ENGR 11 : SAMPLE EXAM 1 Solutions

1. Consider the following snippet of code:

```
1    x = [64:-8:8];
2    y = linspace(4,32,8);
3    I8 = eye(9,9); I8(:,9) = []; I8(9,:) = [];
4    A = [zeros(1,8); ones(1, 8); linspace(2,2,8)];
5    B = I8(:, [4, 8]);
6    C = [y;  A(3,:); B; x];
```

When we execute the code above in MATLAB, we produce an error.

A. (1 points) Which line of this code causes the error? What is the problem with this line?

Line 6 : $B \in \mathbb{R}^{8\times 2}$ while $\vec{y}, \vec{x}, A(3,:) \in \mathbb{R}^{1\times 8}$

B. (1 points) Modify the line of code that contains the error using a single transpose operator ' so that when we execute this code, the matrix $C \in \mathbb{R}^{5\times 8}$ is successfully saved in memory without errors.

```
C = [y; A(3,:); B' ; x];
```

C. (2 points) Assuming you've corrected the error, write the entry-by-entry definition of C.

$$C = \begin{bmatrix} 4 & 8 & 12 & 16 & 20 & 24 & 28 & 32 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 64 & 56 & 48 & 40 & 32 & 24 & 16 & 8 \end{bmatrix}$$

Note: linspace(4,32,8)

4 8 12 16 20 24 28 32

$\Delta = \frac{32-4}{7} = \frac{28}{7}$

2. Consider the following decimal number

$$x = 2^0 + 2^2 + 2^3 + 2^5 + 2^6 + 2^9 + 2^{10} + 2^{12} = (5741)_{10}$$

A. (3 points) Convert this number into an unsigned binary representation. Explain your work.

Descending order: $x = 1\cdot 2^{12} + 0\cdot 2^{11} + 1\cdot 2^{10} + 1\cdot 2^{9} + 0\cdot 2^{8} + 0\cdot 2^{7} + 1\cdot 2^{6} + 1\cdot 2^{5} + \cdots$
$+ 0\cdot 2^{4} + 1\cdot 2^{3} + 1\cdot 2^{2} + 0\cdot 2^{1} + 1\cdot 2^{0}$

$$x = (1|0110|0110|1101)_2$$

$$= (b_{12}\ b_{11}\ b_{10}\ b_9\ b_8\ b_7\ b_6\ b_5\ b_4\ b_3\ b_2\ b_1\ b_0)_2$$

$$= \sum_{k=0}^{12} b_k \cdot 2^k$$

B. (3 points) While explaining your thinking, please predict the output of the following code:

```
x = 5741;
format hex
y = uint16(x)
```

$$y = (0001\ 0110\ 0110\ 1101)_2$$

$$\Rightarrow y = (1\ 6\ 6\ d)_{16}$$

$$= 1\cdot 16^3 + 6\cdot 16^2 + 6\cdot 16^1 + d\cdot 16^0$$

$$= 1\cdot 4096 + 6\cdot 256 + 6\cdot 16 + 13\cdot 1$$

$$= 4096 + 1536 + 96 + 13$$

$$= 5741\ \checkmark$$

3. (14 points) Please fill out the following table of values. The first row of the table is completed for you.

Exam 1, Problem 3: Bounds for Integer data types in MATLAB

Data Type	Lowerbound in hexadecimal	Lowerbound in decimal	Upperbound in decimal	Upperbound in hexadecimal
uint8	00	0	$2^8 - 1 = 255$	ff
uint16	0000	0	$2^{16} - 1$	ff ff
uint32	0000 0000	0	$2^{32} - 1$	ffff ffff
uint64	0000 0000 0000 0000	0	$2^{64} - 1$	ffff ffff ffff ffff
int8	80	-2^7	$2^7 - 1$	7f
int16	8000	-2^{15}	$2^{15} - 1$	7f ff
int32	8000 000	-2^{31}	$2^{31} - 1$	7fff ffff
int64	8000 0000 0000 0000	-2^{63}	$2^{63} - 1$	7fff ffff ffff ffff

4. (4 points) Convert the unsigned hexadecimal integer $x = (abcd)_{16}$ into the unsigned binary and decimal equivalents. Show your work and explain your steps.

$$x = (abcd)_{16}$$

$$= a \cdot 16^3 + b \cdot 16^2 + c \cdot 16^1 + d \cdot 16^0$$

$$= 10 \cdot 16^3 + 11 \cdot 16^2 + 12 \cdot 16 + 13 \cdot 16^0$$

$$= 10 \cdot 4096 + 11 \cdot 256 + 12 \cdot 16 + 13 \cdot 1$$

$$= 40{,}960 + 2816 + 192 + 13$$

$$= (43{,}981)_{10}$$

We also know that in binary, we have

$$x = (1010\ 1011\ 1100\ 1101)_2$$

5. Consider the following lines of code:

```
x = int16(151); y = int16(-151);
format hex
x, y
```

✓ A. (6 points) Write the output that is generated by this code.

Note: $151 = 128 + 16 + 4 + 2 + 1$

$= 2^7 + 2^4 + 2^2 + 2^1 + 2^0$

$= (10010111)_2$ ← unsigned binary encoding

$= (0000\ 0000\ 1001\ 0111)_{2C}$ ← 16-bit range expansion (recall for nonnegative integers, we know the twos complement encoding is identical to the unsigned binary encoding)

$\boxed{0\ 0\ 9\ 7}$ ← output from x in command window

☐ see scratch paper for y output.

