Name: Lesson 9 Solutions, Part I Class #:______

Math 48A, Lesson 9: Radical Functions

1. THE SQUARE ROOT OPERATION

1A. Suppose you're talking to your abuelita (grandma) and she is not familiar with the idea of an "square root". Explain to your abuelita what it means to find the "square root" of a number. Put your description into words.

☐ a square root of a number is any number times itself to get the desired number (the number we are searching for). - Katrina

☐ a root is a number we are trying to take a square root of...

The square root gives the number s.t. when we multiply we get back to thing inside

1B. Show how to calculate the square root of at least two numbers.

Example 1: $\sqrt[2]{4} = 2$ since $2 \cdot 2 = 4$

Example 2: $\sqrt[2]{25} = \underline{\underline{5}}$ since $25 = 5^2$

Anatomy of Roots/Radical

index

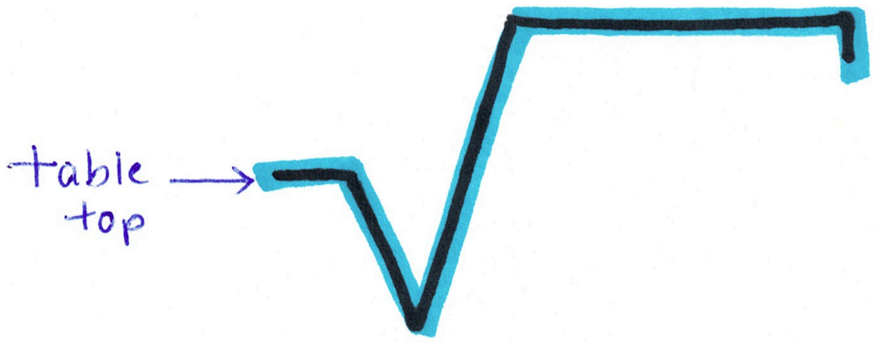
radical symbol

$$\sqrt[n]{a} = b$$

radicand ← must be nonnegative

(the Expression inside the root symbol : argument)

For the root symbol



$$f(\boxed{x}) = \sqrt[n]{\boxed{x}}$$

argument: input to our function
(expression inside our function)

$$g(\boxed{x}) = |\boxed{x}|$$

Example 2:

root symbol (radical symbol)

index

$$\boxed{\sqrt[2]{25}} = \boxed{5}$$

(output of the squareroot)

radicand

(input)

$$\sqrt[2]{25} \neq \frac{25}{2}$$

$$\shortparallel \qquad \qquad \shortparallel$$

$$5 \neq 12.5$$

Note: Square roots are not the same as division

$\Rightarrow \quad 25 = 5 \cdot 5 = \boxed{5}^{2}$

radicand

output raised to the index

- When we take a root, if we take the output raised to the power of the number in the index, we get back to the radicand

-

$$\sqrt[3]{8} = 2$$

radicand

$$8 = 2^3$$

When we see

index

$$\sqrt[3]{8} = \boxed{2}$$

output

radicand

is asking: find an output such that when we multiply that output by itself three times, we get back to 8

* verbal representation *

- When we take an nth root

$$\sqrt[n]{x} = b$$

we are trying to find output

so that when we take that

output to the power of the index

we get back to the radicand

- $\sqrt[n]{x} = \boxed{b} \Rightarrow x = b^{n}$

* symbolic representation *

index

$$\boxed{\sqrt[n]{x} = b \Rightarrow x = b^n}$$

$$\Rightarrow \quad x = \underbrace{b \cdot b \cdot b \cdots b}_{n\text{-times}}$$

- We have to find an output so that can be multiplied by itself the index amount of times so that the product will equal the radicand

Nerdy Definition

- We have to find an output that can be multiplied by itself the amount of times indicated by value of index so that this product is equal to the radicand index

$$\sqrt[5]{32} = \boxed{2}$$

radicand

output

the fifth root of 32

$\Rightarrow$ **what number** can I take to the **5th power** to get **32**?

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1C. What happens when we try to take the square root of a negative number? Explain your reasoning in words and in symbols.

