Name: Lesson 8, Solutions Part 2 ! Class #:______

4. FIND EQUIVALENT REPRESENTATION FOR ABSOLUTE VALUES

4A. Consider the following absolute value function:

$$g(x) = |\,2x + 2\,|$$

Make a conjecture (a mathematical guess) for an equivalent piecewise representation of this function. Call your piecewise guess function $h(x)$.

Let's make a guess:

argument (expression inside) : the stuff inside abs value bars

$$g(x) = \left|\,\overbrace{2x+2}^{\text{argument (expression inside)}}\,\right|$$

left abs value bar

right abs value bar

$$\Rightarrow |2x+2| \stackrel{?}{=} \begin{cases} 2x+2 & \text{if } \boxed{2x+2>0} \\ 0 & \text{if } \boxed{2x+2=0} \\ -(2x+2) & \text{if } 2x+2<0 \end{cases}$$

17

$\boxed{2x + 2 = 0}$ output

slope: $m = \frac{2}{1}$ rise / run

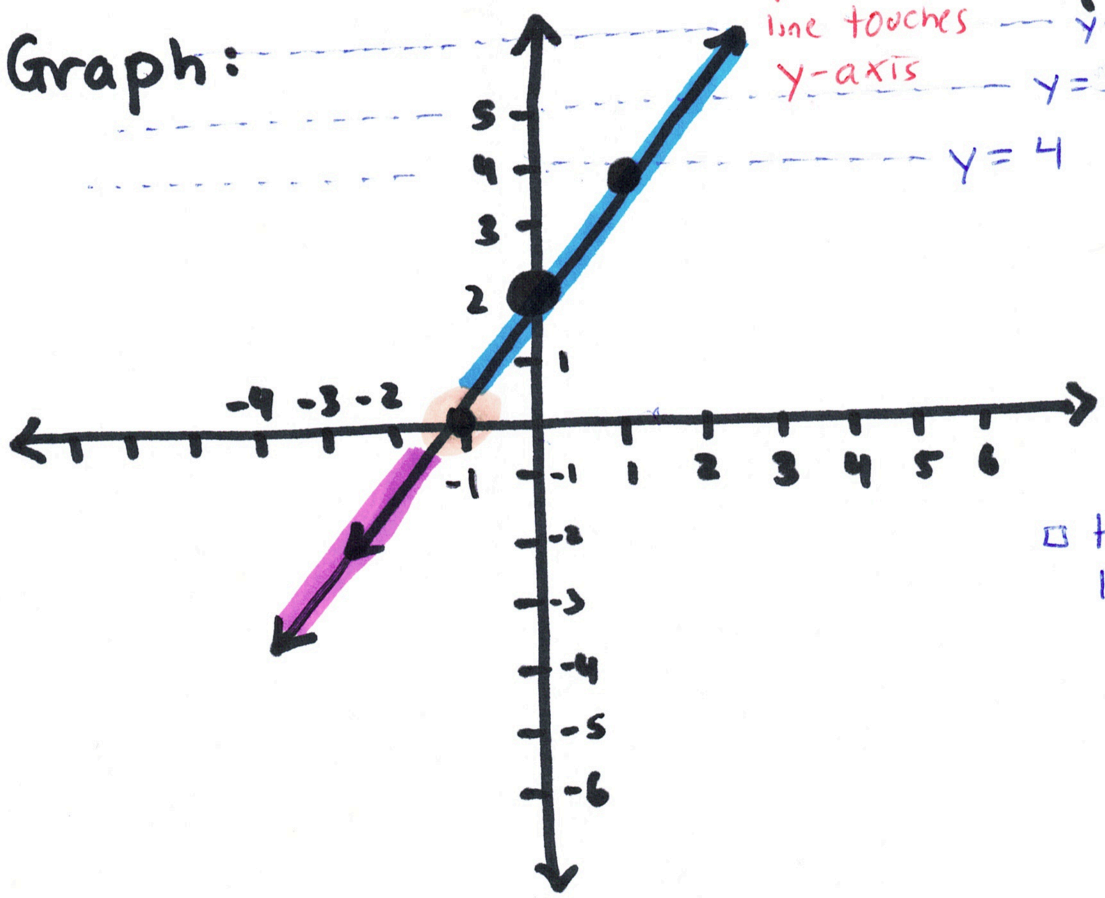
y-intercept: $b = 2$

$(0, 2)$

Output
y-axis

place where line touches y-axis

Graph:



$(x=0)$

the vertical line called y-axis

Graphically we see $\boxed{x = -1}$

$\Rightarrow 2x + 2 = 0$

line touches $y=0$ as an output

Algebraic argument

$$2x+\cancel{2} = 0$$

$$\underbrace{-2}_{+0} \qquad -2$$

cancel out

$$\Rightarrow \quad \frac{\cancel{2}\cdot x}{\cancel{2}_{1}} = \frac{-2}{2}$$

$$\Rightarrow \quad \boxed{x = -1}$$

$$h(x) = \begin{cases} 2x + 2 & \text{if } \boxed{x > -1} \\ 0 & \text{if } \boxed{x = -1} \\ -(2x+2) & \text{if } \boxed{x < -1} \end{cases}$$

