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SECTION 9.5: Expressions Containing Several Radical Terms (p. 698 – 705)

- ☐ Like radicals (p. 698)
- ☐ Adding and subtracting radical expressions (p. 698)
- ☐ Product and quotients of two or more radical terms (p. 699)
- ☐ Negative rational exponents (p. 679)

Simplify each of the following rational expressions.

1. $\sqrt{3} + 6\sqrt{2} - 2\sqrt{2} = \sqrt{3} + (6-2)\sqrt{2}$

$= \boxed{\sqrt{3} + 4 \cdot \sqrt{2}}$

OR

$= \sqrt[2]{3} + 4 \cdot \sqrt[2]{2}$

2. $\sqrt{3} \cdot (7 - \sqrt{2}) + \sqrt{6} = 7 \cdot \sqrt{3} - \sqrt{3} \cdot \sqrt{2} +$

$= 7\sqrt{3} - \sqrt{3 \cdot 2} + \sqrt{}$

$= 7\sqrt{3} - \sqrt{6} + \sqrt{}$

$= \boxed{7\sqrt{3}}$

3. $\sqrt{3} + \sqrt{27}$

$= \sqrt{3} + \sqrt{3 \cdot 9}$

$= \sqrt{3} + \sqrt{3} \cdot \sqrt{9}$

$= \sqrt{3} + 3\sqrt{3}$

$= \boxed{4 \cdot \sqrt{3}}$

4. $2\sqrt[3]{x} + 5x + 4\sqrt[3]{x} - 3$

$= 2\sqrt[3]{x} + 4\sqrt[3]{x} + 5x - 3$

$= (2+4) \cdot \sqrt[3]{x} + 5x - 3$

$= \boxed{6\sqrt[3]{x} + 5x - 3}$

OPTIONAL CHALLENGE PROBLEMS: Simplify the expressions below

5. $\sqrt[3]{2} \cdot (\sqrt[3]{4} - 2\sqrt[3]{32})$

$= \sqrt[3]{2} \cdot \sqrt[3]{4} - 2 \cdot \sqrt[3]{2} \cdot \sqrt[3]{32}$

$= \sqrt[3]{2 \cdot 4} - 2 \cdot \sqrt[3]{2 \cdot 32}$

$= \sqrt[3]{8} - 2 \cdot \sqrt[3]{64}$

$= 2 - 2 \cdot 4$

$= 2 - 8 = \boxed{-6}$

6. $\sqrt{a^2 - 3} - \frac{1}{\sqrt{a^2-3}}$

addition of fractions with a different denominator

(For solution to #6 see Reverse)

$$\sqrt{a^2-3} - \frac{1}{\sqrt{a^2-3}} = \frac{\sqrt{a^2-3}}{1} - \frac{1}{\sqrt{a^2-3}}$$

$$= \frac{\sqrt{a^2-3}}{1} \cdot \frac{\sqrt{a^2-3}}{\sqrt{a^2-3}} - \frac{1}{\sqrt{a^2-3}}$$

$$= \frac{\sqrt{a^2-3} \cdot \sqrt{a^2-3}}{\sqrt{a^2-3}} - \frac{1}{\sqrt{a^2-3}}$$

$$= \frac{\sqrt{(a^2-3) \cdot (a^2-3)}}{\sqrt{a^2-3}} - \frac{1}{\sqrt{a^2-3}}$$

$$= \frac{\sqrt{(a^2-3)^2}}{\sqrt{a^2-3}} - \frac{1}{\sqrt{a^2-3}}$$

$$= \frac{a^2-3 - 1}{\sqrt{a^2-3}}$$

$$= \frac{a^2-4}{\sqrt{a^2-3}}$$

$$= \boxed{\frac{(a-2) \cdot (a+2)}{\sqrt{a^2-3}}}$$

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SECTION 9.6: Solving Radical Equations (p. 707 – 715)

- ☐ Radical equations (p. 707)
- ☐ The Principle of Powers: If $a = b$, then $a^n = b^n$ for any exponent n (p. 707)
- ☐ Check for extraneous solutions to radical equations (p. 707)
- ☐ To solve an equation with a radical term (p. 709)
- ☐ To solve an equation with two or more radical terms (p. 710)

Solve the following equations for the unknown variable using either:

Method 1: Algebraic Techniques

Method 2: Graphical Techniques

In either case, be sure to

- Identify each step you take in the solution.
- EXPLAIN HOW YOUR STEPS RELATE TO THE ORDER OF OPERATION RULES