Recall :

$$\sqrt[2]{-1} = b$$

radicand is negative

$$\Rightarrow \quad -1 = \boxed{b^2}$$

negative

nonegative

all real numbers b when squared their product will be nonnegative (either positive or zero)

this makes no sense : not possible in $\mathbb{R}$

(No square roots of negative numbers)

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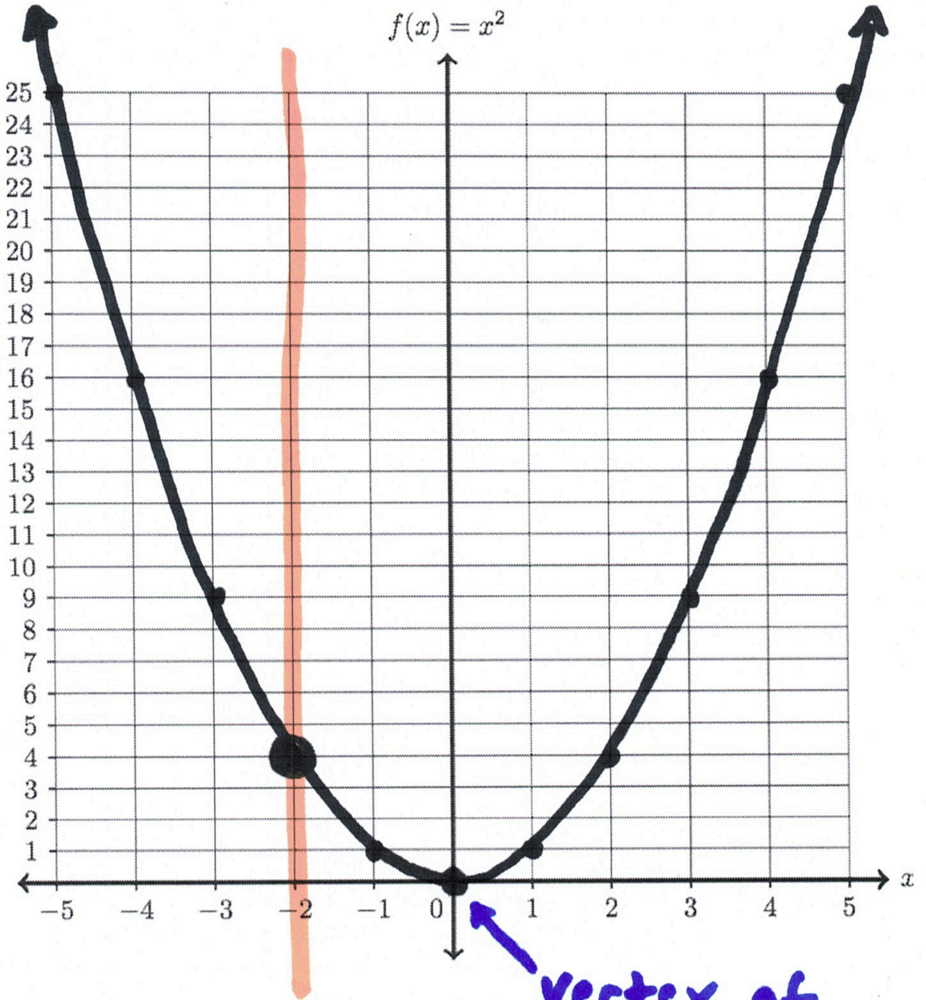
2. GRAPH THE QUADRATIC FUNCTION

Consider the following quadratic function

$$y = x^2 = f(x)$$

Create a table of values and graph the resulting lines on these axes below.

