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Lesson 14 : Multiplying Radical Expressions

- ☐ Product Rule for Radical: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$
- ☐ The Quotient Rule for Radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
- ☐ Using the product rule to simplify
- ☐ Radical Expressions on the TI Calculator:
- ☐ To simplify radical expressions with index n by factoring
- ☐ Identify factors in radicand with exponents that are multiples of n

Recall:

☐ $a^m \cdot b^m = (a \cdot b)^m$

☐ $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

Recall the anatomy of radical expressions:

$$b = \sqrt[n]{a} \qquad\qquad b = a^{\frac{1}{n}}$$

Class 19 : $\sqrt[n]{a} = a^{\frac{1}{n}}$

"the index of the root becomes the denominator of a fractional power"

State the product rule for radicals:

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}} = \sqrt[n]{a \cdot b}$$

$$\Rightarrow \boxed{\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}}$$

multiplication is outside the radicals

multiplication inside the radical

Idea: make math simpler

State the quotient rule for radicals:

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{1/n} = \sqrt[n]{\frac{a}{b}}$$

$$\Rightarrow \boxed{\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}}$$

← division on the inside of the radicals

division on the outside of the radicals

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Use the rules of exponents and radicals to simplify each of the following problems

1. $\sqrt{200} = \sqrt[2]{200}$ ← here, we have a radical with index 2... Thus, we want to find "groups of size two" or perfect square powers inside the radicand

$= \sqrt[2]{100 \cdot 2}$ ← multiplication on inside of radical

$= \sqrt[2]{100} \cdot \sqrt[2]{2} = 10 \cdot \sqrt[2]{2} = \boxed{10\sqrt{2}}$

↑ multiplication on outside of radical

2. $\frac{\sqrt{80}}{\sqrt{5}} = \frac{\sqrt[2]{80}}{\sqrt[2]{5}}$ ← division on the outside

$= \sqrt[2]{\frac{80}{5}}$

$= \sqrt[2]{16} = \boxed{4}$

3. $3\sqrt[3]{25} \cdot 2\sqrt[3]{5} = 3 \cdot \sqrt[3]{25} \cdot 2 \cdot \sqrt[3]{5}$

$= 6 \cdot \sqrt[3]{25} \cdot \sqrt[3]{5}$

↑ multiplication on the outside

$= 6 \cdot \sqrt[3]{25 \cdot 5}$

$= 6 \cdot \sqrt[3]{125} = 6 \cdot 5 = \boxed{30}$

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4. $\frac{\sqrt{75xy}}{3\sqrt{3x}}$ $= \frac{1\cdot\sqrt[2]{75\cdot x\cdot y}}{3\cdot\sqrt[2]{3x}}$

$= \frac{1}{3}\cdot\frac{\sqrt[2]{75xy}}{\sqrt[2]{3x}}$ ← division on outside of radicals

$= \frac{1}{3}\cdot\sqrt[2]{\frac{75\cdot x\cdot y}{3\cdot x}}$ ← division on inside

$= \frac{1}{3}\sqrt[2]{\frac{75}{3}\cdot\frac{x}{x}\cdot\frac{y}{1}}$

$= \frac{1}{3}\cdot\sqrt[2]{25\cdot y}$

multiplication is on inside

$= \frac{1}{3}\cdot\sqrt[2]{25}\cdot\sqrt[2]{y} = \boxed{\frac{5}{3}\cdot\sqrt[2]{y}}$

Note: $162 = 2\cdot 81$

5. $\sqrt[4]{162x^6}$ $= \sqrt[4]{2\cdot(3\cdot3\cdot3\cdot3)\cdot(x\cdot x\cdot x\cdot x)\cdot x\cdot x}$

$= \sqrt[4]{3^4\cdot x^4\cdot 2\cdot x^2}$ ← multiplication inside radicand

perfect 4th powers

$= \sqrt[4]{3^4}\cdot\sqrt[4]{x^4}\cdot\sqrt[4]{2\cdot x^2}$

$= \boxed{3\cdot|x|\cdot\sqrt[4]{2\cdot x^2}}$

Even index $n=4$ "undoes" groups of size 4

even index

idea: $\sqrt[4]{x^4} = |x|$

$x=-1 \Rightarrow \sqrt[4]{x^4} = \sqrt[4]{(-1)^4}$

$= \sqrt[4]{1}$

$= 1$

$= |-1|$

6. $\sqrt[4]{27x^3y^5}\cdot\sqrt[4]{3xy^3}$

multiplication on the outside

$\Rightarrow \sqrt[4]{27\cdot x^3\cdot y^5}\cdot\sqrt[4]{3\cdot x\cdot y^3} = \sqrt[4]{27\cdot x^3\cdot y^5\cdot 3\cdot x\cdot y^3}$