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1. $\sqrt[4]{15x} = 3$

$$\Rightarrow \left(\sqrt[4]{15x}\right)^4 = 3^4$$

$$\Rightarrow 15x = 81$$

$$\Rightarrow \frac{15x}{15} = \frac{81}{15}$$

$$\Rightarrow x = \frac{3 \cdot 27}{3 \cdot 5} = \frac{3}{3} \cdot \frac{27}{5} = \boxed{\frac{27}{5} = 5.4}$$

2. $5\sqrt[2]{15x - 3} - 2 = 13$

$$\Rightarrow 5\sqrt[2]{15x-3} = 15$$

$$\Rightarrow \sqrt{15x - 3} = 3$$

$$\Rightarrow \left(\sqrt{15x-3}\right)^2 = 3^2$$

$$\Rightarrow 15x - 3 = 9$$

$$\Rightarrow 15x = 12$$

$$\Rightarrow x = \frac{12}{15} = \frac{3 \cdot 4}{3 \cdot 5}$$

$$\Rightarrow \boxed{x = \frac{4}{5} \approx 0.8}$$

Class 18 Handout, Problem 2

Let's solve the algebraic equation

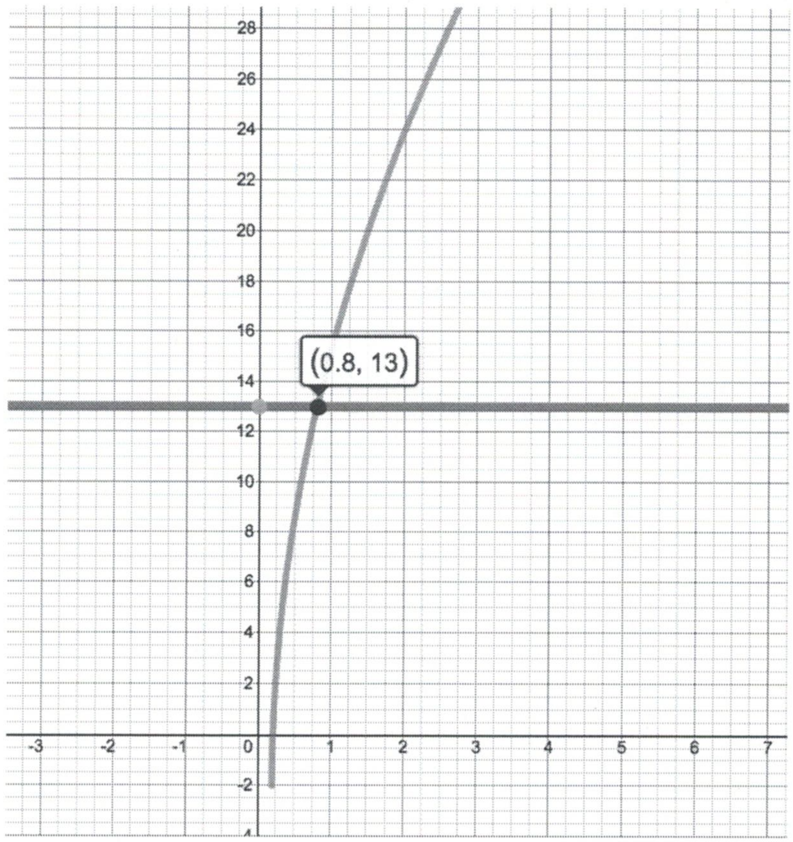
$$5\sqrt{15x-3} - 2 = 13$$

using our graphical method. To do so, we will graph both the left-hand and right-hand side. Let

$y_1 = 5\sqrt{15x-3} - 2$

$y_2 = 13$

$y_1 = 5\sqrt{15x-3} - 2$



$y_2 = 13$

We find the point of intersection between curves y_1 and y_2.

We see this occurs at

$$(0.8, 13)$$

The first-coordinate $\boxed{x = 0.8 = \frac{4}{5}}$ is our solution.