Input x	*Output* y
−5	25
−4	16
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9
4	16
5	25



vertex of parabola (minimum value)

this is a function because it passes the vertical line test

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3. GRAPH A QUADRATIC RELATION (Square root relation)

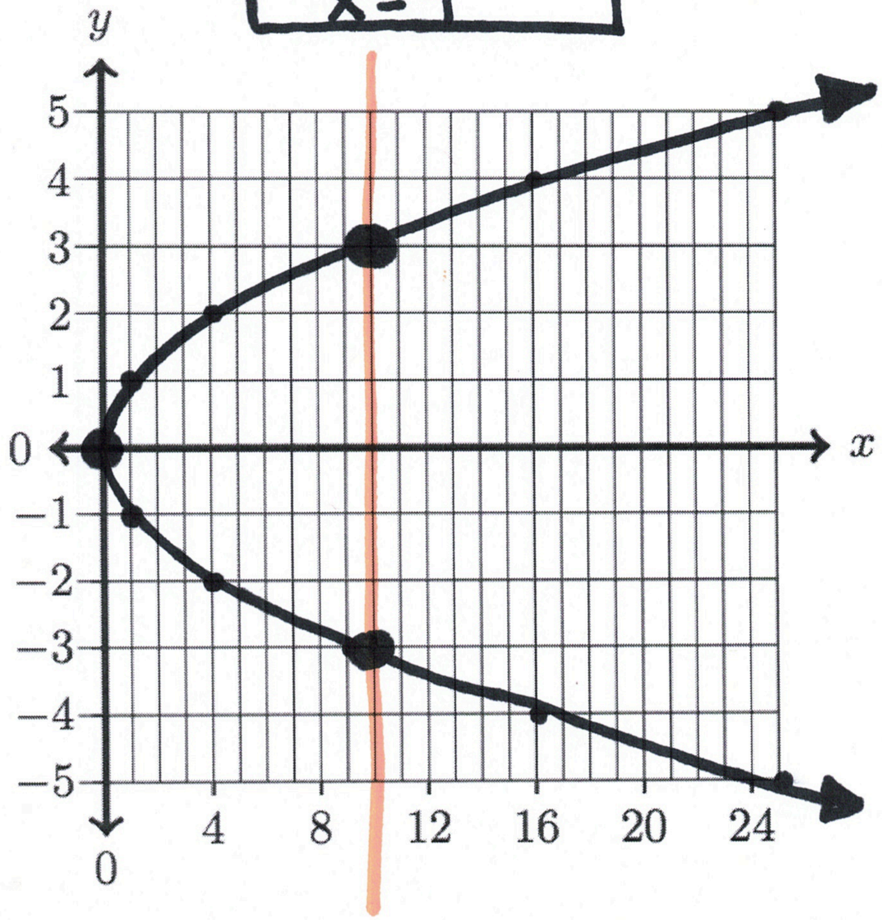
Consider the quadratic relationship given by

implicit relation $y^2 = x$ relation (NOT a function)

Fill out the table below. Then use that table to graph the quadratic relation on the axes given below.

vertical line $x=9$

Output x	*Input* y
25	−5
16	−4
9	−3
4	−2
1	−1
0	0
1	1
4	2
9	3
16	4
25	5



This graph fails the vertical line test:

there is at least one vertical line that intersects the graph in two spots

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4A. Compare and contrast problem 2 with problem 3. What do you notice about the role of the input and output variables in the two problems?

Note:
- the input/output relationships are switched
- the x-values in $y = x^2$ become the y-values in $x = y^2$
- the y-values in $y = x^2$ become x-values in $x = y^2$
- we are swapping inputs and outputs: replacing x with y and y with x

