Name : ____________________ **Class Number:** _____

ENGR 11: Lesson 4 Suggested Problems

Theoretic Problems: Discussed in notes

---

1. **Review of essential terms and concepts**

   A. How do we add unsigned binary integers? When doing so, what is a *carry bit*?

   B. If we add two $m-$bit unsigned binary integers, how many bits might the sum be?

   C. What does the term *overflow* refer to?

   D. What happens if we try to add $(215 + 89)_{10}$ using MATLAB's `uint8` data class? Why does this happen? What can we do to make sure our addition is calculated properly?

   E. What does the term *range expansion* refer to? How can we use MATLAB's `uint` data types to properly expand the range of unsigned integers? How do we know when we should do such an operation?

   F. What are signed integers?

   G. What is the *signed-magnitude representation* for signed integers? What are the two major drawbacks of this signed-magnitude representation?

   H. What is the *twos-complement representation* for signed integers?

   I. Does MATLAB use the signed-magnitude or twos complement representation for the four `int` data types? Why? How do you know this?

   J. What algorithm can we use to convert signed decimal integers into twos complement? Describe the steps in detail.

   K. What algorithm can we use to negate signed integers in twos complement form? Describe the steps in detail.

   L. How can we use each of the following data types in MATLAB: `int8`, `int16`, `int32`, `int64`?

   M. How does range expansion work in twos complement?

Problems Solved in Jeff's Notes

---

2. Complete each of the following sums in both decimal and binary

   ☐ Example 4.1: $(139 + 91)_{10}$

   ☐ Example 4.2: $(215 + 89)_{10}$

---

3. Recall the general notation for an $(m + 1)-$bit twos complement representation of the signed integer $x \in \mathbb{Z}$ given by $x = (b_m b_{m-1} ... b_2 b_1 b_0)_{2C}$ where each digit b_i is either 0 or 1, if we interpreted x as an unsigned binary integer, then we could write the following sum

$$x = -b_m \cdot 2^m + b_{m-1} \cdot 2^{m-1} + \cdots + b_1 \cdot 2^1 + b_0 \cdot 2^0 = -b_m \cdot 2^m \sum_{i=0}^{m-1} b_i \cdot 2^i$$

   Explicitly convert each of the following twos complement representations into decimal form.

   ☐ Example 4.3: $x = (1000\,0000)_{2C}$

- ☐ Example 4.4: $x = (1111\,1111)_{2C}$
- ☐ Example 4.5: $x = (0101\,1001)_{2C}$

---

4. Convert each of the following decimal representations into twos complement representations:
   - ☐ Example 4.6: $x = (-53)_{10}$
   - ☐ Example 4.7: $x = (-120)_{10}$
   - ☐ Create a table of all possible 8−bit twos complement signed integers

---

5. Negate each of the following signed integers in twos complement representations:
   - ☐ Example 4.8: $x = (-53)_{10}$
   - ☐ Example 4.9: $x = (-18)_{10}$
   - ☐ Example: $x = (-128)_{10}$
   - ☐ Example: $x = (0)_{10}$

---

6. Suppose B is a twos complement signed integer. Let $\overline{B}$ be the bitwise complement of B. Prove that

$$B + (\overline{B} + 1) = 0$$

   What does this proof say about how we negate twos complement numbers?

Suggested Problems

---

20. Consider each of the following signed decimal integers:

    A. $(-49)_{10}$ B. $(-75)_{10}$ C. $(1011)_{10}$ D. $(-31876)_{10}$

    i. Convert these numbers into twos complement representations using paper and pencils analysis
    ii. Check your work using MATLAB

---

21. Consider each of the following signed binary integers in twos complement:

    A. $(1111\,1111)_{2C}$ B. $(1000\,0001)_{2C}$ C. $(0101\,0101\,1110)_{2C}$ D. $(1001\,1110\,0110\,0101)_{2C}$

    i. Convert these numbers into signed decimal integers using paper and pencils analysis
    ii. Check your work using MATLAB