

Math 105 Exam 2 Review Solutions

1-3 Simplify, state values where undefined

$$\begin{aligned} \textcircled{1} \quad & \frac{8x^2}{24x}, x \neq 0 \\ & = \frac{\cancel{8} \cdot x \cdot \cancel{x}}{\cancel{8} \cdot 3 \cdot \cancel{x}}, x \neq 0 \\ & = \frac{x}{3}, x \neq 0 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad & \frac{6x+12}{7x+14} \\ & = \frac{6(x+2)}{7(x+2)}, x \neq -2 \\ & = \frac{6}{7}, x \neq -2 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad & \frac{x^2-1}{x^2+3x+2} \\ & = \frac{(x+1)(x-1)}{(x+1)(x+2)}, x \neq -1, -2 \\ & = \frac{x-1}{x+2}, x \neq -1, -2 \end{aligned}$$

4-11 Perform the indicated operation

$$\begin{aligned} \textcircled{4} \quad & \frac{3}{x-1} + \frac{2}{(x-1)^2} \\ & = \frac{3(x-1)}{(x-1)^2} + \frac{2}{(x-1)^2} \\ & = \frac{3x-1}{(x-1)^2} + \frac{2}{(x-1)^2} \\ & = \frac{3x-1+2}{(x-1)^2} \\ & = \frac{3x+1}{(x-1)^2}, x \neq 1 \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad & \frac{10x+20}{2x^2-3x+1} \cdot \frac{x^2-1}{5x+10} \\ & = \frac{10(x+2)}{(2x-1)(x-1)} \cdot \frac{(x-1)(x+1)}{5(x+2)} \\ & = \frac{20(x+2)(x-1)(x+1)}{5(2x-1)(x-1)(x+2)} \\ & = \frac{2(x+1)}{2x-1}, x \neq \frac{1}{2}, 1, -2 \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad & (x+2) \div \frac{x+3}{x-1} \\ & = \frac{x+2}{1} \cdot \frac{x-1}{x+3} \\ & = \frac{(x+2)(x-1)}{x+3}, x \neq 1, -3 \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad & \frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2} \\ & = \frac{x}{(x+2)(x+3)} - \frac{2}{(x+2)(x+1)} \\ & = \frac{x(x+1)}{(x+1)(x+2)(x+3)} - \frac{2(x+3)}{(x+2)(x+1)(x+3)} \\ & = \frac{x^2+x}{(x+1)(x+2)(x+3)} - \frac{2x+6}{(x+2)(x+1)(x+3)} \\ & = \frac{x^2+x-(2x+6)}{(x+1)(x+2)(x+3)} \\ & = \frac{x^2+x-2x-6}{(x+1)(x+2)(x+3)} \\ & = \frac{x^2-x-6}{(x+1)(x+2)(x+3)} \\ & = \frac{(x-3)(x+2)}{(x+1)(x+2)(x+3)} \\ & = \frac{x-3}{(x+1)(x+3)}, x \neq -1, -2, -3 \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad & \frac{5x}{x-7} - \frac{3x}{7-x} \\ & = \frac{5x}{x-7} - \frac{(-1)(3x)}{(-1)(7-x)} \\ & = \frac{5x}{x-7} - \frac{-3x}{-7+x} \\ & = \frac{5x}{x-7} - \frac{-3x}{x-7} \\ & = \frac{5x - (-3x)}{x-7} \\ & = \frac{5x+3x}{x-7} \\ & = \frac{8x}{x-7}, x \neq 7 \end{aligned}$$

$$\begin{aligned} \textcircled{9} \quad & \frac{2a}{a^2-1} + \frac{1}{a^2+a} \\ & = \frac{2a}{(a+1)(a-1)} + \frac{1}{a(a+1)} \\ & = \frac{(2a)(a)}{(a+1)(a-1)(a)} + \frac{1 \cdot (a-1)}{a(a+1)(a-1)} \\ & = \frac{2a^2}{(a+1)(a-1)(a)} + \frac{a-1}{a(a+1)(a-1)} \\ & = \frac{2a^2 + (a-1)}{(a+1)(a-1)(a)} \\ & = \frac{2a^2+a-1}{a(a+1)(a-1)} \\ & = \frac{(2a-1)(a+1)}{a(a+1)(a-1)} \\ & = \frac{2a-1}{a(a-1)}, a \neq 0, 1, -1 \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad & \frac{a^2+3a+2}{a^2+4} \div (5a^2+10a) \\ & = \frac{a^2+3a+2}{a^2+4} \cdot \frac{1}{5a^2+10a} \\ & = \frac{(a+1)(a+2)}{a^2+4} \cdot \frac{1}{5a(a+2)} \\ & = \frac{(a+1)(\cancel{a+2})}{(a^2+4)(5a)(\cancel{a+2})} \\ & = \frac{a+1}{(5a)(a^2+4)}, a \neq -2, 0 \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad & \frac{x^2-2x-3}{x^2-4} \cdot \frac{x+5}{x+1} \\ & = \frac{(x-3)(x+1)}{(x-2)(x+2)} \cdot \frac{(x+5)}{(x+1)} \\ & = \frac{(x-3)(\cancel{x+1})(x+5)}{(x-2)(x+2)(\cancel{x+1})} \\ & = \frac{(x-3)(x+5)}{(x-2)(x+2)}, x \neq -1, -2, 2 \end{aligned}$$

