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SECTION 8.3: Addition, Subtraction and Least Common Denominators (p. 584 – 593)

- ☐ The sum of two rational expressions (p. 584)
- ☐ The difference of two rational expressions (p. 585)
- ☐ Least common multiple (LCM) (p. 586)
- ☐ Least common denominator (LCD) (p. 586)
- ☐ To find the Least common denominator (LCD) (p. 587)

Fundamental Principle of Fractions: $\frac{AB}{AC} = \frac{A}{A} \cdot \frac{B}{C} = 1 \cdot \frac{B}{C} = \frac{B}{C}$

Addition of Fractions: $\frac{A}{D} + \frac{B}{D} = \frac{A+B}{D}$

Addition of Fractions: $\frac{A}{D} - \frac{B}{D} = \frac{A-B}{D}$

For problems 1 – 6, start with the given expression and use a series of operations to create the **equivalent** expressions to combine these fractions together. Remember, you can change the way a number looks without changing the VALUE by multiplying or dividing by 1 (in any form you want).

1. $\frac{3a+5}{7} + \frac{11a+16}{7} = \frac{(3a+5)+(11a+16)}{7}$

common denominator

$= \frac{14a+21}{7}$

$= \frac{7(2a+3)}{7 \cdot 1}$

$= \frac{7}{7} \cdot \frac{2a+3}{1}$

$= \boxed{2a+3}$ ✓

2. $\frac{4x}{2(x^2-1)} - \frac{4}{2(x^2-1)}$

$= \frac{4x-4}{2(x^2-1)}$

$= \frac{4 \cdot (x-1)}{2 \cdot (x-1) \cdot (x+1)}$

$= \frac{4}{2} \cdot \frac{(x-1)}{(x-1)} \cdot \frac{1}{(x+1)}$

$= 2 \cdot 1 \cdot \frac{1}{x+1} = \boxed{\frac{2}{x+1}}$ ✓

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3. $\frac{t^2-5t}{t-1}+\frac{5t-t^2}{t-1}$

$= \frac{t^2-5t+5t-t^2}{t-1}$

$= \frac{(t^2-t^2)-5t+5t}{t-1}$

$= \frac{0+0}{t-1}$

$= \frac{0}{t-1}$

$= 0$

4. $\frac{2a^2+15}{a^2-7a+12}-\frac{11a}{a^2-7a+12}$

$= \frac{2a^2+15-11a}{a^2-7a+12}$

$= \frac{2a^2-11a+15}{a^2-7a+12}$

$= \frac{(2a-5)\cdot(a-3)}{(a-4)\cdot(a-3)}$

$= \frac{2a-5}{a-4}\cdot\frac{a-3}{a-3}$

$= \frac{2a-5}{a-4}\cdot 1$ $a\neq$

$= \frac{2a-5}{a-4}$

5. $\frac{t^2-3t}{t^2+6t+9}+\frac{2t+12}{t^2+6t+9}$

$= \frac{t^2-3t+2t+12}{(t+3)^2}$

$= \frac{t^2-t+12}{(t+3)^2}$

6. $\frac{5-3x}{x^2+3x-4}-\frac{x+1}{x^2+3x-4}$

$= \frac{(5-3x)-(x+1)}{x^2+3x-4}$

$= \frac{5-3x-x-1}{x^2+3x-4}$

$= \frac{4-4x}{(x-1)\cdot(x+4)}$

$= \frac{4(1-x}{(x+4)\cdot(x-$

$= \frac{4}{x+4}\cdot\frac{1-}{x-}$

$= \frac{-4}{x+4}$ $+1$

if $x\neq$

For problems 7 – 12, find the LCM between the two numbers using any method.