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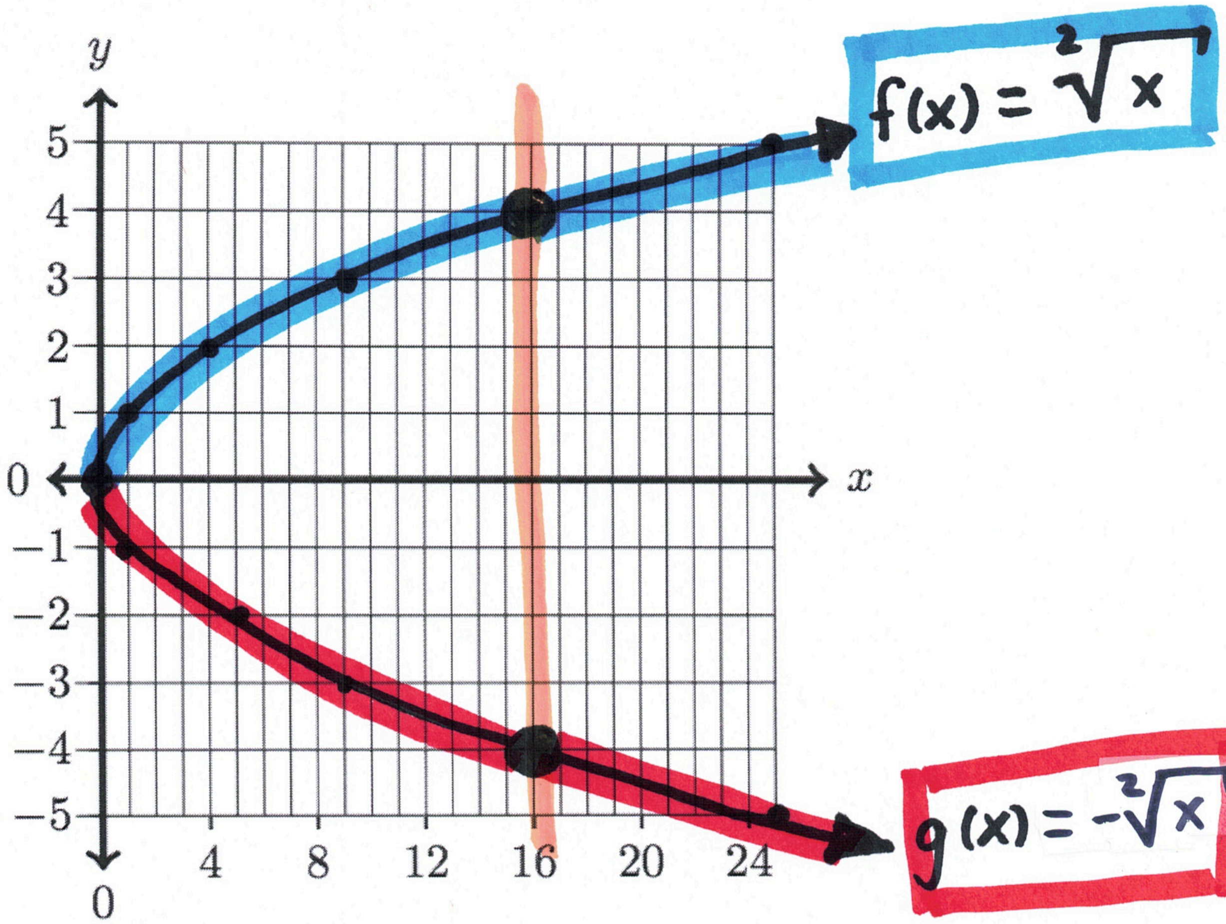
4B. Redraw your graph of the quadratic relationship $y^2 = x$ from problem 3 on the axes below. Does this graph represent a function? Why or why not? Explain your answer.



This is not a function since there is at least one input value with two different outputs.

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4C. Suppose you are looking at the quadratic relationship $y^2 = x$.
Also, suppose that I tell you that on this relationship, I want to find y-values such that $x = 16$. What y-value satisfies this relationship?
How easy is to give a unique "answer" to this question?

$$y^2 = x \Rightarrow y^2 = 16$$

$$\Rightarrow y = +4 \quad \text{or} \quad y = -4$$

$$y^2 = 16 \Rightarrow y = \sqrt[2]{16}$$

$$\Rightarrow 4 = \sqrt[2]{16} = -4$$

$$\overset{?}{\Rightarrow} 4 \neq -4$$

4D. Why do mathematicians (like you and me) prefer to work with functions?

In Math, we try to avoid ambiguity!

we want all mathematicians to be able to converge on same "answer" at the same time for functions.