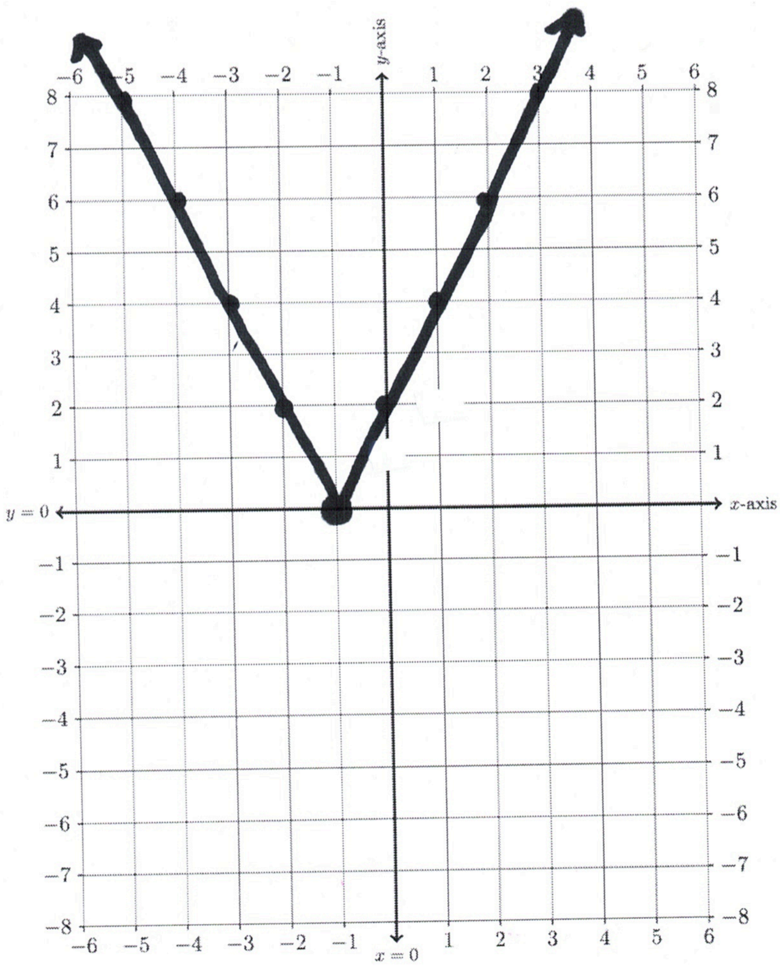
Name:______________________ Class #:______

4B. Test the conjecture you made in problem 4A by creating a graph of your test piecewise function $h(x)$ and also the absolute value function

$$g(x) = |2x + 2| = |2(x+1)|$$

Try to adapt your conjecture until you get your guess for $h(x)$ to match identically the graph of $h(x)$. Capture this equivalent representation.

x	$g(x)$.	$h(x)$.
−6	10	10
−5	8	8
−4	6	6
−3	4	4
−2	2	2
−1	0	0
0	2	2
1	4	4
2	6	6
3	8	8
4	10	10
5	12	12
6	14	14



