

SECTION 8.6: Rational Equations (p. 612 – 619)

- Rational Equation (p. 612)
- To solve a rational equation (p. 612)
- Connecting the concepts: expressions versus equations (p. 617)

Use your understanding of fractions and the process of finding least common denominators to solve for x . If no solution exists, state this.

Note: $\frac{24}{9} = \frac{8 \cdot 8}{8 \cdot 3} = \frac{8}{3}$

1. $\frac{2}{3x} + \frac{1}{x} = 10$

$$\Rightarrow \frac{2}{3x} + \frac{1}{x} \cdot \frac{3}{3} = 10$$

$$\Rightarrow \frac{2}{3x} + \frac{3}{3x} = 10$$

$$\Rightarrow \frac{2+3}{3x} = 10$$

$$\Rightarrow \frac{5}{3x} = 10$$

$$\Rightarrow \frac{5}{1} \cdot \frac{1}{3x} = \frac{10}{1} \cdot \frac{3x}{1}$$

$$\Rightarrow \frac{5}{30} = \frac{30x}{30}$$

$$\Rightarrow \boxed{x = 1/6}$$

2. $\frac{x-1}{6x} = \frac{4}{9}$

$$\Rightarrow \frac{\cancel{6x}}{1} \cdot \frac{x-1}{\cancel{6x}} = \frac{4}{9} \cdot \frac{6x}{1}$$

$$\Rightarrow (x-1) = \frac{24x}{9} = \frac{8x}{3}$$

$$\Rightarrow \frac{3}{1} \cdot \frac{(x-1)}{1} = \frac{8x}{3} \cdot \frac{3}{1}$$

$$\Rightarrow 3(x-1) = 8x$$

$$\Rightarrow 3x - 3 = 8x$$

$$\Rightarrow -3 = 5x$$

$$\Rightarrow \boxed{x = -\frac{3}{5}}$$

$x^2 - 25 = (x-5)(x+5)$
 difference of squares

3. $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$ (solve this equation for variable x)

$$\Rightarrow \frac{2}{(x+5)} \cdot \frac{(x-5)}{(x-5)} + \frac{1}{(x-5)} \cdot \frac{(x+5)}{(x+5)} = \frac{16}{(x-5)(x+5)}$$

$$\Rightarrow \frac{2(x-5) + 1(x+5)}{(x+5)(x-5)} = \frac{16}{(x-5)(x+5)}$$

$$\Rightarrow 2(x-5) + 1(x+5) = 16$$

$$\Rightarrow 2x - 10 + x + 5 = 16$$

$$\Rightarrow 3x - 5 = 16$$

$$\Rightarrow 3x = 21 \Rightarrow \boxed{x = 7}$$

4. $\frac{1}{m-3} + 2 = \frac{1}{m-3}$

option 1: $-\frac{1}{m-3} - \frac{1}{m-3}$

$$\Rightarrow 2 = \frac{1}{m-3} - \frac{1}{m-3} \neq 0$$

NOSOLUTION

option 2:

$$\Rightarrow \frac{1}{m-3} + \frac{2(m-3)}{1(m-3)} = \frac{1}{m-3}$$

$$\Rightarrow \frac{1 + 2(m-3)}{m-3} = \frac{1}{m-3}$$

$$\Rightarrow 1 + 2(m-3) = 1$$

$$\Rightarrow 2(m-3) = 0$$

$$\Rightarrow 2m - 6 = 0$$

$$\Rightarrow 2m = 6 \Rightarrow \boxed{m \neq 3}$$

NOSOLUTION

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5. $\frac{1}{3} - \frac{1}{x+2} = \frac{x+14}{3x^2-12}$

$$\Rightarrow \frac{1}{3} \cdot \frac{(x+2)}{(x+2)} - \frac{3}{3} \cdot \frac{1}{x+2} = \frac{x+14}{3(x^2-4)}$$

$$\Rightarrow \frac{x+2}{3(x+2)} - \frac{3}{3(x+2)} = \frac{x+14}{3(x-2)(x+2)}$$

$$\Rightarrow \frac{(x+2) - 3}{3(x+2)} = \frac{x+14}{3(x+2)(x-2)}$$

$$\Rightarrow \frac{x-1}{3(x+2)} = \frac{x+14}{3(x+2)(x-2)}$$

$$\Rightarrow \frac{(x-1)}{3(x+2)} \cdot \frac{(x-2)}{(x-2)} = \frac{x+14}{3(x+2)(x-2)}$$

$$\Rightarrow (x-1) \cdot (x-2) = x+14$$

continued on scratch

7. Let $f(x) = x + \frac{6}{x}$. Find all values of a for which $f(a) = 5$.

output $y = f(x)$ input
function notation

$f(a) = 5$ ← output
input $x=a$

$$a + \frac{6}{a} = 5$$

$$a(a + \frac{6}{a}) = a \cdot 5$$

$$a^2 + 6 = 5a$$

$$a^2 - 5a + 6 = 0$$

$$(a-3) \cdot (a-2) = 0$$

$a=3$ or $a=2$ ✓

6. $\frac{x^2}{x-3} = \frac{9}{x-3}$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x^2 - 9 = 0$$

$$\Rightarrow (x-3) \cdot (x+3) = 0$$

$$\Rightarrow (x-3) = 0 \text{ or } x+3 = 0$$

$$\Rightarrow x \neq 3 \text{ or } \boxed{x = -3}$$

Extraneous Solution

$$\Rightarrow x^2 - 3x + 2 = x + 14$$

Let's get RHS equal 0

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$\Rightarrow (x - 6)(x + 2) = 0$$

$$\Rightarrow x - 6 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow \boxed{x = 6} \checkmark \text{ or } \boxed{x = -2}$$

Extraneous
Solution
↗

When solving rational equation
always check solutions
for validity