12-13. Solve each of the following equations with rational expressions

12. $1 + \frac{3}{x} = \frac{28}{x^2}$ $x \neq 0$ LCD = x^2

$$\frac{1 \cdot x^2}{1 \cdot x^2} + \frac{3 \cdot x}{x \cdot x} = \frac{28}{x^2}$$

$$\frac{x^2}{x^2} + \frac{3x}{x^2} = \frac{28}{x^2}$$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x + 7)(x - 4) = 0$$

$$x + 7 = 0 \text{ or } x - 4 = 0$$

$$x = -7 \text{ or } x = 4$$

13. $\frac{36}{p^2 - 9} = \frac{2p}{p + 3} - 1$

$$\frac{36}{(p+3)(p-3)} = \frac{2p}{p+3} - 1 \quad p \neq 3, -3 \text{ LCD: } (p+3)(p-3)$$

$$\frac{36}{(p+3)(p-3)} = \frac{2p(p-3)}{(p+3)(p-3)} - \frac{1(p+3)(p-3)}{1(p+3)(p-3)}$$

$$\frac{36}{(p+3)(p-3)} = \frac{2p^2 - 6p}{(p+3)(p-3)} - \frac{p^2 - 9}{(p+3)(p-3)}$$

$$36 = 2p^2 - 6p - p^2 + 9$$

$$36 = p^2 - 6p + 9$$

$$0 = p^2 - 6p - 27$$

$$0 = (p - 9)(p + 3)$$

$$p - 9 = 0 \text{ or } p + 3 = 0$$

$$p = 9$$

$$p = -3$$

is an extraneous solution it makes the denominator zero

only solution $p = 9$

14-16. Evaluate each of the following.

14. $\sqrt{(-3)^2} = \sqrt{9} = 3$

15. $\sqrt[3]{-8} = -2$

16. $\sqrt[4]{-16} = \text{DNE}$
does not exist.

17-19. Use your calculator to approximate each of the following. Round your answer to 4 decimal places.

17. $\sqrt{15} = 3.8730$

18. $\sqrt[3]{-19} = -2.6684$

19. $\sqrt{-39} = -1.6877$

20-22. Simplify each of the following radical expressions. Use absolute value notation when appropriate.

20. $\sqrt{25x^2} = \sqrt{25} \sqrt{x^2}$
 $= 5|x|$

21. $\sqrt[3]{27x^3} = \sqrt[3]{27} \sqrt[3]{x^3}$
 $= 3x$

22. $\sqrt[5]{32y^{10}x^7} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot y^5 \cdot y^5 \cdot x^5 \cdot x^2}$
 $= \sqrt[5]{2^5} \sqrt[5]{y^5} \sqrt[5]{y^5} \sqrt[5]{x^5} \sqrt[5]{x^2}$
 $= 2 \cdot y \cdot y \cdot x \cdot \sqrt[5]{x^2}$
 $= 2y^2x\sqrt[5]{x^2}$

23. Write the following using radical notation

$$2x^{3/5} = 2 \cdot x^{3/5} = 2 \cdot (x^3)^{1/5} = 2\sqrt[5]{x^3}$$

24. Write the following using rational exponents

$$\sqrt[3]{x^5} = (x^5)^{1/3} = |x^{5/3}|$$

25-27. Simplify each of the following

25. $\sqrt[3]{\sqrt{x}} = \sqrt[3]{x^{1/2}}$
 $= (x^{1/2})^{1/3}$
 $= x^{1/6}$

26. $y^{2/5} y^{3/5} = y^{2/5 + 3/5}$
 $= y^{5/5}$
 $= y^1 \text{ or } y$

27. $\left(\frac{\sqrt[3]{x^{-2}}}{\sqrt[4]{y^{-3}}}\right)^{12} = \left(\frac{(x^{-2})^{1/3}}{(y^{-3})^{1/4}}\right)^{12}$
 $= \left(\frac{x^{-2}}{y^{-3/4}}\right)^{12}$
 $= \left(\frac{y^{3/4}}{x^2}\right)^{12}$
 $= \frac{(y^{3/4})^{12}}{(x^2)^{12}} = \frac{y^9}{x^{24}} = \frac{y^9}{x^{24}}$