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3. $\sqrt[2]{x-2} - 7 = -4$ $\Rightarrow \sqrt[2]{x-2} = 3$

$\Rightarrow \left(\sqrt{x-2}\right)^2 = 3^2$

$\Rightarrow x - 2 = 9$

$\Rightarrow \boxed{x = 11}$

4. $3+\sqrt{5-x} = x$ $\Rightarrow \sqrt{5-x} = x - 3$

$\Rightarrow \left(\sqrt{5-x}\right)^2 = (x-3)^2$

$\Rightarrow 5 - x = (x-3)\cdot(x-3)$

$\Rightarrow 5 - x = x^2 - 6x + 9$

$\Rightarrow \cancel{5 - x =}\ 0 = x^2 - 5x + 4$

$\Rightarrow (x-4)\cdot(x-1) = 0$

$\Rightarrow (x-4) = 0$ OR $x - 1 = 0$

$\Rightarrow \boxed{x = 4}$ or $x \neq 1$

extraneous solution

Check: $x = 4$

$3 + \sqrt{5-4} = 3 + \sqrt{1} = 4$ ✓

Check: $x = 1$

$3 + \sqrt{5-1} = 3 + \sqrt{4} = 5 \neq 1$

Class 18 Handout, Problem 3

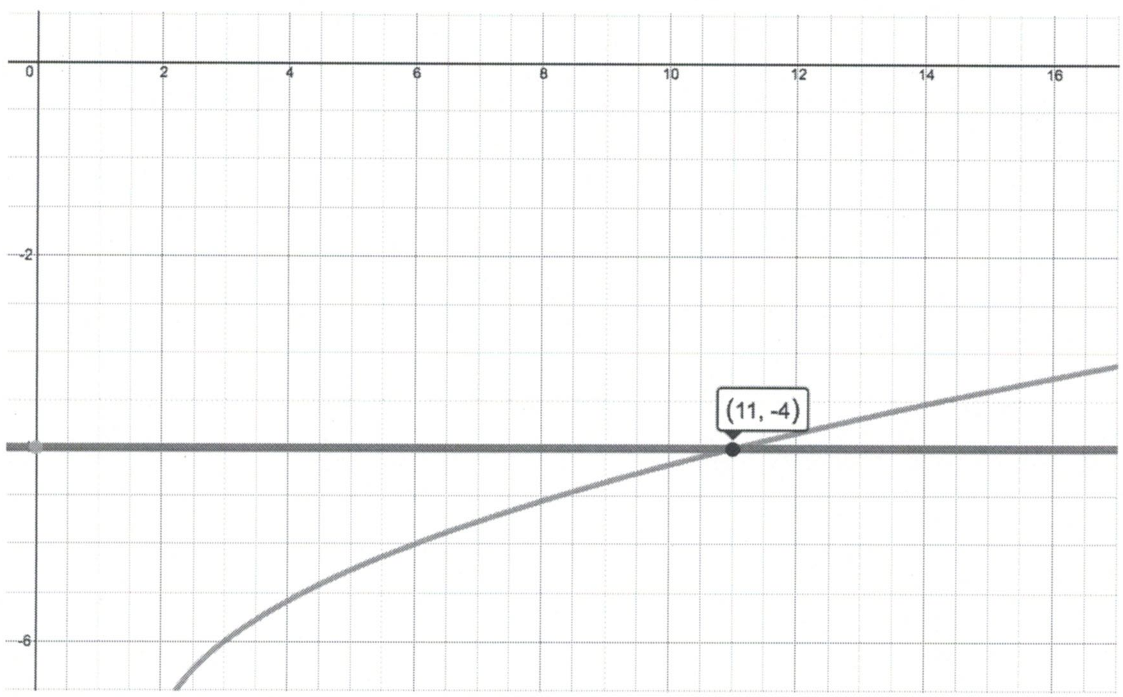
Let's solve the equation $\sqrt{x-2} - 7 = -4$ using our graphical method. To do this, we let

$$y_1 = \sqrt{x-2} - 7$$

$$y_2 = -4$$

We graph both of these curves on the axis below.

$y_2 = -4$



$y_1 = \sqrt{x-2} - 7$

Let's find the point of intersection between curves y_1 and y_2.

