Name: Answer Key Class #: ∞

SECTION 10.2: The Quadratic Formula (p. 754 – 760)

- ☐ Quadratic equation in standard form: $ax^2 + bx + c = 0$ (p. 754)
- ☐ Quadratic formula for the solution of a quadratic equation in standard form (p. 754)

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{OR} \quad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

- ☐ To solve a Quadratic Equation (p. 755)
- ☐ Connecting the Concepts: The Four Methods to Solve Quadratic Equations (p. 758)

Problem 1A – 1C: Solve the following quadratic equation using three different methods:

$$x^2 = x + 6$$

1A. Method 1: Solve by factoring

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)\cdot(x+2) = 0$

$\Rightarrow x - 3 = 0$ OR $x + 2 = 0$

$\Rightarrow \boxed{x = 3}$ or $\boxed{x = -2}$

1B. Method 2: Complete the Square

side note: $\left[\frac{b}{2}\right]^2 = \left[\frac{-1}{2}\right]^2 = \frac{1}{4}$

$\Rightarrow x^2 - x = 6$

$a = 1,\ b = -1$

$\Rightarrow x^2 - x + \frac{1}{4} = 6 + \frac{1}{4} = \frac{24}{4} + \frac{1}{4} = \frac{25}{4}$

$\Rightarrow (x - 1/2)^2 = \frac{25}{4}$

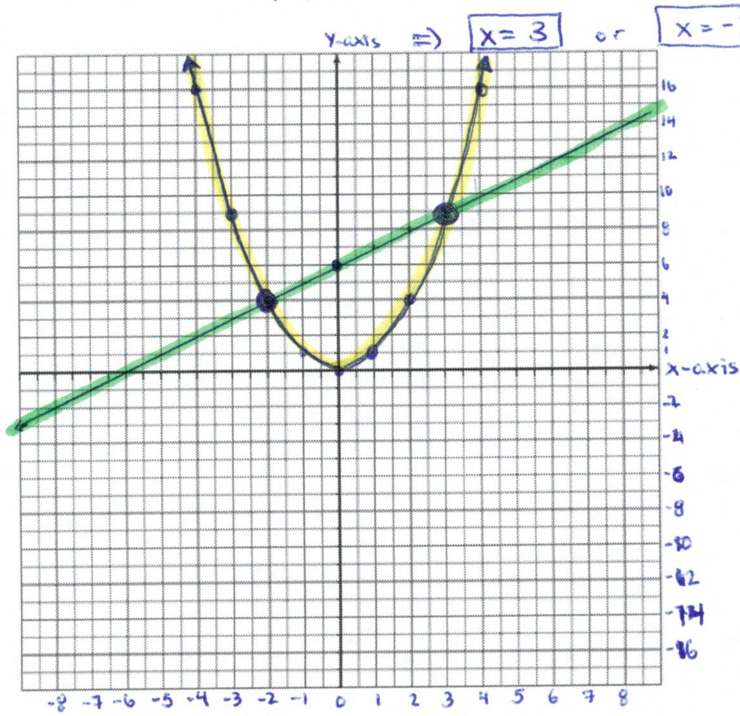
$\Rightarrow \sqrt{(x-1/2)^2} = \sqrt{\frac{25}{4}}$

$\Rightarrow |x - 1/2| = \frac{5}{2}$

$\Rightarrow x - \frac{1}{2} = \frac{5}{2}$ or $x - \frac{1}{2} = -\frac{5}{2}$

$\Rightarrow x = \frac{6}{2}$ or $x = \frac{-4}{2}$

$\Rightarrow \boxed{x = 3}$ or $\boxed{x = -2}$



1C. Method 3: Solve Graphically

To solve this algebraic equation graphically we start by graphing the LHS and RHS of our algebraic equation. To this end, we let

$y_1 = x^2$ and $y_2 = x + 6$

We know our solutions occur at the points of intersection

$(-2, 4)$ and $(3, 9)$

We also know that the 1st coordinates of these points are the solutions w/

$\boxed{x = -2}$ or $\boxed{x = 3}$

Name: ______________________ Class #: ______

Problems 2 – 5: Solve the following quadratic equations for x using the method of completing the square.