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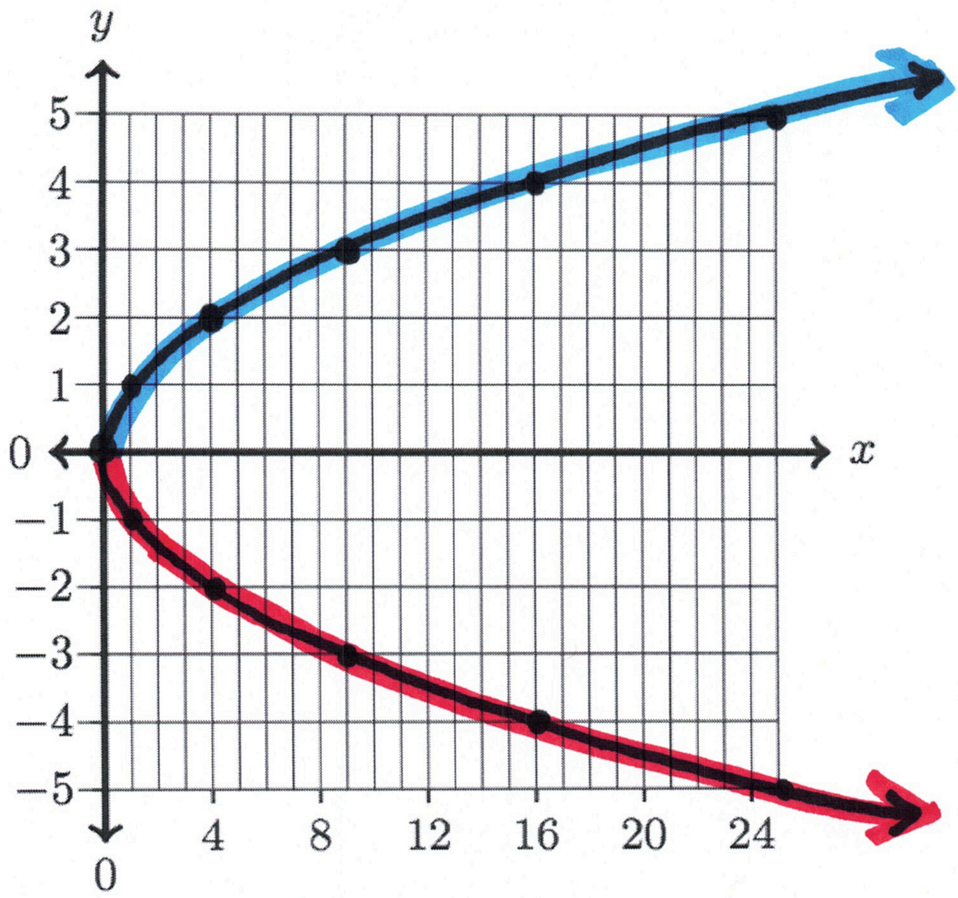
4E. Redraw your graph of the quadratic relationship $y^2 = x$ from problem 3 on the axes below. How could you turn this graph into a function?



We can break the graph of the relation $y^2 = x$ into two pieces

- the top piece where $y \geq 0$
- the bottom piece where $y \leq 0$

Problem 4E

To counter act this issue, we make a special agreement:

$$y^2 = 16 \Rightarrow \text{Either } y = \boxed{\sqrt[2]{16}}$$

OR

$$y = \boxed{-\sqrt[2]{16}}$$

The square root symbol takes only the positive y values that satisfy our relation:

$$y^2 = x \rightarrow y = \boxed{+\sqrt[2]{x}}$$

$$y^2 = x \rightarrow y = \boxed{-\sqrt[2]{x}}$$

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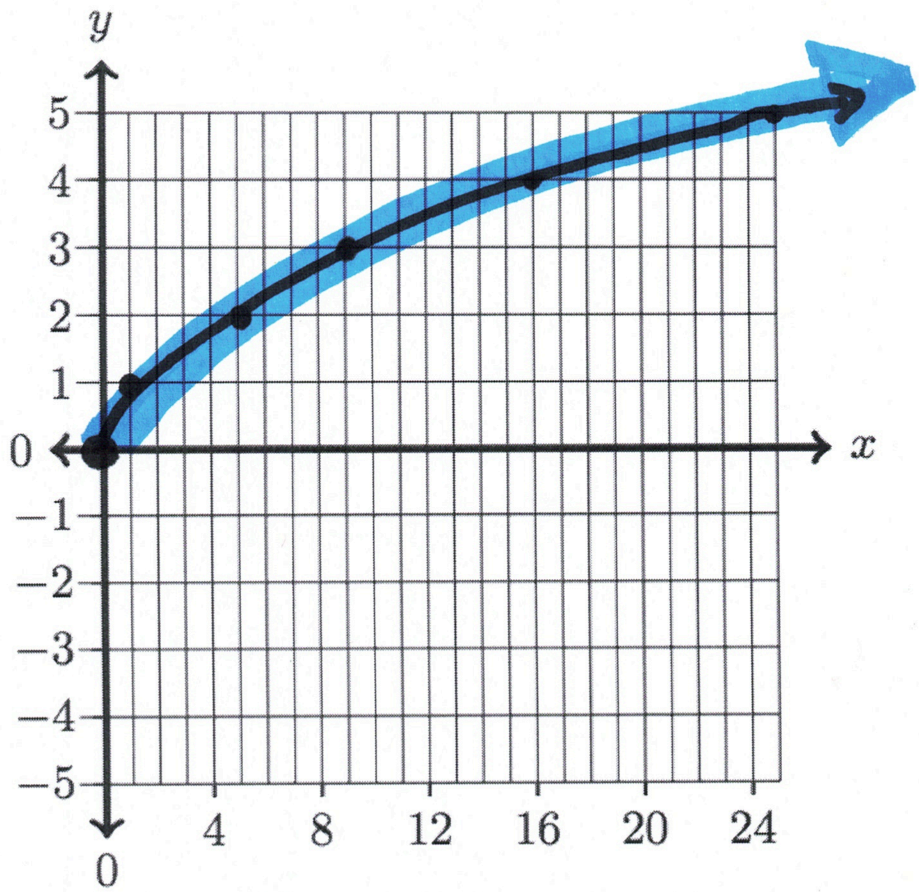
5. SQUARE ROOT FUNCTION	TYPE OF ROOT FUNCTION

5A. Consider the square root function

$$\boxed{f(x) = \sqrt[2]{x}}$$

Fill out the table below. Then use that table to graph the square root function.

Input	*Output*
x	$f(x) = \sqrt[2]{x}$
-1	Error
0	0
1	1
4	2
9	3
16	4
25	5



Notice that when we write

$$\sqrt[2]{x} = y \quad \Rightarrow \quad y^2 = x \quad \& \quad \boxed{y \geq 0}$$

we're looking for $\boxed{\text{non-negative values}}$ of y that, when squared, produce x.