We see this intersection occurs at

$$(11, -4)$$

The solution to our equation is the first coordinate $\boxed{x = 11}$

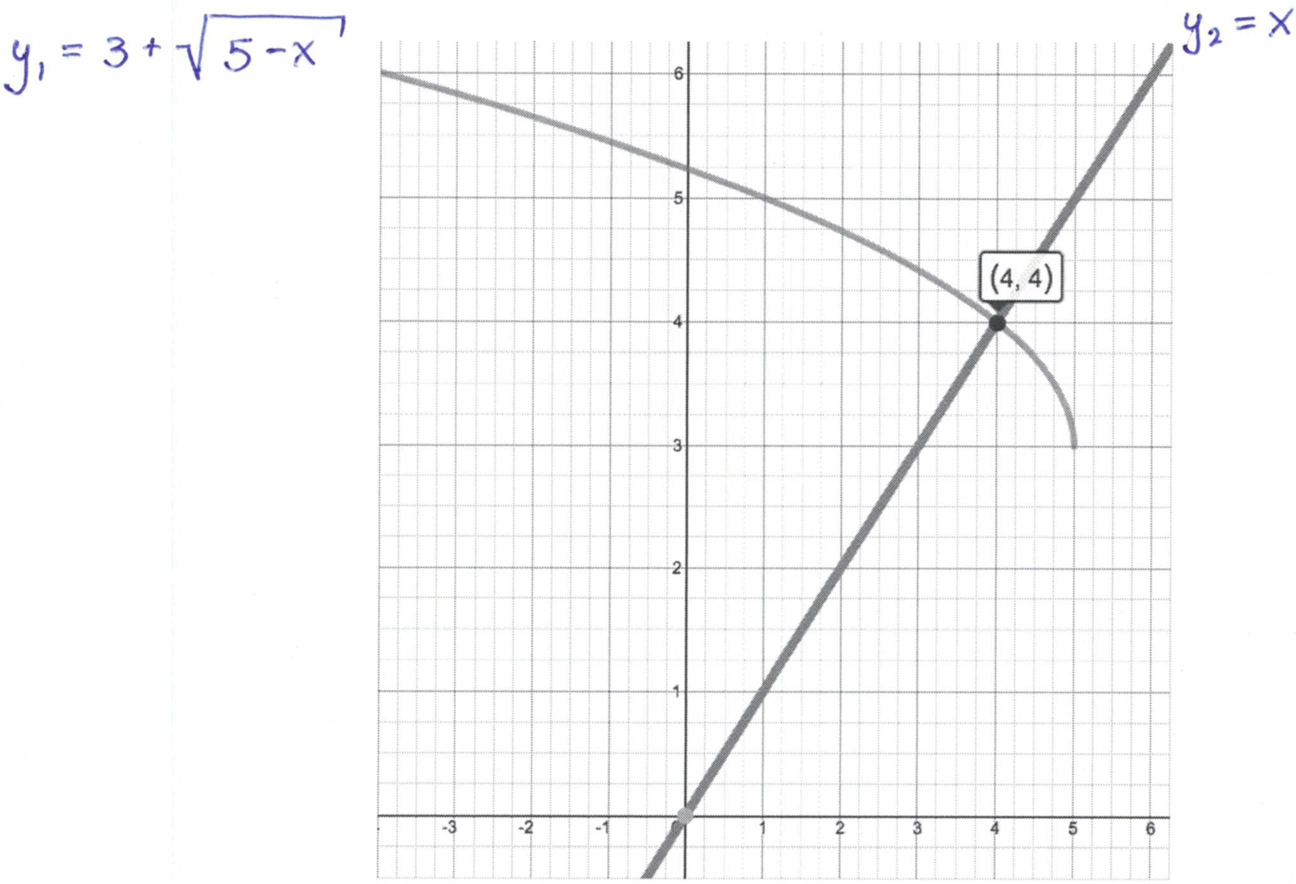
Class 18 Handout, Problem 4

To solve the equation in this problem, we set

$$y_1 = 3 + \sqrt{5 - x} \qquad (\text{expression on left-hand side})$$

$$y_2 = x \qquad (\text{expression on right-hand side})$$

We graph both of these functions on the axis below



To solve our equation

$$3 + \sqrt{5 - x} = x$$

we locate the point of intersection at $(4, 4)$. The solution to our equation is the 1st-coordinate $\boxed{x = 4}$.

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5. $\sqrt[3]{3y+6} + 7 = 8$

$$\Rightarrow \sqrt[3]{3y+6} = 1$$

$$\Rightarrow \left(\sqrt[3]{3y+6}\right)^3 = 1^3$$

$$\Rightarrow 3y + 6 = 1$$

$$\Rightarrow 3y = -5$$

$$\Rightarrow \boxed{y = -5/3}$$

OPTIONAL CHALLENGE PROBLEMS: Solve the following equations for the unknown variable:

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6. $\sqrt[2]{4x-3} = 2 + \sqrt[2]{2x-5}$

$$\Rightarrow \left(\sqrt[2]{4x-3}\right)^2 = \left(2 + \sqrt{2x-5}\right)^2$$

$$\Rightarrow 4x - 3 = \left(2 + \sqrt{2x-5}\right) \cdot \left(2 + \sqrt{2x-5}\right)$$

$$\Rightarrow 4x - 3 = 4 + 4\sqrt{2x-5} + 2x - 5$$

$$\Rightarrow 4x - 3 = 2x - 1 + 4\sqrt{2x-5}$$

$$\Rightarrow 2x - 2 = 4\sqrt{2x-5}$$

$$\Rightarrow (2x-2)^2 = \left(4 \cdot \sqrt{2x-5}\right)^2$$

$$\Rightarrow 4x^2 - 8x + 4 = 16 \cdot \left(\sqrt{2x-5}\right)^2$$

$$\Rightarrow 4x^2 - 8x + 4 = 16 \cdot (2x-5)$$

$$\Rightarrow 4x^2 - 8x + 4 = 32x - 80$$

$$\Rightarrow 4x^2 - 40x + 84 = 0$$

$$\Rightarrow 4 \cdot (x^2 - 10x + 21) = 0$$

$$\Rightarrow 4 \cdot (x-3) \cdot (x-7) = 0$$

$$\Rightarrow x - 3 = 0 \quad \text{OR} \quad x - 7 = 0$$

$$\Rightarrow \boxed{x = 3} \quad \text{OR} \quad \boxed{x = 7}$$

Check: $x = 3$ $\quad \sqrt{4 \cdot 3 - 3} = \sqrt{9} \stackrel{\checkmark}{=} 2 + \sqrt{6-1}$

Check: $x = 7$ $\quad \sqrt{4 \cdot 7 - 3} = \sqrt{25} \stackrel{\checkmark}{=} 2 + \sqrt{14-5}$

