Name: Lesson 16 Solutions

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LESSON 16: Quadratic Equations

- □ General form of quadratic function: $f(x) = ax^2 + bx + c$
- □ Parabola- the graph of a quadratic function
- □ Standard form of quadratic equation: $ax^2 + bx + c = 0$
- □ Three scenarios for x-intercepts of parabola
  - No x-intercepts: no real solution to equation $ax^2 + bx + c = 0$
  - One x-intercept: One solution to equation $ax^2 + bx + c = 0$
  - Two x-intercepts: Two solution to equation $ax^2 + bx + c = 0$
- □ Principle of Square Roots: If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$
- □ Method of completing the square
- □ To complete the square for $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$
- □ To solve quadratic equation by completing the square

In problems 1 – 4, add a constant to make the expression a perfect square trinomial.

- ➢ Identify each step you take in the solution.
- ➢ EXPLAIN WHY you are taking each step

Recall the standard form of a quadratic function is $y = ax^2 + bx + C$

1. $w^2 + 6w$ ← in this case, we have $a = 1$ and $b = 6$. To transform this expression into a perfect-square trinomial, we must add $c = \left[\frac{b}{2}\right]^2 = \left[\frac{6}{2}\right]^2 = 3^2 = 9$

$$\Rightarrow w^2 + 6w + 9 = w^2 + 3w + 3w + 9$$
$$= w(w+3) + 3(w+3)$$
$$= (w+3)\cdot(w+3) = \boxed{(w+3)^2}$$

this is a perfect square !!

2. $t^2 - 7t$ ← in this case, we have $a = 1$ and $b = -7$. To transform this expression into a perfect square trinomial, we must add

$$c = \left[\frac{b}{2}\right]^2 = \left[\frac{-7}{2}\right]^2 = \frac{-7}{2}\cdot\frac{-7}{2} = \frac{49}{4}$$

$$\Rightarrow t^2 - 7t + \frac{49}{4} = t^2 - \frac{7}{2}\cdot t - \frac{7}{2}t + \frac{49}{4}$$
$$= t\cdot\left(t - \frac{7}{2}\right) - \frac{7}{2}\left(t - \frac{7}{2}\right)$$
$$= \left(t - \frac{7}{2}\right)\cdot\left(t - \frac{7}{2}\right) = \boxed{\left(t - \frac{7}{2}\right)^2}$$

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5. $x^2 - \frac{11}{2}x$ ← in this case, we have $a=1$ and $b = \frac{-11}{2}$. To transform this into a perfect-square trinomial, we add

$$c = (b \div 2)^2 = \left[\frac{-11}{2} \div 2\right]^2 = \left[\frac{-11}{2} \cdot \frac{1}{2}\right]^2 = \left[\frac{-11}{4}\right]^2 = \frac{-11}{4} \cdot \frac{-11}{4} = \frac{121}{16}$$

$$\Rightarrow x^2 - \frac{11}{2}x + \frac{121}{16} = x^2 - \frac{11}{4}x - \frac{11}{4}x + \frac{121}{16}$$

$$= x \cdot \left(x - \frac{11}{4}\right) - \frac{11}{4} \cdot \left(x - \frac{11}{4}\right)$$

$$= \left(x - \frac{11}{4}\right) \cdot \left(x - \frac{11}{4}\right) = \boxed{\left(x - \frac{11}{4}\right)^2}$$

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6. $m^2 + \frac{5}{4}m$ ← in this problem, we have $a=1$ and $b = \frac{5}{4}$. To produce a perfect square trinomial, we add

$$c = (b \div 2)^2 = \left[\frac{5}{4} \div \frac{2}{1}\right]^2 = \left[\frac{5}{4} \cdot \frac{1}{2}\right]^2 = \left[\frac{5}{8}\right]^2 = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$$

$$\Rightarrow m^2 + \frac{5}{4}m + \frac{25}{64} = m^2 + \frac{5}{8}m + \frac{5}{8}m + \frac{25}{64}$$

$$= m \cdot \left(m + \frac{5}{8}\right) + \frac{5}{8}\left(m + \frac{5}{8}\right)$$

$$= \left(m + \frac{5}{8}\right) \cdot \left(m + \frac{5}{8}\right)$$

$$= \boxed{\left(m + \frac{5}{8}\right)^2}$$

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Mini-Lecture: Solve the following quadratic equation using three different methods:

$$\underbrace{x^2 + 6x}_{\text{LHS of equation}} \overset{\text{equals sign}}{=} \underbrace{16}_{\text{RHS of equation}}$$