2. $x^2 = 4x - 4$

side note: $\left[\frac{b}{2}\right]^2 = \left[\frac{-4}{2}\right]^2 = (-2)^2 = 4$

$\Rightarrow x^2 - 4x = -4$

$a=1,\ b=-4$

$\Rightarrow x^2 - 4x + 4 = -4 + 4$

$\Rightarrow \underbrace{x^2 - 4x + 4}_{\text{perfect-square trinomial}} = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow \sqrt{(x-2)^2} = \sqrt{0}$

$\Rightarrow |x-2| = 0$

this quadratic equation has one solution

$\Rightarrow x - 2 = 0 \Rightarrow \boxed{x = 2}$

3. $3p^2 = 18p - 6$

Side Note: $\left[\frac{b}{2}\right]^2 = \left[\frac{-6}{2}\right]^2 = (-3)^2 = 9$

$\Rightarrow 3p^2 - 18p = -6$

$\Rightarrow p^2 - 6p = -2$

$a=1,\ b=-6$

$\Rightarrow p^2 - 6p + 9 = -2 + 9$

$\Rightarrow \underbrace{p^2 - 6p + 9}_{\text{perfect-square trinomial}} = 7$

$\Rightarrow (p-3)^2 = 7$

$\Rightarrow \sqrt{(p-3)^2} = \sqrt{7}$

$\Rightarrow |p-3| = \sqrt{7}$

$\Rightarrow p - 3 = +\sqrt{7}$ OR $p - 3 = -\sqrt{7}$

$\Rightarrow \boxed{p = 3 + \sqrt{7} \text{ OR } p = +3 - \sqrt{7}}$

4. $x^2 = 3x + 5$

Side Note: $\left[\frac{b}{2}\right]^2 = \left[\frac{-3}{2}\right]^2 = \frac{9}{4}$

$\Rightarrow x^2 - 3x = 5$

$a=1,\ b=-3$

$\Rightarrow x^2 - 3x + \frac{9}{4} = 5 + \frac{9}{4} = \frac{20}{4} + \frac{9}{4} = \frac{29}{4}$

$\Rightarrow \left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$

$\Rightarrow \sqrt{(x - 3/2)^2} = \sqrt{\frac{29}{4}}$

$\Rightarrow \left|x - \frac{3}{2}\right| = \frac{\sqrt{29}}{2}$

$\Rightarrow x - \frac{3}{2} = +\frac{\sqrt{29}}{2}$ OR $x - \frac{3}{2} = -\frac{\sqrt{29}}{2}$

$\Rightarrow x = \frac{3}{2} + \frac{\sqrt{29}}{2}$ OR $x = \frac{3}{2} - \frac{\sqrt{29}}{2}$

$\Rightarrow \boxed{x = \frac{3 \pm \sqrt{29}}{2}}$

5. $5x^2 = 13x + 18$

Side Note: $\left[\frac{b}{2}\right]^2 = \left[\frac{-13}{5} \div \frac{2}{1}\right]^2 = \left[\frac{-13}{5} \cdot \frac{1}{2}\right]^2 = \left[\frac{-13}{10}\right]^2 = \frac{169}{100}$

$\Rightarrow 5x^2 - 13x = 18$

$\Rightarrow x^2 - \frac{13}{5}x = \frac{18}{5}$

$\Rightarrow x^2 - \frac{13}{5}x + \frac{169}{100} = \frac{18}{5} + \frac{169}{100}$

$\Rightarrow \left(x - \frac{13}{10}\right)^2 = \frac{529}{100}$

$\Rightarrow \sqrt{\left(x - \frac{13}{10}\right)^2} = \sqrt{\frac{529}{100}}$

$\Rightarrow \left|x - \frac{13}{10}\right| = \frac{23}{10}$

$\Rightarrow x - \frac{13}{10} = +\frac{23}{10}$ OR $x - \frac{13}{10} = -\frac{23}{10}$

$\Rightarrow x = \frac{13+23}{10}$ OR $x = \frac{13}{10} - \frac{23}{10}$

$\Rightarrow \boxed{x = 3.6}$ OR $\boxed{x = -1}$

Recall the standard form of a quadratic equation

$$ax^2 + bx + c = 0$$

where we have
- coefficient of x^2 term: a
- coeffocient of x term: b
- constant term: c

In standard form, we set the RHS of the equation to zero and write our polynomial in descending form.