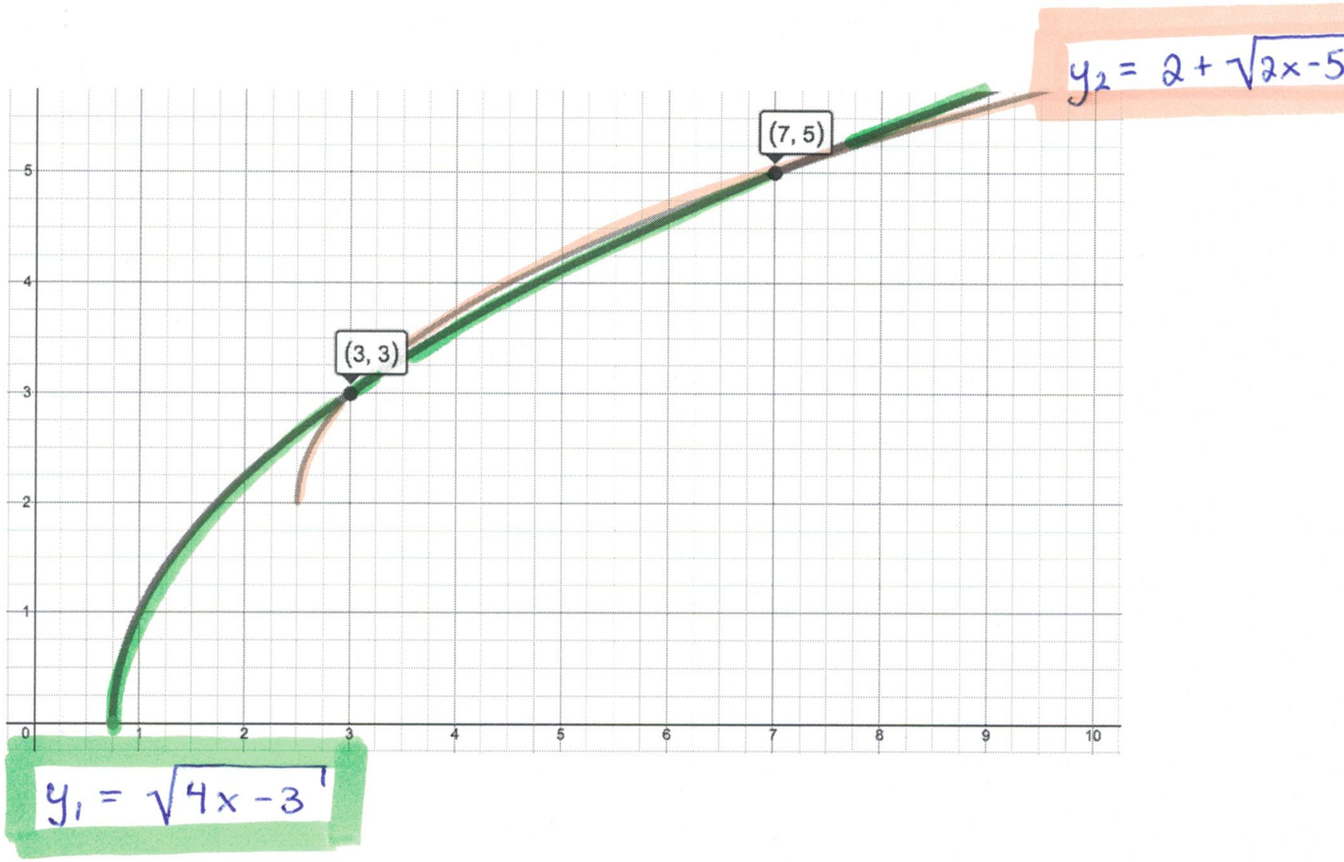
Class 18 Handout, Problem 6

Let's solve the equation $\sqrt{4x-3} = 2 + \sqrt{2x-5}$ using our graphical method. We set

$$y_1 = \sqrt{4x-3}$$

$$y_2 = 2 + \sqrt{2x-5}$$

and graph these curves on the axis below



Our solutions occur at the point of intersections

(3, 3) and (7, 5)

The 1st-coordinates of these P.O.I. given by

$\boxed{x=3}$ and $\boxed{x=7}$

Check: $x \cancel{=} 7$ $\quad \sqrt{7+2} + \sqrt{21+4} = \sqrt{9} + \sqrt{25} = 3+5 \neq 2$

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$x = -1 \quad \sqrt{-1+2} + \sqrt{-3+4} = \sqrt{1} + \sqrt{1} = 2$ ✓

7. $\sqrt{x+2} + \sqrt{3x+4} = 2$

$\Rightarrow \sqrt{x+2} = 2 - \sqrt{3x+4}$

$\Rightarrow \left(\sqrt{x+2}\right)^2 = \left(2 - \sqrt{3x+4}\right)^2$

$\Rightarrow x+2 = \left(2 - \sqrt{3x+4}\right) \cdot \left(2 - \sqrt{3x+4}\right)$

$\Rightarrow x+2 = 4 - 4\sqrt{3x+4} + (3x+4)$

$\Rightarrow x+2 = 3x + 8 - 4\sqrt{3x+4}$

$\Rightarrow -2x - 6 = -4\sqrt{3x+4}$

$\Rightarrow 2x + 6 = 4\sqrt{3x+4}$

$\Rightarrow x+3 = 2\sqrt{3x+4}$

$\Rightarrow (x+3)^2 = \left(2 \cdot \sqrt{3x+4}\right)^2$

$\Rightarrow (x+3) \cdot (x+3) = 4 \cdot (3x+4)$

$\Rightarrow x^2 + 6x + 9 = 12x + 16$

$\Rightarrow x^2 - 6x - 7 = 0$

$\Rightarrow (x-7) \cdot (x+1) = 0$

$\Rightarrow x - 7 = 0$ OR $x + 1 = 0$

$\Rightarrow x \cancel{=} 7$ or $\boxed{x = -1}$

Extraneous solution

8. $\sqrt{6x+7} - \sqrt{3x+3} = 1$

$\Rightarrow \sqrt{6x+7} = 1 + \sqrt{3x+3}$

$\Rightarrow \left(\sqrt{6x+7}\right)^2 = \left(1 + \sqrt{3x+3}\right)^2$

$\Rightarrow 6x + 7 = \left(1 + \sqrt{3x+3}\right) \cdot \left(1 + \sqrt{3x+3}\right)$