Method 1: Solve by factoring

$x^2 + 6x = 16$

$\Rightarrow x^2 + 6x - 16 = 0$

$a = 1,\ b = 6,\ c = -16$

Mult -16, Add $+6$: $+8$, -2

$\Rightarrow x^2 + 8x - 2x - 16 = 0$

$\Rightarrow x \cdot (x+8) - 2(x+8) = 0$

$\Rightarrow (x-2) \cdot (x+8) = 0$

$\Rightarrow x - 2 = 0$ OR $x + 8 = 0$

$\Rightarrow \boxed{x = 2}$ OR $\boxed{x = -8}$

Method 2: Complete the Square

$$x^2 + 6x \overset{\text{equal sign}}{=} 16$$

$$\Rightarrow \underbrace{x^2 + 6x + 9}_{\text{perfect-square trinomial}} = 16 + 9 \qquad \left[\frac{b}{2}\right]^2 = \left[\frac{6}{2}\right]^2 = 3^2 =$$

$a = 1$, $b = 6$

$\Rightarrow (x+3)^2 = 25$

index $n = 2$ even

$\Rightarrow \sqrt[2]{(x+3)^2} = \sqrt[2]{25}$

$\Rightarrow |x+3| = 5$

$x + 3 = -5$ → $\boxed{x = -8}$

OR

$x + 3 = +5$ → $\boxed{x = 2}$

Method 3: Solve Graphically

Step 1: Graph LHS $y_1 = x^2 + 6x$

Step 2: Graph RHS $\boxed{y_2 = 16}$

Step 3: Find Point(s) of intersection between two graphs

Step 4: Write each point as an ordered pair

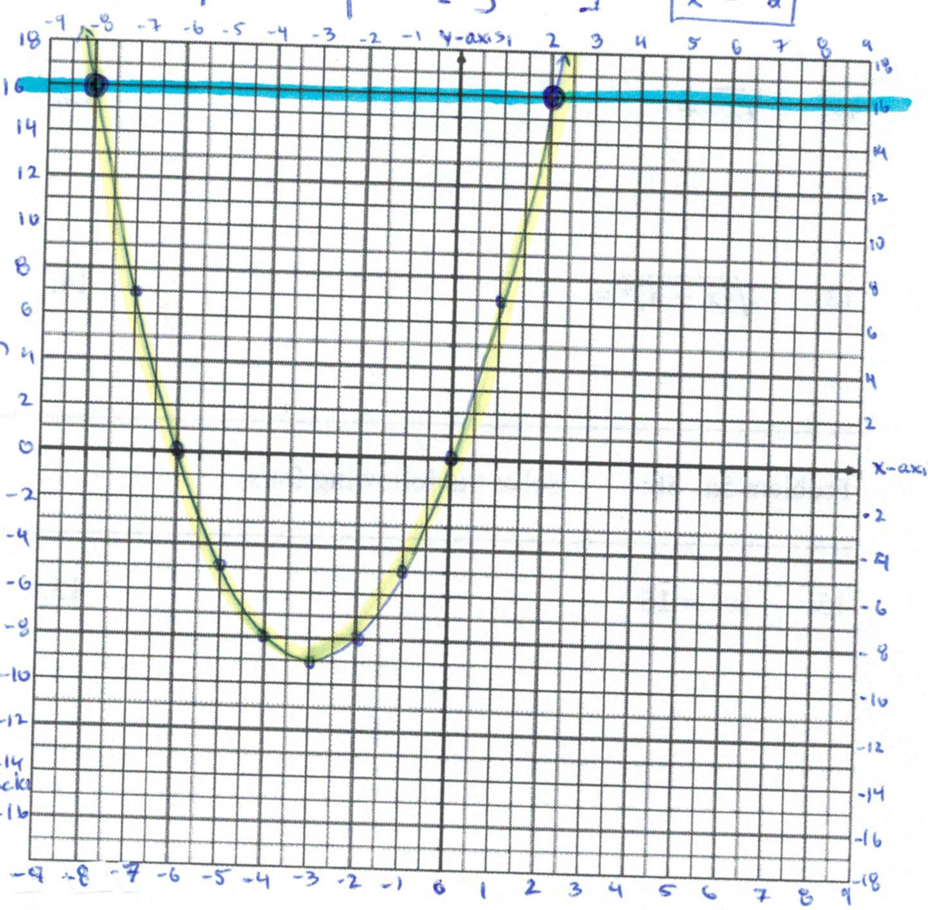
Left POI: $(-8, 16)$

Righ POI: $(2, 16)$

x-coordinates

Step 5: Solutions are the x-coordinates

$\boxed{x = -8}$ or $\boxed{x = 2}$



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17. Solve the following quadratic equation using three different methods:

equal sign

$$\underbrace{x^2 - 12x}_{\text{LHS of equation}} = \underbrace{-32}_{\text{RHS of equation}}$$