$= \sqrt[4]{81\cdot x^4\cdot y^8}$

multiplication on inside

$= \sqrt[4]{81}\cdot\sqrt[4]{x^4}\cdot\sqrt[4]{y^8}$

$= 3\cdot|x|\cdot y^{2/1} = 3|x|\cdot y^2$

note: $(b^n)^m = b^{n\cdot m}$

$\sqrt[4]{y^8} = (y^8)^{\frac{1}{4}}$

$= y^{\frac{8}{1}\cdot\frac{1}{4}}$

$= y^{8/4} = y^2$

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7. $\frac{\sqrt[3]{189\cdot x^5\cdot y^7}}{\sqrt[3]{7\cdot x^2\cdot y^2}} = \sqrt[3]{\frac{189\cdot x^5\cdot y^7}{7\cdot x^2\cdot y^2}}$

division outside the radical

$= \sqrt[3]{\frac{189}{7}\cdot\frac{x^5}{x^2}\cdot\frac{y^7}{y^2}}$

$= \sqrt[3]{27\cdot x^3\cdot y^5}$

$= \sqrt[3]{27}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{y^5}$

$= 3\cdot x\cdot\sqrt[3]{(y\cdot y\cdot y)y\cdot y}$

$= 3\cdot x\,\sqrt[3]{y^3\cdot y^2}$

$= 3\cdot x\cdot\sqrt[3]{y^3}\cdot\sqrt[3]{y^2}$

$\boxed{= 3\cdot x\cdot y\cdot\sqrt[3]{y^2}}$

multiplication outside radical

8. $\sqrt[5]{16\,w^4\,b^5}\cdot\sqrt[5]{4\,w\,b^6} = \sqrt[5]{16\cdot w^4\cdot b^5\cdot 4\cdot w\cdot b^6}$

mult. inside radical

$= \sqrt[5]{64\cdot w^5\cdot b^{11}}$

$= \sqrt[5]{(2\cdot 2\cdot 2\cdot 2\cdot 2)2\cdot w^5\cdot b^5\cdot b^5\cdot b}$

$= \sqrt[5]{2^5\cdot w^5\cdot b^5\cdot b^5\cdot(2\cdot b)}$

perfect fifth powers

$= \sqrt[5]{2^5}\cdot\sqrt[5]{w^5}\cdot\sqrt[5]{b^5}\cdot\sqrt[5]{b^5}\cdot\sqrt[5]{2}$

$= 2\cdot w\cdot b\cdot b\cdot\sqrt[5]{2\cdot b}$

$\boxed{= 2\cdot w\cdot b^2\cdot\sqrt[5]{2b}}$

9. $\sqrt[5]{\frac{64\cdot a^{11}\cdot b^{28}}{2\cdot a\cdot b^{-2}}} = \sqrt[5]{32\cdot a^{10}\cdot b^{30}}$

side note: $\frac{b^{28}}{b^{-2}} = b^{28}\div b^{-2}$

$= \frac{b^{28}}{1}\div\frac{1}{b^2}$

$= \frac{b^{28}}{1}\cdot\frac{b^2}{1}$

$= b^{30}$

$= \sqrt[5]{32}\cdot\sqrt[5]{a^{10}}\cdot\sqrt[5]{b^{30}}$

$= 2\cdot\sqrt[5]{(a\cdot a\cdot a\cdot a\cdot a)(a\cdot a\cdot a\cdot a\cdot a)}\cdot\sqrt[5]{b^{30}}$

$= 2\cdot\sqrt[5]{a^5\cdot a^5}\cdot\sqrt[5]{b^5\cdot b^5\cdot b^5\cdot b^5\cdot b^5\cdot b^5}$

$= 2\cdot\sqrt[5]{a^5}\cdot\sqrt[5]{a^5}\cdot\left(\sqrt[5]{b^5}\right)^6$

$\boxed{= 2\cdot a^2\cdot b^6}$