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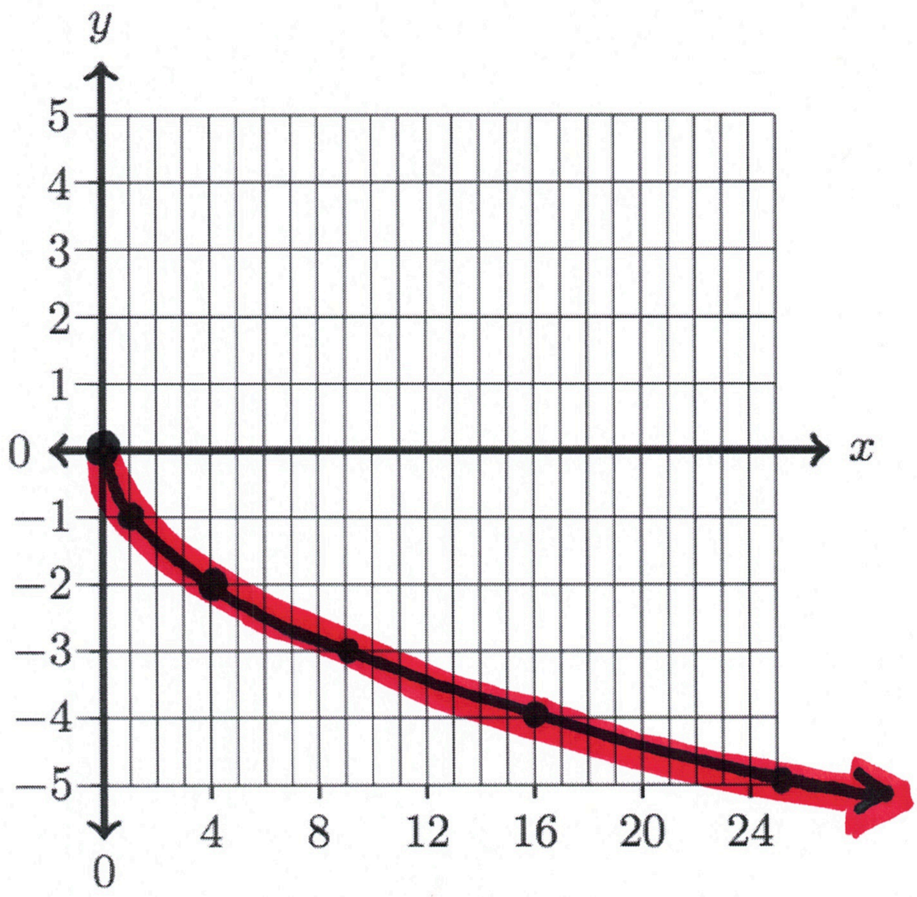
5B. Consider the square root function

$$g(x) = -\sqrt[2]{x}$$

Fill out the table below. Then use that table to graph the square root function.

Input	*Output*
x	$g(x) = -\sqrt[2]{x}$
-1	Error
0	0
1	-1
4	-2
9	-3
16	-4
25	-5



When we write the statement

$$-\sqrt[2]{x} = y \Rightarrow y^2 = x \text{ AND } \boxed{y \leq 0}$$

We're looking for non-positive values of y that, when squared, produce x.

Thus, in both cases, we have

$$\pm\sqrt[2]{x} = b \Rightarrow b^2 = x$$

But the reverse is not true:

if $b^2 = x$, then $\begin{cases} x = \boxed{\sqrt[2]{b}} \\ x = \boxed{-\sqrt[2]{b}} \end{cases}$

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5C. How are the graphs from problems 5A and 5B related to the work you did in problem 4A – 4E?

The graph of the quadratic relation

$$y^2 = x$$

does not represent a function.

We break this graph into two parts

Top : $y^2 = x$ for $y \geq 0$

Bottom : $y^2 = x$ for $y \leq 0$

Each part now represents a function with a unique answer to the question "what number, when squared, produces x as an output?"

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5D. What are the similarities and differences between the quadratic relation $y^2 = x$ and the square root function $y = \sqrt[2]{x}$?

The funcion $y = \sqrt[2]{x}$ is asking

what nonnegative number $y \geq 0$ exists

s.t. when we square y we get back

to the value of x.

In symbols, we write

$$\boxed{y = \sqrt[2]{x}} \Rightarrow y^2 = x \text{ with } \boxed{y \geq 0}$$

The function $y = -\sqrt[2]{x}$ is asking what $\boxed{\text{nonpositive number } y \leq 0}$ exists such that when we take y to the second power, we get back to the value of x.

In symbols, we write :

$$\boxed{y = -\sqrt[2]{x}} \Rightarrow y^2 = 0 \text{ with } \boxed{y \leq 0}$$

The quadratic relation $y^2 = x$ is asking to find all possible real numbers y (either positive or negative) such that when we multiply y by itself, we get back to the value of x:

In symbols, we write

$$y^2 = x \Rightarrow \begin{cases} \boxed{y = \sqrt[2]{x}} \text{ with } \boxed{y \geq 0} \\ \text{OR} \\ \boxed{y = -\sqrt[2]{x}} \text{ with } \boxed{y \leq 0} \end{cases}$$