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Lesson 13 Rational Numbers as Exponents

- ☐ Definition and properties of integer exponents
- ☐ Rational exponents: $a^{1/n} = \sqrt[n]{a}$
- ☐ Positive rational exponents
- ☐ Negative rational exponents
- ☐ Laws of exponents for real number exponents
- ☐ To simplify radical expressions

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1. List the Laws of Exponents (see p. 343 and p. 680)

"b multiplied by itself n times"

Exponent notation: $\boxed{b^n = b \cdot b \cdot \cdots \cdot b}$

eg $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

$b^n$ ← exponents

base b

(exponents count the number of multiplications)

One as an exponent: $\boxed{b^1 = b}$ "b multiplied by itself 1 time"

eg: $1^1 = 1$

$2^1 = 2$

$(-475)^1 = -475$

Zero as an exponent: $\boxed{b^0 = 1}$ "Any nonzero base raised to the zeroth power equals one"

eg: $2^0 = 1$

$6^0 = 1$

$32^0 = 1$

Negative exponent: $\boxed{b^{-n} = \frac{1}{b^n}}$ "b to the minus n"

eg: $5^{-6} = \frac{1}{5^6}$

· Any negative exponent can be written as a positive exponent in reciprocal

Product Rule: $\boxed{b^n \cdot b^m = b^{n+m}}$

· multiplication of exponents with same base turns into addition of exponent values

eg: $7^3 \cdot 7^2 = (7 \cdot 7 \cdot 7)(7 \cdot 7)$
$= 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$
$= 7^5 = 7^{3+2}$

Quotient Rule: $\boxed{\frac{b^n}{b^m} = b^{n-m}}$

· division of exponents with the same base turns into subtraction of exponent values

eg: $\frac{6^4}{6^3} = \frac{6 \cdot 6 \cdot 6 \cdot 6}{6 \cdot 6 \cdot 6}$
$= \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{6} \cdot \frac{6}{1}$
$= 1 \cdot 1 \cdot 1 \cdot 6 = 6^1 = 6^{4-3}$

The Power Rule: $\boxed{(b^n)^m = b^{n \cdot m}}$

if a raise an exponent to another exponent, I leave the base alone and multiply the exponent values

eg: $(3^2)^3 = 3^2 \cdot 3^2 \cdot 3^2$
$= (3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$
$= 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
$= 3^6 = 3^{3 \cdot 2}$

Raising a product to a power: $(a \cdot b)^n$

$\boxed{(a \cdot b)^n = a^n \cdot b^n}$ "exponentiation goes through multiplication"

eg: $(7 \cdot 9)^4 = (7 \cdot 9) \cdot (7 \cdot 9) \cdot (7 \cdot 9) \cdot (7 \cdot 9)$
$= 7 \cdot 9 \cdot 7 \cdot 9 \cdot 7 \cdot 9 \cdot 7 \cdot 9 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
$= 7^4 \cdot 9^4$

Raising a quotient to a power: $\left(\frac{a}{b}\right)^n$

$\boxed{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$

eg: $\left(\frac{19}{37}\right)^2 = \frac{19}{37} \cdot \frac{19}{37} = \frac{19 \cdot 19}{37 \cdot 37} = \frac{19^2}{37^2}$

Factors and Negative Exponents: $(a \cdot b)^{-n}$

$\boxed{(a \cdot b)^{-n} = \frac{1}{(a \cdot b)^n} = \frac{1}{a^n \cdot b^n}}$

Reciprocals and Negative Exponents: $\frac{b^{-n}}{b^{-m}} = b^{-n-(-m)} = b^{m-n}$

$\frac{b^{-n}}{b^{-m}} = b^{-n} \div b^{-m}$

$= \frac{1}{b^n} \div \frac{1}{b^m}$

$= \frac{1}{b^n} \cdot \frac{b^m}{1} = \boxed{\frac{b^m}{b^n}}$

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Use the rules of exponents to solve each of the following problems

4. $x^{1/2} \cdot x^{1/2} = x^{\frac{1}{2}+\frac{1}{2}}$

$= x^{2/2}$

$= x^{1} = \boxed{x}$

5. $x^{1/3} \cdot x^{1/3} \cdot x^{1/3} = x^{1/3 + 1/3 + 1/3}$

$= x^{3/3}$

$= x^{1} = \boxed{x}$

6. $(x^{1/2})^2 = x^{\frac{1}{2}\cdot\frac{2}{1}}$

$= x^{2/2}$

$= x^{1} = \boxed{x}$

7. $(x^{1/5})^5 = x^{\frac{1}{5}\cdot\frac{5}{1}}$

$= x^{5/5}$

$= x^{1} = \boxed{x}$

Use what you know about radicals to solve each of the following problems

8. $\sqrt{x}\sqrt{x} = x$

Recall that the square root of x, denoted at $\sqrt{x}$, is the number such that if we multiply $\sqrt{x}$ by itself twice we get back to radicand. This is what we've done in this problem.