$\Rightarrow 6x + 7 = 1 + 2 \cdot \sqrt{3x+3} + 3x + 3$

$\Rightarrow 6x + 7 = 3x + 4 + 2\sqrt{3x+3}$

$\Rightarrow 3x + 3 = 2\sqrt{3x+3}$

$\Rightarrow (3x+3)^2 = \left(2 \cdot \sqrt{3x+3}\right)^2$

$\Rightarrow (3x+3) \cdot (3x+3) = 4 \cdot (3x+3)$

$\Rightarrow 9x^2 + 18x + 9 = 12x + 12$

$\Rightarrow 9x^2 + 6x - 3 = 0$

$\Rightarrow 3 \cdot (3x^2 + 2x - 1) = 0$

$\Rightarrow 3 \cdot (3x-1) \cdot (x+1) = 0$

$\Rightarrow 3x - 1 = 0$ OR $x + 1 = 0$

$\Rightarrow \boxed{x = 1/3}$ or $\boxed{x = -1}$

Class 18 Handout, Problem 7

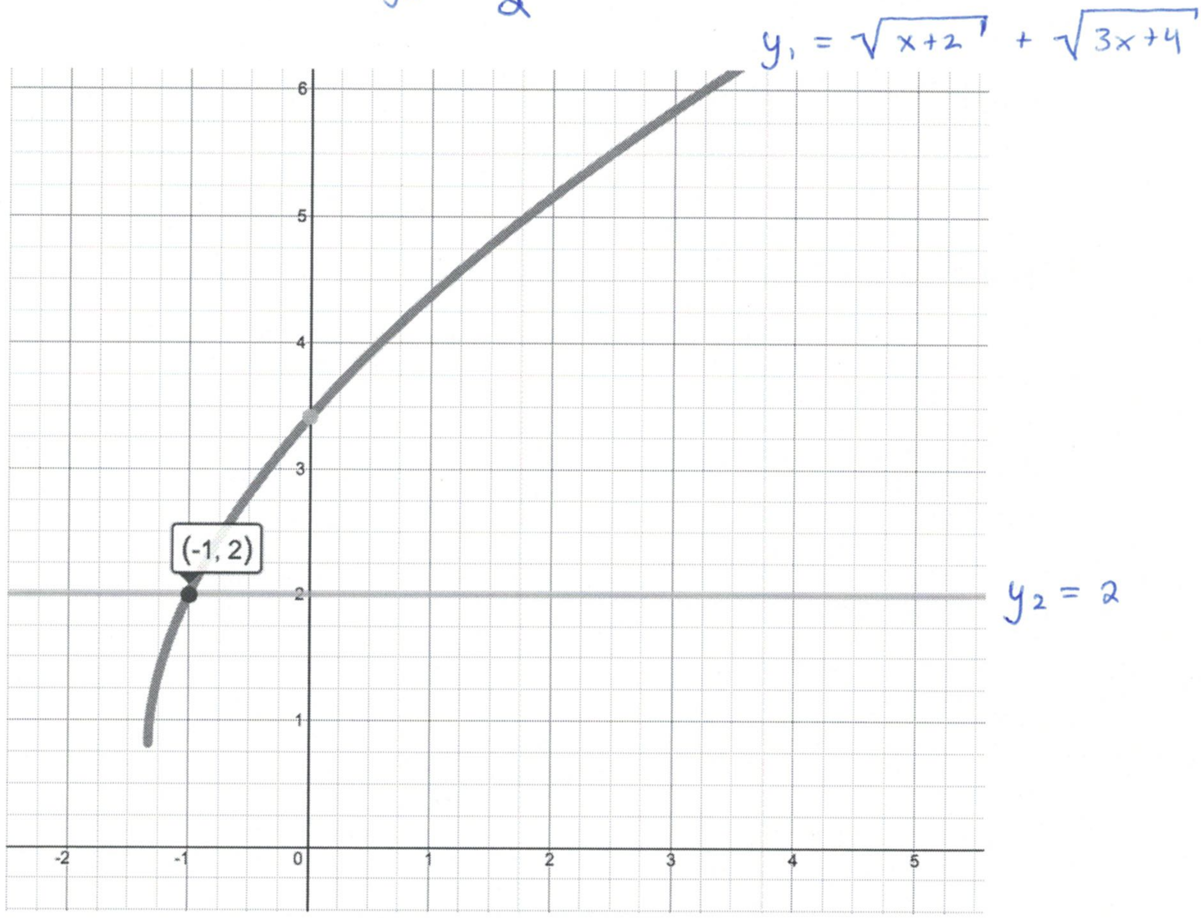
Let's solve the algebraic equation

$$\sqrt{x+2} + \sqrt{3x+4} = 2$$

To do so, we graph the left-hand side and right-hand side separately setting

$$y_1 = \sqrt{x+2} + \sqrt{3x+4}$$

$$y_2 = 2$$



We see the two curve intersect at point $(-1,2)$.

The first coordinate of this point

$$\boxed{x = -1}$$

is the solution we desired.