Method 1: Solve by factoring

$x^2 - 12x = -32$

$\Rightarrow x^2 - 12x + 32 = 0$ RHS is zero

$\Rightarrow x^2 - 4x - 8x + 32 = 0$

$\Rightarrow x \cdot (x-4) - 8(x-4) = 0$

$\Rightarrow (x-8) \cdot (x-4) = 0$

$\Rightarrow x - 8 = 0$ OR $x - 4 = 0$

$\Rightarrow \boxed{x = 8}$ OR $\boxed{x = 4}$

Method 2: Complete the Square

equals sign

$x^2 - 12x = -32$

$\Rightarrow \underbrace{x^2 - 12x + 36}_{\text{perfect-square trinomial}} = -32 + 36$ $a = 1$ $b = -12$ $\left[\frac{b}{2}\right]^2 = \left[\frac{-12}{2}\right]^2 = 6^2 = 36$

$\Rightarrow (x-6)^2 = 4$

$\Rightarrow \sqrt[2]{(x-6)^2} = \sqrt[2]{4}$

$\Rightarrow |x - 6| = 2$

$\Rightarrow x - 6 = -2$ OR $x - 6 = 2$ $\boxed{x = 4}$ OR $\boxed{x = 8}$

Method 3: Solve Graphically

Step 1: Graph LHS $y_1 = x^2 - 12x$

Step 2: Graph RHS $y_2 = -32$

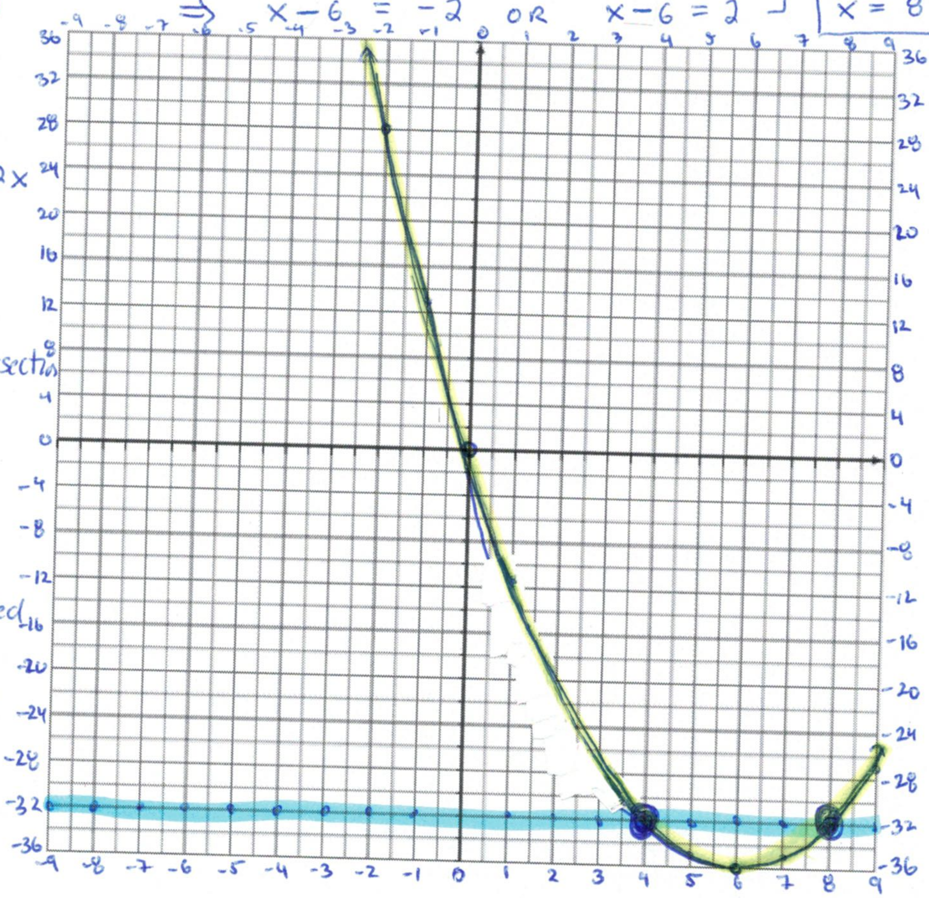
Step 3: Find the point(s) of intersection between the two graphs.

Step 4: Write each point of intersection as an ordered pair

Left P.o.I: $(4, -32)$

x-coordinates

Right P.o.I: $(8, -32)$



Step 5: The solutions are the x-coordinates

$\boxed{x = 4}$ or $\boxed{x = 8}$

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Problem 18 - 21: Solve each of the following quadratic equations by completing the square.