9. $\sqrt[3]{x}\sqrt[3]{x}\sqrt[3]{x} = x$

Recall that the cube root of x, denoted as $\sqrt[3]{x}$, is the number such that if we multiply $\sqrt[3]{x}$ by itself thrice, we get back to the radicand

10. $(\sqrt{x})^2 = x$

Recall that $\sqrt[2]{x} = b$

$\Rightarrow b^2 = x$

$\Rightarrow (\sqrt{x})^2 = x$

11. $(\sqrt[5]{x})^5 = x$

Recall that if $\sqrt[5]{x} = b$

$\Rightarrow b^5 = x$

$\Rightarrow (\sqrt[5]{x})^5 = x$

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Identify the connection between radicals and exponential notation? fractional power.

index radical symbol

$$b = \sqrt[n]{a}$$

radicand

"b equals the nth root of a"

$$b = a^{\frac{1}{n}}$$

← denominator of fractional power (hint: index)

base a

"b equals a to the power of one divided by n"

Rewrite using radical notation:

8. $x^{\frac{1}{5}}$ ← "x to the one fifth power"

- variable base x
- fractional power: $\frac{1}{5}$
- denominator of fractional power is $n = 5$ (index)

$\Rightarrow \boxed{x^{\frac{1}{5}} = \sqrt[5]{x}}$

9. $w^{\frac{2}{3}}$ ← "variable w to the two thirds power"

- variable base w
- fractional power $\frac{2}{3}$
- denominator of fractional power is $n = 3$ (index)

$\Rightarrow w^{2/3} = w^{\frac{2}{1}\cdot\frac{1}{3}} = (w^2)^{1/3} = \boxed{\sqrt[3]{w^2}}$

Rewrite using exponent notation:

10. $\sqrt[5]{y}$ ← "the fifth root of y"

- index $n = 5$
- radicand y

$\Rightarrow \boxed{\sqrt[5]{y} = y^{\frac{1}{5}}}$

11. $\sqrt[7]{x^3}$ ← "the seventh root of variable x cubed"

- index $n = 7$
- radicand $x^3$

$\Rightarrow \sqrt[7]{x^3} = (x^3)^{1/7}$

$= x^{\frac{3}{1}\cdot\frac{1}{7}}$

$= \boxed{x^{3/7}}$

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Use your calculator to evaluate following mathematical expressions (with 6 digits after the decimal):

12. $\sqrt{8} \approx 2.828427$

13. $\frac{2\sqrt{18}}{3} \approx 2.828427$

14. $\sqrt[4]{64} \approx 2.828427$

Revisit the radicals above. Simplify use the strategies we discussed last time.

15. $\sqrt{8}$

$$= (8)^{1/2}$$
$$= (4 \cdot 2)^{1/2}$$
$$= 4^{1/2} \cdot 2^{1/2}$$
$$= \sqrt{4} \cdot \sqrt{2} = \boxed{2 \cdot \sqrt{2}} \checkmark$$

16. $\frac{2\sqrt{18}}{3}$

$$= \frac{2}{3} \cdot \sqrt{18}$$
$$= \frac{2}{3} \cdot (18)^{1/2}$$
$$= \frac{2}{3} \cdot (9 \cdot 2)^{1/2}$$
$$= \frac{2}{3} \cdot 9^{1/2} \cdot 2^{1/2}$$
$$= \frac{2}{3} \cdot \sqrt{9} \cdot \sqrt{2}$$
$$= \frac{2}{3} \cdot \frac{3}{1} \cdot \sqrt{2}$$
$$= \boxed{2 \cdot \sqrt{2}} \checkmark$$

17. $\sqrt[4]{64}$

$$= (64)^{1/4}$$
$$= (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)^{1/4}$$
$$= (2^6)^{1/4}$$
$$= 2^{6/4}$$
$$= 2^{3/2}$$
$$= 2^{1 + 1/2}$$
$$= 2^1 \cdot 2^{1/2}$$
$$= \boxed{2 \cdot \sqrt{2}} \checkmark$$

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Use rational exponents to simplify each of the following radical expressions .

24. $\sqrt[5]{\sqrt[2]{x}}$ $= \left(\sqrt[2]{x}\right)^{1/5}$

$= \left(x^{1/2}\right)^{1/5}$

$= x^{\frac{1}{2}\cdot\frac{1}{5}}$

$= x^{\frac{1}{10}}$

$\boxed{= \sqrt[10]{x}}$

25. $\sqrt[6]{(12x)^3}$ $= \left((12x)^3\right)^{1/6}$

$= (12x)^{3/6}$

$= (12x)^{1/2}$

$\boxed{= \sqrt[2]{12x}}$