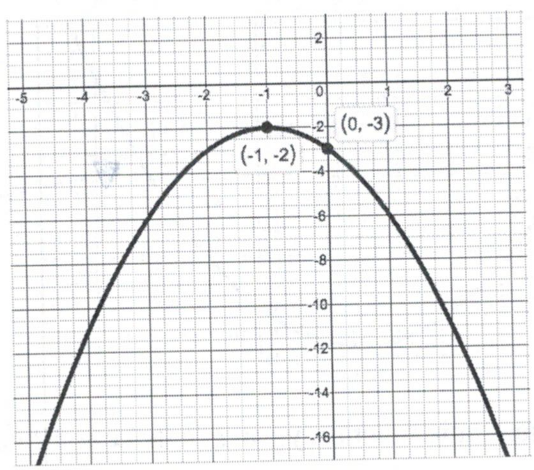
$\Rightarrow t = \dfrac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot 3}}{2\cdot 1}$

$\Rightarrow t = \dfrac{-2 \pm \sqrt{4-12}}{2}$

$\Rightarrow t = \dfrac{-2 \pm \sqrt{-8}}{2}$ NOT Possible in $\mathbb{R}$

$\Rightarrow$ No solution

Consider the graph of the quadratic function $y_1 = -x^2 - 2x - 3$ given below.



The parabola never crosses the line $y=0$ (doesn't touch x-axis

3B. How many x-intercepts does the quadratic function $y_1 = -x^2 - 2x - 3$ have?

NONE : The parabola doesn't cross x-axis (output never equals zero).

3C. Look at the discriminant from part 2A and the quadratic formula. Why does your answer to 2B make sense?

$b^2 - 4ac = 2^2 - 4\cdot 1\cdot 3 = 4 - 12 = -8 < 0$

It is not possible to take square root of a negative... Thus, no solution indicating no x-intercepts.