7. 12, 30

$12 = 2\cdot 2\cdot 3$

$30 = 2\cdot 3\cdot 5$

$LCM = 2\cdot 2\cdot 3\cdot 5$

$= 10\cdot 6 = 60$

8. 15, 50 150

$15 = 3\cdot 5$

$50 = 2\cdot 5\cdot 5$

$LCM = 2\cdot 3\cdot 5\cdot 5 = 150$

9. $2(y-3), 6(y-3)$

$2\cdot(y-3) = 2\cdot(y-3)$

$6\cdot(y-3) = 2\cdot 3\cdot(y-3)$

$LCM = 2\cdot 3\cdot(y-3)$

$= 6\cdot(y-3)$

10. x^2-4, x^2+5x+6

$(x+2)\cdot(x-2)\cdot(x+3)$

$x^2-4 = (x-2)\cdot(x+2)$

$x^2+5x+6 = (x+2)\cdot(x+3)$

$+3 \times +2 = 6$, $+5$

$LCM = (x-2)\cdot(x+2)\cdot(x+$

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OPTIONAL CHALLENGE PROBLEMS

11. $y^3 - y,\ y^4 - y^2$

$y^3 - y = y \cdot (y^2 - 1) = y(y-1)(y+1)$

$y^4 - y^2 = y^2(y^2 - 1) = y^2(y-1)(y+1)$

LCM = $\boxed{y^2 \cdot (y-1)(y+1)}$

12. $6x^3 - 24x^2 + 18x,\ 4x^5 - 24x^4 + 20x^3$

$6x^3 - 24x^2 + 18x = 6x \cdot (x^2 - 4x + 3)$

$= 6x \cdot (x-3)(x-1)$

$4x^5 - 24x^4 + 20x^3 = 4x^3 (x^2 - 6x + 5)$

$= 4x^3 (x-1)(x-5)$

LCM = $\boxed{24x^3 \cdot (x-1)(x-3)(x-5)}$

SECTION 8.4: Addition and Subtraction with Unlike Denominators (p. 593 – 601)

- ☐ The add or subtract rational expressions having different denominators (p. 593)
- ☐ When factors are opposite: (a – b) = -1 (b – a) (p. 596)

For problems 13 – 18, find the least common denominator. Then, add or subtract the fractions below.

13. $\frac{5x^2}{8} + \frac{7x}{12} = \frac{5x^2}{8} \cdot \frac{3}{3} + \frac{7x}{12} \cdot \frac{2}{2}$

No common denom

$= \frac{15x^2}{24} + \frac{14x}{24}$

$= \frac{15x^2 + 14x}{24}$

$= \boxed{\frac{x(15x+14)}{24}}$

14. $\frac{2a}{a^2-1} + \frac{1}{a^2+a} =$

no common denom

$= \frac{2a}{(a-1)(a+1)} \cdot \frac{a}{a} + \frac{1}{a(a+1)} \cdot \frac{(a-1)}{(a-1)}$

$= \frac{2a^2 + a - 1}{a(a+1)(a-1)}$

$= \frac{(2a-1)(a+1)}{a \cdot (a-1) \cdot (a+1)} = \frac{2a-1}{a \cdot (a-1)} \cdot \frac{a+1}{a+1}$

$= \boxed{\frac{2a-1}{a(a-1)}}$

15. $\frac{7}{x^2-64} + \frac{3}{x+8} =$

$= \frac{7}{(x-8)(x+8)} + \frac{3}{(x+8)} \cdot \frac{(x-8)}{(x-8)}$

$= \frac{7 + 3(x-8)}{(x+8)(x-8)}$

$= \frac{7 + 3x - 24}{(x+8)(x-8)}$

$= \boxed{\frac{3x - 17}{(x+8)(x-8)}}$

16. $\frac{5}{x^2-25} - \frac{x}{5x-25} =$

$= \frac{5}{(x-5)(x+5)} - \frac{x}{5(x-5)}$

$= \frac{5}{5} \cdot \frac{5}{(x-5)(x+5)} - \frac{x}{5(x-5)} \frac{(x+5)}{(x+5)}$

$= \frac{25 - x^2 - 5x}{5(x-5)(x+5)}$

$= \boxed{\frac{-x^2 - 5x + 25}{5(x-5)(x+5)}}$

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17. $\frac{x}{x+5} - \frac{2}{x-3} =$

$= \frac{x}{(x+5)} \cdot \frac{(x-3)}{(x-3)} - \frac{2}{(x-3)} \cdot \frac{(x+5)}{(x+5)}$

$= \frac{x^2 - 3x}{(x+5)\cdot(x-3)} - \frac{(2x+10)}{(x+5)\cdot(x-3)}$

$= \frac{x^2 - 3x - (2x+10)}{(x+5)\cdot(x-3)}$

$= \boxed{\frac{x^2 - 5x - 10}{(x+5)\cdot(x-3)}}$

18. $\frac{6}{x-2} + \frac{3}{2-x} =$

$= \frac{6}{(x-2)} + \frac{3}{(2-x)} \cdot \frac{-1}{-1}$

$= \frac{6}{(x-2)} + \frac{-3}{(x-2)}$

$= \frac{6 + -3}{x-2}$

$= \boxed{\frac{3}{x-2}}$

OPTIONAL CHALLENGE PROBLEMS

19. $\frac{1}{x+1} - \frac{x}{x-2} + \frac{x^2+2}{x^2-x-2}$

$= \frac{1}{(x+1)} - \frac{x}{(x-2)} + \frac{x^2+2}{(x-2)(x+1)}$

$= \frac{1}{(x+1)} \cdot \frac{(x-2)}{(x-2)} - \frac{x}{(x-2)} \cdot \frac{(x+1)}{(x+1)} + \frac{x^2+2}{(x-2)(x+1)}$