28-30. Perform the indicated operations and simplify if possible. Assume that the radicand was not formed by raising negative quantities to even powers.

$$\begin{aligned} (28) \quad \frac{\sqrt{125p^3}}{\sqrt{5p}} &= \sqrt{\frac{125p^3}{5p}} \\ &= \sqrt{25p^2} \\ &= \boxed{5p} \end{aligned}$$

$$\begin{aligned} (29) \quad (3\sqrt{2x})(5\sqrt{6x^3}) &= (3)(\sqrt{2x})(5)(\sqrt{6x^3}) \\ &= (3)(5)(\sqrt{2x})(\sqrt{6x^3}) \\ &= 15\sqrt{(2x)(6x^3)} \\ &= 15\sqrt{12x^4} \\ &= 15\sqrt{4 \cdot 3 \cdot x^2 \cdot x^2} \\ &= 15\sqrt{4} \sqrt{3} \cdot \sqrt{x^2} \sqrt{x^2} \\ &= 15 \cdot 2 \cdot \sqrt{3} \cdot x \cdot x \\ &= \boxed{30x^2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} (30) \quad (\sqrt{5} - 3\sqrt{2})(2\sqrt{5} - \sqrt{2}) &= (\sqrt{5})(2\sqrt{5}) - (\sqrt{5})(\sqrt{2}) - (3\sqrt{2})(2\sqrt{5}) + (3\sqrt{2})(\sqrt{2}) \\ &= 2\sqrt{5} \cdot 5 - \sqrt{5} \cdot 2 - 6\sqrt{2} \cdot 5 + 3\sqrt{2} \cdot 2 \\ &= 2\sqrt{25} - \sqrt{10} - 6\sqrt{10} + 3\sqrt{4} \\ &= 2 \cdot 5 - \sqrt{10} - 6\sqrt{10} + 3 \cdot 2 \\ &= 10 - \sqrt{10} - 6\sqrt{10} + 6 \\ &= 10 + 6 - \sqrt{10} - 6\sqrt{10} \\ &= \boxed{16 - 7\sqrt{10}} \end{aligned}$$

31-32. Solve each of the following equations with radical expressions.

$$\begin{aligned} (31) \quad (w-1)^{1/3} - 2 &= 2 \\ \sqrt[3]{w-1} - 2 &= 2 \\ \sqrt[3]{w-1} &= 4 \\ (\sqrt[3]{w-1})^3 &= (4)^3 \\ w-1 &= 64 \\ \boxed{w} &= \boxed{65} \end{aligned}$$

check:

$$\begin{aligned} (65-1)^{1/3} - 2 &\stackrel{?}{=} 2 \\ (64)^{1/3} - 2 &\stackrel{?}{=} 2 \\ 4 - 2 &= 2 \checkmark \end{aligned}$$

$$\begin{aligned} (32) \quad y + \sqrt{y-2} &= 8 \\ \sqrt{y-2} &= 8-y \\ (\sqrt{y-2})^2 &= (8-y)^2 \\ y-2 &= 64 - 16y + y^2 \\ y &= 66 - 16y + y^2 \\ 0 &= 66 - 17y + y^2 \\ 0 &= y^2 - 17y + 66 \\ 0 &= (y-11)(y-6) \\ y-11=0 \text{ or } y-6=0 \\ y &= 11 \text{ or } y=6 \\ \swarrow & \\ \text{extraneous} & \\ \text{solution} & \\ \boxed{y=6} & \text{ is the only solution} \end{aligned}$$

check:

$$\begin{aligned} 11 + \sqrt{11-2} &\stackrel{?}{=} 8 \\ 11 + \sqrt{9} &\stackrel{?}{=} 8 \\ 11 + 3 &\neq 8 \\ 6 + \sqrt{6-2} &\stackrel{?}{=} 8 \\ 6 + \sqrt{4} &= 8 \\ 6 + 2 &= 8 \checkmark \end{aligned}$$