When solving this equation, we are looking for the points of intersection between the graphs of functions

$$y_1 = ax^2 + bx + c \qquad \text{and} \qquad y_2 = 0$$

In other words, we want to find where the parabola given by equation $y_1 = ax^2 + bx + c$ crosses the x-axis. These points are also known as the x-intercepts of y_1 (or the zeros of y_1).

The quadratic formula is a closed-form formula used to quickly identify the x-intercepts ~~with~~ that arises from the method of completing the square.

The quadratic formula is given as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We can use this to locate the x-intercept(s) for the quadratic function $y = ax^2 + bx + c$.

In this case, we say the 1-st coordinate of each intercept is given below:

1st-coordinate of Left intercept

$$x_1 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

~~2nd~~ 1st coordinate of right Intercept

$$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

To solve a quadratic equation using the quadratic formula, we follow the steps below:

Step 1: Get the quadratic equation into standard form

$$ax^2 + bx + c = 0$$

by getting the right-hand side to zero and combine like-terms of left-hand side (writing 2nd degree polynomial in descending order).

Step 2: Specifically identify the values of coefficients a, b, c for the quadratic function in standard form $\underline{\underline{a}}\,x^2 + \underline{\underline{b}}x + \underline{\underline{c}}$

Step 3: Substitute the values of a, b, c into the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

1st coordinate of left intercept

$$x_1 = \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$$

2st coordinate of right intercept

$$x_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2 \cdot a}$$

Step 4: Do arithmetic and simplify (if possible)

STEP 1: Consider the quadratic equation

$$5x^2 + 8x + 3 = 0$$

This is in standard form with right-hand side equal to zero.

STEP 2: We see $a = 5$, $b = 8$, $c = 3$

STEP 3: $$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 5 \cdot 3}}{2 \cdot 5}$$

STEP 4: $$x = \frac{-8 \pm \sqrt{64 - 60}}{10}$$

$$\Rightarrow x = \frac{-8 \pm \sqrt{4}}{10} = \frac{-8 \pm 2}{10}$$

$$\Rightarrow x_1 = \frac{-8 - 2}{10} \quad \text{OR} \quad x_2 = \frac{-8 + 2}{10}$$

$$\Rightarrow \boxed{x_1 = \frac{-10}{10} = -1} \quad \text{OR} \quad \boxed{x_2 = \frac{-6}{10} = 0.6}$$

Name:________________________ Class #:______

Problem 5 – 6: Derive the quadratic formula.

5. $5x^2 + 8x + 3 = 0$

$$\Rightarrow 5x^2 + 8x = -3$$

$$\Rightarrow x^2 + \frac{8}{5}x = \frac{-3}{5}$$

$$\Rightarrow x^2 + \frac{8}{5}x + \frac{8^2}{(2\cdot 5)^2} = \frac{-3}{5} + \frac{8^2}{(2.5)^2}$$

$$\Rightarrow \left(x + \frac{8}{2\cdot 5}\right)^2 = \frac{8^2}{4\cdot 5^2} - \frac{3}{5}$$

$$\Rightarrow \left(x + \frac{8}{2\cdot 5}\right)^2 = \frac{8^2}{4\cdot 5^2} - \frac{3}{5}\cdot\frac{4}{4}\cdot\frac{5}{5}$$

$$\Rightarrow \left(x + \frac{8}{2\cdot 5}\right)^2 = \frac{8^2 - 4\cdot 5\cdot 3}{4\cdot 5^2}$$

$$\Rightarrow \sqrt{\left(x + \frac{8}{2\cdot 5}\right)^2} = \sqrt{\frac{8^2 - 4\cdot 5\cdot 3}{4\cdot 5^2}}$$

$$\Rightarrow \left|x + \frac{8}{2\cdot 5}\right| = \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{\sqrt{4\cdot 5^2}} = \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{\sqrt{4}\cdot\sqrt{5^2}}$$