$= \frac{(x-2) - x\cdot(x+1) + x^2 + 2}{(x-2)\cdot(x+1)}$

$= \frac{x - 2 - x^2 - x + x^2 + 2}{(x-2)\cdot(x+1)}$

$= \frac{0}{(x-2)(x+1)} = \boxed{0}$ if $x \neq 2$, $x \neq -1$

20. $\frac{1}{x^2+5x+6} - \frac{2}{x^2+3x+2} + \frac{1}{x^2-3x-4}$

$= \frac{1}{(x+2)\cdot(x+3)} - \frac{2}{(x+1)(x+2)} + \frac{1}{(x-4)(x+1)}$

$= \ldots$ see next page

Problem 20) Let's find the optimal form of the LCM for our common denominator

$$x^2 + 5x + 6 = (x+2)\cdot(x+3)$$

$$x^2 + 3x + 2 = (x+1)\cdot(x+2)$$

$$x^2 - 3x - 4 = (x+1)\cdot(x-4)$$

Then, we see our LCM is given by

$$\text{LCM} = (x+1)\cdot(x+2)\cdot(x+3)\cdot(x-4)$$

Let's use this to find our common denominator

$$= \frac{1}{(x+2)\cdot(x+3)} - \frac{2}{(x+1)(x+2)} + \frac{1}{(x-4)(x+1)}$$

$$= \frac{1}{(x+2)\cdot(x+3)}\cdot\frac{(x+1)}{(x+1)}\cdot\frac{(x-4)}{(x-4)} - \frac{2}{(x+1)(x+2)}\cdot\frac{(x+3)}{(x+3)}\cdot\frac{(x-4)}{(x-4)} + \frac{1}{(x+1)(x-4)}\cdot\frac{(x+2)}{(x+2)}\cdot\frac{(x+3)}{(x+3)}$$

$$= \frac{(x^2 - 3x - 4) - 2\cdot(x^2 - x - 12) + (x^2 + 5x + 6)}{(x+2)\cdot(x+3)\cdot(x+1)(x-4)}$$

$$= \frac{x^2 - 3x - 4 - 2x^2 + 2x + 24 + x^2 + 5x + 6}{(x+1)(x+2)(x+3)(x-4)}$$

$$= \frac{(x^2 - 2x^2 + x^2) + (-3x + 2x + 5x) + (-4 + 24 + 6)}{(x+1)(x+2)(x+3)(x-4)}$$

$$= \frac{0 + 4x + 26}{(x+1)(x+2)(x+3)(x-4)}$$

$$= \boxed{\frac{4x + 26}{(x+1)(x+2)(x+3)(x-4)}}$$

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SECTION 8.4: Addition and Subtraction with Unlike Denominators (p. 593 – 601)

- ☐ The add or subtract rational expressions having different denominators (p. 593)
- ☐ When factors are opposite: (a – b) = -1 (b – a) (p. 596)

For problems 1 – 8, find the least common denominator. Then, add or subtract the fractions below.

1. $\frac{5}{x^2-25} - \frac{x}{5x-25}$

$= \frac{5}{(x-5)\cdot(x+5)} - \frac{x}{5(x-5)}$

$= \frac{5}{(x-5)(x+5)} \cdot \frac{5}{5} - \frac{x}{5\cdot(x-5)} \cdot \frac{(x+5)}{(x+5)}$

$= \frac{25 - (x^2+5x)}{5\cdot(x-5)\cdot(x+5)}$

$= \frac{25 - x^2 - 5x}{5\cdot(x+5)\cdot(x-5)} = \boxed{\frac{-x^2-5x+25}{5\cdot(x+5)\cdot(x-5)}}$

2. $\frac{x}{x+5} - \frac{2}{x-3}$

$= \frac{x}{(x+5)} \cdot \frac{(x-3)}{(x-3)} - \frac{2}{(x-3)} \cdot \frac{(x+5)}{(x+5)}$

$= \frac{x\cdot(x-3) - 2\cdot(x+5)}{(x-3)\cdot(x+5)}$

$= \frac{x^2-3x-2x-10}{(x-3)\cdot(x+5)}$

$= \boxed{\frac{x^2-5x-10}{(x-3)\cdot(x+5)}}$

3. $\frac{6}{x-2} + \frac{3}{2-x}$ Notice: $2-x = -x+2 = -1\cdot(x-2)$

$= \frac{6}{x-2} + \frac{3}{-x+2}$

$= \frac{6}{x-2} + \frac{-1}{-1} \cdot \frac{3}{(-x+2)}$

$= \frac{6}{x-2} + \frac{-3}{x-2}$

$= \frac{6-3}{x-2} = \boxed{\frac{3}{x-2}}$

4. $\frac{c-5}{c^2-64} - \frac{5-c}{64-c^2}$ Notice: $c^2-64 = -64+c^2 = -1\cdot(64-c^2)$