Name:______________________ Class #:______

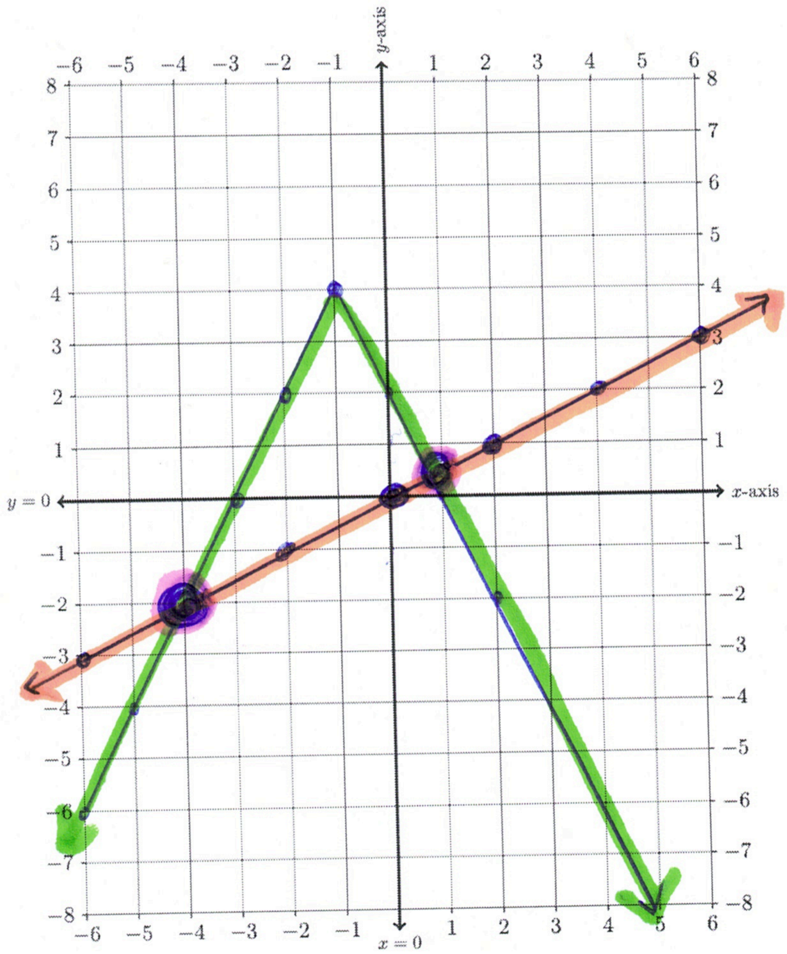
5. SOLVE ABSOLUTE VALUE EQUATIONS

5A. Consider the following absolute value equation:

$$\boxed{4 - |2x + 2|} = \boxed{\frac{1}{2}x}$$

Use the left-hand side (LHS) and right-hand side of this equation to a table of values and draw the resulting graph on the axes below.

x	LHS $4-\|2x+2\|$	RHS $\frac{1}{2}x$
-6	-6	-3
-5	-4	-2.5
-4	-2	-2
-3	0	-1.5
-2	2	-1
-1	4	-0.5
0	2	0
$\frac{4}{5}$	0.4	0.4
1	0	0.5
2	-2	1
3	-4	1.5
4	-6	2
5	-8	2.5
6	-10	3



Note: $y = mx + b$ (slope-intercept)

slope, y-intercept

$$\frac{1}{2}x = \frac{1}{2}x + 0$$

$$\Rightarrow m = \frac{1}{2}, \quad b = 0$$

y-intercept $(0, 0)$

22

5B The two point of intersectors are

Point 1: $(-4, -2)$

Point 2: $(\frac{4}{5}, 0.4)$

$\Rightarrow$ We have two solutons

$\boxed{x = -4}$ and $\boxed{x = \frac{4}{5}}$

Name:____________________ Class #:______

5C. Now let's solve the absolute value equation

$$4 - |2x + 2| = \frac{1}{2}x$$

using an algebraic technique. Hint: Use your work in problems 4A – 4B.

$$+\cancel{4} - |2x+2| = \frac{1}{2}x$$

$$-4 \qquad\qquad\qquad\qquad -4$$

$$0$$

$$\Rightarrow -1\cdot(-|2x+2|) = -1\left(\frac{1}{2}x - 4\right)$$

$$\Rightarrow |2x+2| = -\frac{1}{2}x + 4$$

Option 1: $2x+2 = -\frac{1}{2}x + 4$

Option 2: $-(2x+2) = -\frac{1}{2}x + 4$

Option 1:

$$2x + \cancel{2} = -\frac{1}{2}x + \cancel{4}$$
$$\quad -2 \qquad\qquad\qquad -2$$
$$0$$

$$\Rightarrow \quad 2x = -\frac{1}{2}x + 2$$

$$\Rightarrow \quad 2 \cdot 2x = 2 \cdot \left(-\frac{1}{2}x + 2\right)$$

$$\Rightarrow \quad 4x = -\cancel{x} + 4$$
$$+x \qquad\qquad +x$$
$$0$$

$$\Rightarrow \quad 5x = 4$$

$$\Rightarrow \quad \boxed{x = \frac{4}{5}}$$

Option 2:

$$-(2x+2) = -\frac{1}{2}x + 4$$

$$\Rightarrow -2x - 2 = -\frac{1}{2}x + 4$$

$$\quad +2 \qquad\qquad +2$$

$$\Rightarrow -2x = -\frac{1}{2}x + 6$$

$$+\frac{1}{2}x \qquad +\frac{1}{2}x$$

<u>Note:</u>

$$\frac{-2}{1}x + \frac{1}{2}x = \frac{-4}{2}x + \frac{1}{2}x$$

$$= \frac{-4x + 1x}{2}$$

$$= \frac{-3}{2}x$$

25

$$\Rightarrow \quad -\frac{3}{2}x = 6$$

$$\Rightarrow \quad -\frac{2}{3} \cdot \frac{3}{2} \cdot x = -\frac{2}{3} \cdot \frac{6}{1}$$

1

$$\Rightarrow \quad \boxed{x = -\frac{12}{3} = -4}$$