6. $ax^2 + bx + c = 0$ (assume $a>0$)

$$\Rightarrow ax^2 + bx = -c$$

$$\Rightarrow \cancel{a}\ x^2 + \frac{b}{a}x = \frac{-c}{a}$$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{b^2}{(2\cdot a)^2} = -\frac{c}{a} + \frac{b^2}{(2\cdot a)}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4\cdot a^2} - \frac{c}{a}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4\cdot a^2} - \frac{c}{a}\cdot\frac{4}{4}\cdot$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4\,a\cdot c}{4a^2}$$

$$\Rightarrow \sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{(b^2 - 4ac)}{4a^2}}$$

$$\Rightarrow \left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2 - 4\cdot a\cdot c}}{\sqrt{4\cdot a^2}}$$

$$\Rightarrow \left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2 - 4\cdot a\cdot c}}{\sqrt{4}\cdot\sqrt{a^2}}$$

$$\Rightarrow \left|x + \frac{8}{2\cdot 5}\right| = \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

$$\Rightarrow \left|x + \frac{b}{2a}\right| = \frac{\sqrt{b^2 - 4\cdot a\cdot c}}{2a}$$

$$\Rightarrow x + \frac{8}{2\cdot 5} = + \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

OR

$$x + \frac{8}{2\cdot 5} = - \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

$$\Rightarrow x + \frac{b}{2a} = + \frac{\sqrt{b^2 - 4ac}}{2a}$$

OR

$$x + \frac{b}{2a} = - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-8}{2\cdot 5} + \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

OR

$$x = \frac{-8}{2\cdot 5} - \frac{\sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

$$\Rightarrow x = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$$

OR

$$x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-8 + \sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

OR

$$x = \frac{-8 - \sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$$

$$\Rightarrow x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

OR

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \boxed{x = \frac{-8 \pm \sqrt{8^2 - 4\cdot 5\cdot 3}}{2\cdot 5}}$$

$$\Rightarrow \boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Name:______________________ Class #:______

Problem 7 – 10: Solve the following quadratic equations using the quadratic formula.

7. $x^2 = 4x - 4$

$\Rightarrow x^2 - 4x + 4 = 0$

We see $a=1$, $b=-4$, $c=4$ and by the quadratic formula we have:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\cdot 1\cdot 4}}{2\cdot 1}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16-16}}{2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{0}}{2} = \boxed{2}$$

8. $3p^2 = 18p - 6$

$\Rightarrow 3p^2 - 18p + 6 = 0$

We have $a=3$, $b=-18$, $c=6$ and we s

$$p = \frac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$$

$$= \frac{-(-18) \pm \sqrt{(-18)^2 - 4\cdot 3\cdot 6}}{2\cdot 3}$$

$$= \frac{18 \pm \sqrt{324 - 72}}{6}$$

$$= \frac{18 \pm \sqrt{252}}{6}$$

$$= \frac{18 \pm \sqrt{36\cdot 7}}{6}$$

$$= \frac{18 \pm \sqrt{36}\sqrt{7}}{6}$$

$$= \frac{18 \pm 6\sqrt{7}}{6}$$

$$= \boxed{3 \pm \sqrt{7}}$$

9. $x^2 = 3x + 5$

$\Rightarrow x^2 - 3x - 5 = 0$

We see $a=1$, $b=-3$, $c=-5$ and we have

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4\cdot 1\cdot (-5)}}{2\cdot 1}$$

$$x = \frac{\overset{3}{\cancel{9}} \pm \sqrt{9+20}}{2}$$

$\cancel{= 9 \pm}$

$$\boxed{x = \frac{3 \pm \sqrt{29}}{2}} \overset{\text{OR}}{\Rightarrow} \boxed{x_1 = \frac{3-\sqrt{29}}{2} \quad \text{OR} \quad x_2 = \frac{3+\sqrt{29}}{2}}$$

10. $5x^2 = 13x+18$

$\Rightarrow 5x^2 - 13x - 18 = 0$

We notice $a=5$, $b=-13$, $c=-18$ and we Kn

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4\cdot 5\cdot (-18)}}{2\cdot 5}$$

$$x = \frac{13 \pm \sqrt{169 + 360}}{10}$$

$$x = \frac{13 \pm \sqrt{529}}{10}$$

$$x = \frac{13 \pm 23}{10} \Rightarrow \boxed{x_1 = -1} \text{ or } \boxed{x_2 = 3.}$$