B. (2 points)Suppose we attempt to store x using MATLAB's `int8` data class. What happens? Why?

Over flow :

x1 = int8(151); y1 = int8(-151)

format hex

x, y

7f ← output from x

80 ← output from y

6. What decimal value does the raw, uninterpreted 10−bit binary string 10 1011 1101 have if:

A. (3 points) we interpret this number as an unsigned binary integer?

$x = (10\ 1011\ 1101)_2$

value box

2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	1	1	1	0	1

$= 2^9 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$

$= 512 + 128 + 32 + 16 + 8 + 4 + 1$

$\boxed{= (701)_{10}}$

B. (3 points) we interpret this number as a signed integer in signed-magnitude representation?

$x = (10\ 1011\ 1101)_{SM}$

value box

-1	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	0	1	0	1	1	1	1	0	1

$= -1 \cdot (2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0)$

$= -1\ (128 + 32 + 16 + 8 + 4 + 1)$

$= -1 \cdot 189$ $\boxed{= (-189)_{10}}$

C. (3 points) we interpret this number as a signed integer in twos complemment representation?

$x = (10\ 1011\ 1101)_{2C}$

value box

-2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	1	0	1	1	1	1	0	1

$= -2^9 + 2^7 + 2^5 + 2^4 + 2^3 + 2^2 + 2^0$

$= -512 + 189$

$\boxed{= (-323)_{10}}$

Problem 5 continued ...

Let $B = (0000\ 0000\ 1001\ 1011)_{2C} = (151)_{10}$

To find $y = -151$, we complete the following steps:

Step 1: Take the bit-wise complement of B

$\bar{B} = \bar{b}_{15}\,\bar{b}_{14} \cdots \bar{b}_2\,\bar{b}_1\,\bar{b}_0$

$= \overline{0}\overline{0}\overline{0}\overline{0}\ \overline{0}\overline{0}\overline{0}\overline{0}\ \overline{1}\overline{0}\overline{0}\overline{1}\ \overline{0}\overline{1}\overline{1}\overline{1}$

$= 1111\ 1111\ 0110\ 1000$

Step 2: Calculate $\bar{B}+1$ as an unsigned binary add

$$
\begin{array}{rr}
\bar{B}+1 = & 1111\ 1111\ 0110\ 1000 \\
+ & 1 \\
\hline
= & \boxed{1111\ 1111\ 0110\ 1001}
\end{array}
$$

Step 3: Convert to hex $= \boxed{(ff69)}$ ← output in command window for Y

7. (5 points) Suppose $x = -1305$. Let $m \in \mathbb{N}$ be the smallest number of bits that can store x in twos complement, where m can be any positive integer and is not necessarily a power of 2. With this in mind, find m. Explain your reasoning.

$$x = -1305 \Rightarrow |x| = (+1305)_{10}$$

$$= 1024 + 256 + 16 + 8 + 1$$

$$= 2^{10} + 2^{8} + 2^{4} + 2^{3} + 2^{0}$$

$$= (101\ 0001\ 1001)_2$$

$$= (0101\ 0001\ 1001)_{2C}$$

$\Rightarrow$ if $B = |x| = 0101\ 0001\ 1001$, then we can negate B by finding

$$\begin{array}{rcl} \overline{B} & & 1010\ 1110\ 0110 \\ +\ 1 & & +\qquad\qquad\qquad 1 \\ \hline -x & = & (1010\ 1110\ 0111)_{2C} \end{array}$$

$\Rightarrow$ $\boxed{m = 12 \text{ bits}}$

Challenge Problem

8. (Optional, Extra Credit, Challenge Problem: 5 points) Recall that in Lesson 4, we stated that to negate a signed integer $x \in \mathbb{Z}$ that is stored in twos complement, we take the bitwise complement of each bit of the twos complement representation of x and then add 1. With this in mind, determine if the following statement is true or false. Explain your reasoning and be sure to use the Lesson 4 notation for twos complement representation.

   Conjecture: For a signed integer $x \in \mathbb{Z}$ that is stored using an $m-$bit twos complement representation given by

   $$x = (b_{m-1}b_{m-2}...b_2b_1b_0)_{2C}$$

   we can negate x by treating the bit string $b_{m-1}b_{m-2}...b_2b_1b_0$ as an unsigned integer and calculating the difference

   $$2^m - b_{m-1}b_{m-2}...b_2b_1b_0.$$

   The resulting binary string is the twos complement representation of $-x$.

Exam 1, Problem 7 continued...

Let's convert $|x| = +1305$ to binary using our alternative conversion algorithm from pp. 32 – 34 in Lesson 3:

Let $N_0 = 1305$.

Digit Number	Integer Division	Remainder	Position
LSB → 1	$\frac{N_0}{2} = \frac{1305}{2} = 652$	$\boxed{1}$	0 → position 0 ⇒ scaled by 2^0
2	$\frac{652}{2} = 326$	0	1
3	$\frac{326}{2} = 163$	0	2
4	$\frac{163}{2} = 81$	1	3
5	$\frac{81}{2} = 40$	1	4
6	$\frac{40}{2} = 20$	0	5
7	$\frac{20}{2} = 10$	0	6
8	$\frac{10}{2} = 5$	0	7
9	$\frac{5}{2} = 2$	1	8
10	$\frac{2}{2} = 1$	0	9
MSB → 11	$\frac{1}{2} = 0$	$\boxed{1}$	10

position 10 ⇒ Scaled by 2^{10}