$= \frac{(c-5)}{(c^2-64)} - \frac{(5-c)}{(64-c^2)} \cdot \frac{-1}{-1}$

$= \frac{(c-5)}{(c^2-64)} - \frac{(c-5)}{(c^2-64)}$

$= \frac{(c-5) - (c-5)}{c^2-64}$

$= \frac{c-5-c+5}{c^2-64}$

$= \frac{0}{c^2-64} = \boxed{0 \text{ if } x \neq 8, -8}$

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5. $\frac{3a}{4a-20}+\frac{9a}{6a-30}$

$= \frac{3a}{4\cdot(a-5)} + \frac{9a}{6\cdot(a-5)}$

$= \frac{3a}{4\cdot(a-5)}\cdot\frac{3}{3} + \frac{9a}{6\cdot(a-5)}\cdot\frac{2}{2}$

$= \frac{9a}{12\cdot(a-5)} + \frac{18a}{12\cdot(a-5)}$

$= \frac{9a+18a}{12\cdot(a-5)} = \frac{27a}{12\cdot(a-5)} = \boxed{\frac{9a}{4\cdot(a-5)}}$

6. $\frac{t-3}{t^3-1}-\frac{2}{1-t^3}$

$= \frac{(t-3)}{(t^3-1)} - \frac{2}{(1-t^3)}\cdot\frac{-1}{-1}$

$= \frac{t-3}{t^3-1} - \frac{-2}{t^3-1}$

$= \frac{t-3-(-2)}{t^3-1}$

$= \frac{t-3+2}{t^3-1}$

$= \frac{t-1}{t^3-1} = \frac{(t-1)\cdot 1}{(t-1)(t^2+t+1)} = \boxed{\frac{1}{t^2+t+1} \text{ if } t\neq 1}$

OPTIONAL CHALLENGE PROBLEMS

7. $\frac{2x}{x^2-4}+\frac{5}{2-x}-\frac{1}{x+2}$

$= \frac{2x}{(x-2)\cdot(x+2)} + \frac{5}{(2-x)} - \frac{1}{(x+2)}$

$= \frac{2x}{(x-2)(x+2)} + \frac{5}{(2-x)}\cdot\frac{-1}{-1}\cdot\frac{(x+2)}{(x+2)} - \frac{1}{(x+2)}\cdot\frac{(x-2)}{(x-2)}$

$= \frac{2x}{(x-2)\cdot(x+2)} + \frac{-5(x+2)}{(x-2)\cdot(x+2)} - \frac{1\cdot(x-2)}{(x+2)\cdot(x-2)}$

$= \frac{2x-5\cdot(x+2)-(x-2)}{(x-2)\cdot(x+2)}$

$= \frac{2x-5x-10-x+2}{(x-2)\cdot(x+2)}$

$= \frac{-4x-8}{(x-2)\cdot(x+2)} = \frac{-4\cdot(x+2)}{(x-2)\cdot(x+2)} = \boxed{\frac{-4}{(x-2)} \text{ if } x\neq -2}$

8. $\frac{x+5}{x+3}+\frac{x+7}{x+2}-\frac{7x+19}{x^2+5x+6}$

• For solution, see next pages.

$$= \frac{(x+5)}{(x+3)} + \frac{(x+7)}{(x+2)} - \frac{7x+19}{(x+2)\cdot(x+3)}$$

$$= \frac{(x+5)}{(x+3)} \cdot \frac{(x+2)}{(x+2)} + \frac{(x+7)}{(x+2)} \cdot \frac{(x+3)}{(x+3)} - \frac{7x+19}{(x+2)\cdot(x+3)}$$

$$= \frac{(x^2+7x+10) + (x^2+10x+21) - (7x+19)}{(x+2)\cdot(x+3)}$$

$$= \frac{(x^2+x^2) + (7x+10x-7x) + (10+21-19)}{(x+2)\cdot(x+3)}$$

$$= \frac{2x^2+10x+12}{(x+2)\cdot(x+3)}$$

$$= \frac{2\cdot(x^2+5x+6)}{(x+2)\cdot(x+3)}$$

$$= \frac{2}{1} \cdot \frac{(x+2)\cdot(x+3)}{(x+2)\cdot(x+3)}$$

$$= \boxed{2 \quad \text{if} \quad x \neq -2, -3}$$