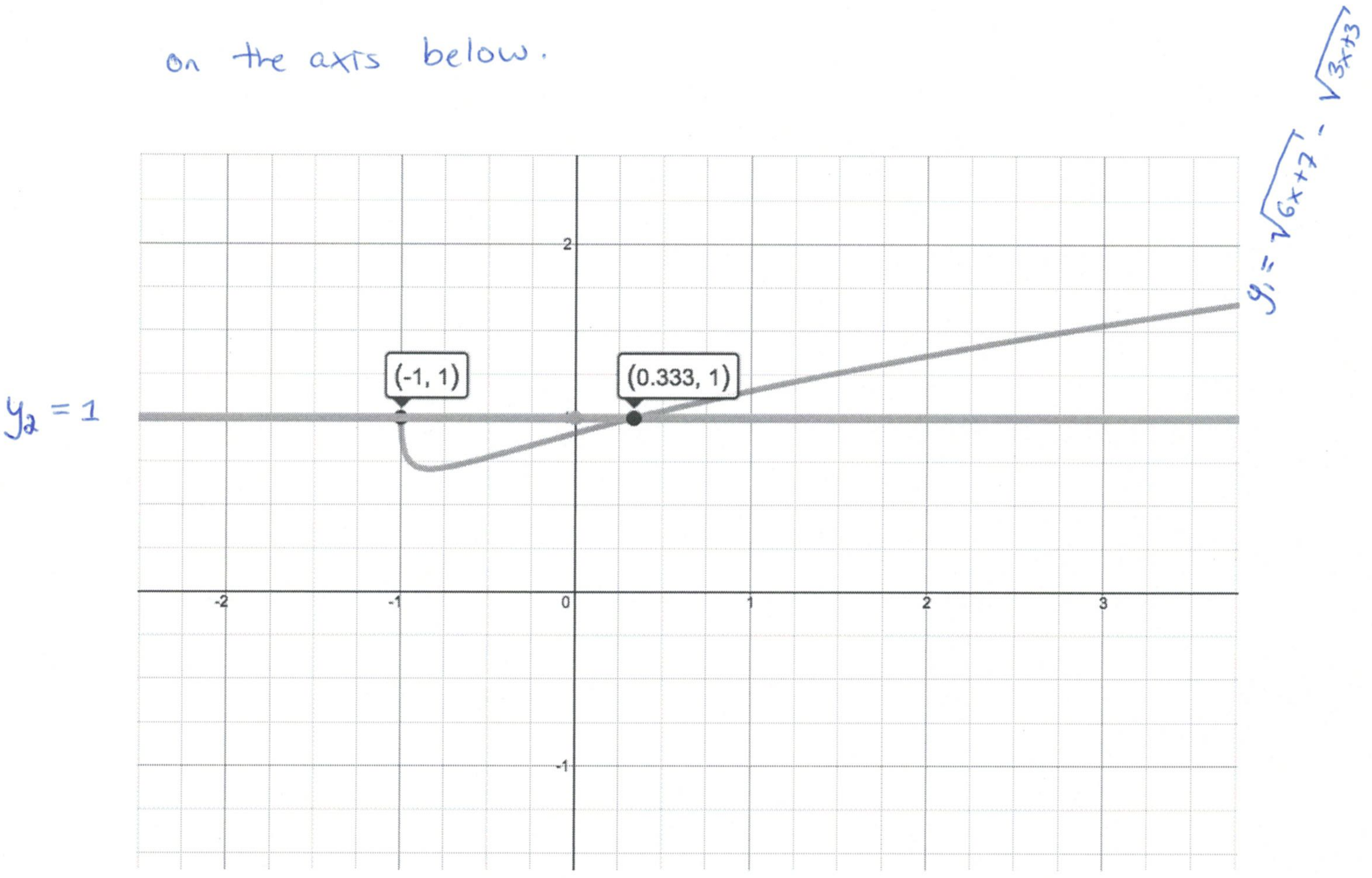
Class 18 Handout, Problem 8

To find the solution to equation $\sqrt{6x+7} - \sqrt{3x+3} = 1$

we graph the Left-hand side and right hand side

$y_1 = \sqrt{6x+7} - \sqrt{3x+3}$ and $y_2 = 1$

on the axis below.



The two curves intersect at points

$(-1, 1)$ and $(\frac{1}{3}, 1)$

corresponding to solutions

$\boxed{x = -1}$ and $\boxed{x = \frac{1}{3}}$