18. $x^2 - 6x = -1$

$a=1,\ b=-6$

$\left[\frac{b}{2}\right]^2 = \left[\frac{-6}{2}\right]^2 = (-3)^2 = 9$

$$x^2 - 6x = -1$$

$$\Rightarrow \underbrace{x^2 - 6x + 9}_{\text{perfect square trinomial}} = -1 + 9$$

$$\Rightarrow x^2 - 3x - 3x + 9 = 8$$

$$\Rightarrow \overbrace{(x-3)^2}^{\text{perfect square}} = 8$$

$$\Rightarrow \sqrt[2]{(x-3)^2} = \sqrt[2]{8}$$

$$\Rightarrow |x-3| = \sqrt{4}\cdot\sqrt{2}$$

$$\Rightarrow x - 3 = -2\sqrt{2} \quad \text{OR} \quad x - 3 = +2\sqrt{2}$$

$$\Rightarrow \boxed{x = 3 - 2\sqrt{2}} \quad \text{OR} \quad \boxed{x = 3 + 2\sqrt{2}}$$

19. $t^2 - 8t = 9$

$a=1,\ b=-8$

$\left[\frac{b}{2}\right]^2 = \left[\frac{-8}{2}\right]^2 = [-4]^2 = 1$

$$t^2 - 8t = 9$$

$$\Rightarrow \underbrace{t^2 - 8t + 16}_{\text{perfect square trinomial}} = 9 + 16$$

$$\Rightarrow \underbrace{(t-4)^2}_{\text{perfect square}} = 25$$

$$\Rightarrow \sqrt[2]{(t-4)^2} = \sqrt[2]{25}$$

$$\Rightarrow |t-4| = 5$$

$$\Rightarrow t - 4 = -5 \quad \text{or} \quad t - 4 = +5$$

$$\Rightarrow \boxed{t = -1} \quad \text{or} \quad \boxed{t = 9}$$

20. $x^2 + 5x = -3$

$a=1,\ b=5$

$\left[\frac{b}{2}\right]^2 = \left[\frac{5}{2}\right]^2 = \frac{25}{4}$

$$\Rightarrow x^2 + 5x + \frac{25}{4} = -3 + \frac{25}{4}$$

$$\Rightarrow \left(x + \frac{5}{2}\right)^2 = \frac{-12+25}{4}$$

$$\Rightarrow \left(x + \frac{5}{2}\right)^2 = \frac{13}{4}$$

$$\Rightarrow \sqrt[2]{\left(x+\frac{5}{2}\right)^2} = \sqrt{\frac{13}{4}} = \frac{\sqrt{13}}{2}$$

$$\Rightarrow \left|x + \frac{5}{2}\right| = \frac{\pm\sqrt{13}}{2}$$

$$\Rightarrow x + \frac{5}{2} = \frac{-\sqrt{13}}{2} \quad \text{or} \quad x + \frac{5}{2} = \frac{\sqrt{13}}{2}$$

$$\Rightarrow \boxed{x = \frac{-5}{2} - \frac{\sqrt{13}}{2}} \quad \text{or} \quad \boxed{x = \frac{-5}{2} + \frac{\sqrt{13}}{2}}$$

21. $3t^2 + 7t - 2 = 0$

careful: $a \neq 1$

$$\Rightarrow \frac{3t^2 + 7t - 2}{3} = \frac{0}{3}$$

let's divide both sides by 3 to get $a=1$

$$\Rightarrow \frac{3t^2}{3} + \frac{7}{3}t - \frac{2}{3} = 0$$

$$\Rightarrow t^2 + \frac{7}{3} = \frac{2}{3}$$

$a=1,\ b=\frac{7}{3}$

$\left[\frac{b}{2}\right]^2 = \left[\frac{7}{3} \div \frac{2}{1}\right]^2 = \left[\frac{7}{3}\cdot\frac{1}{2}\right]^2 = \left[\frac{7}{6}\right]^2 = \frac{49}{36}$

$$\Rightarrow \overbrace{t^2 + \frac{7}{3}t + \frac{49}{36}}^{\text{perfect-square trinomial}} = \frac{2}{3} + \frac{49}{36}$$

$$\Rightarrow \underbrace{\left(t + \frac{7}{6}\right)^2}_{\text{perfect square}} = \frac{2}{3}\cdot\frac{12}{12} + \frac{49}{36}$$

$$\Rightarrow \left(t + \frac{7}{6}\right)^2 = \frac{24+49}{36} = \frac{73}{36}$$